Name: _____ **Student Number:** _____

Math 324B **Advanced Multivariable Calculus I** **Spring 1997**

FINAL EXAM: 6/9/97

- There are 8 problems at 20 points each, for a total of 160 points.
- You may use two $8\frac{1}{2}'' \times 11''$ one-sided pages (or one sheet written on both sides) of your own handwritten notes (no photocopied or printed material).
- *Show your work:* If your work demonstrates that you had the right idea, you might get partial credit even if your answer is wrong. On the other hand, if your answer is not justified by your work, you might lose points even if your answer is correct. If you need more room, continue your work on the back of the page.
- Use the boxes for answers when possible. Measurements must be made with a ruler, and lines and vectors must be drawn with a ruler.
- Simplify algebraic and numerical expressions in answers to the point where they can be evaluated by a student in a precalculus course with a calculator, but you do not need to take them any further. Leave answers such as $\sqrt{7 + (6.8)^2}$ as they are, but work out vector expressions such as dot products and cross products. Calculators are allowed, but you won't need one; it is preferable to leave numerical answers in unsimplified form. Laptop computers are not allowed.
- If you have a question, raise your hand and I will come to your seat.

Graded exams and course grades will be available at 6:00 PM Friday, June 13. If you don't choose any of the options below for getting your test back, you may pick it up personally from me (in Padelford C-546) after July 7.

☐ I want my exam and course grade left outside Professor Lee's office (Padelford C-546).

☐ I want my exam and course grade mailed to me. I am providing a large, self-addressed envelope with 55 cents postage.

☐ I want my exam score and course grade posted on the Web (by student number only).

Signed:_____

1:		5:	
2:		6:	
3:		7:	
4:		8:	
Exam score:			
Course grade:			

Good luck, and have a happy summer!

1. g is the function of two variables defined by $g(x,y) = x^2 - 2y^2$, and $\overrightarrow{F} = \operatorname{grad} g$.

(a) If $\vec{r}(t) = (x(t), y(t))$ is a flow line of $\overrightarrow{F}$, write down the system of ordinary differential equations that $x(t)$ and $y(t)$ must satisfy.

ODE:

(b) Find a parameter representation of the flow line that passes through the point $(2,1)$ at time $t = 0$.

$x(t) =$

$y(t) =$

(c) Is there a time t when this flow line is tangent to a level curve of g? If there is such a time, write it in the box below. If not, write NONE and **explain why in the space below**.

$t =$

2. S is the surface in space described by $z = xy - x + y$.

(a) Write a parameter representation for S using u and v as parameters.

$\vec{r}(u, v) =$ [ ]

(b) The point $P(1, 2, 3)$ lies on S. Write the components of a normal vector $\vec{n}$ to S at P.

$\vec{n} =$ [ ]

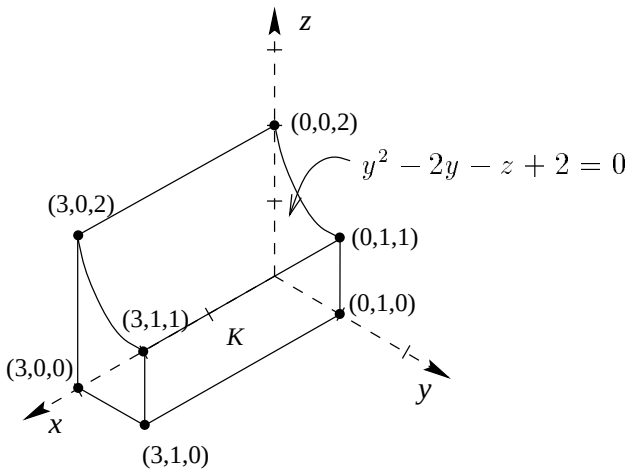
(c) Through $P(1, 2, 3)$ passes a u-coordinate curve (that is, a coordinate curve defined by setting $v =$ constant). Write a parameter representation for this curve.

$x =$ [ ]

$y =$ [ ]

$z =$ [ ]

3. K is the solid shown below. Its front, back, and side walls are vertical, and its top surface is described by the equation $y^2 - 2y - z + 2 = 0$.

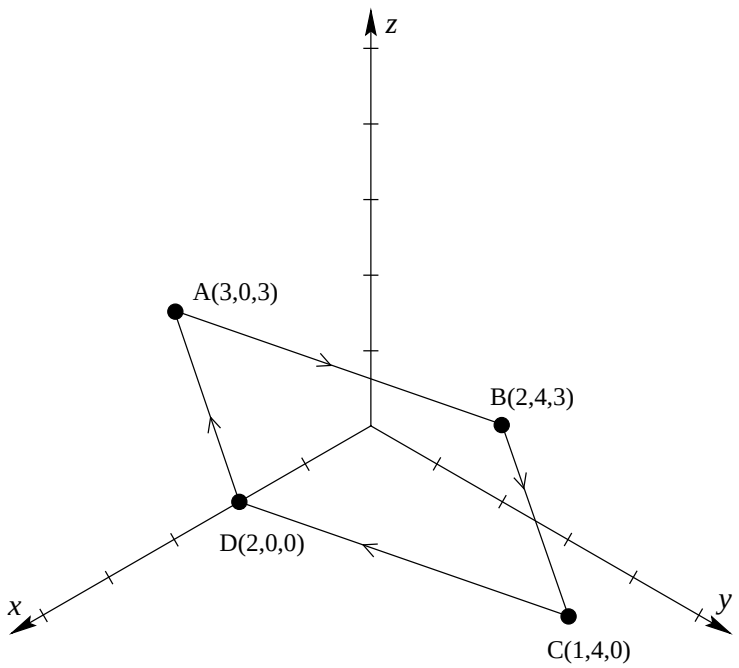


Set up (but do not evaluate) the triple integral $\iiint_K g(x,y,z)\,dV$, where $g(x,y,z)$ is the function $g(x,y,z) = y^2 - 2y$. You may use any order of integration. In the limits, you must name the variables.

$\int$ $\int$ $\int$ $\square$ $d\square$ $d\square$ $d\square$

$\square$ $\square$ $\square$

4. We are given a parallelogram $ABCDA$ in space, shown below. $\overrightarrow{F}$ is the vector field $\overrightarrow{F} = (z\cos x, z+y, \sin x)$.



(a) Compute $\int_M \overrightarrow{F} \cdot d\vec{r}$, where M is the line segment CD oriented as shown.

$$\int_M \overrightarrow{F} \cdot d\vec{r} = \boxed{}$$

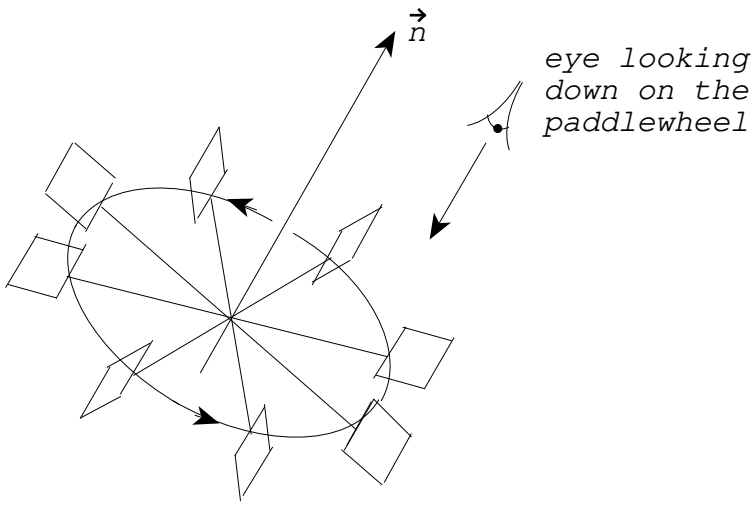
(b) Compute $\int_E \overrightarrow{F} \cdot d\vec{r}$, where E is the closed parallelogram $ABCDA$, oriented as shown. [Hint: curl $\overrightarrow{F} = (-1,0,0)$.]

$$\int_E \overrightarrow{F} \cdot d\vec{r} = \boxed{}$$

5. The vector field $\overrightarrow{F}$ is defined by $\overrightarrow{F} = (8y + 12z, 2x^2 - y^2z, -yz^2 - x^2z)$. You may or may not need part or all of the following information:

$$\text{div } \overrightarrow{F} = -4yz - x^2, \qquad \text{curl } \overrightarrow{F} = (y^2 - z^2, 2xz + 12, 4x - 8).$$

We put a small paddlewheel into the flow. The axis of the paddlewheel points in the direction of $\vec{n}$, as shown below. We look at the paddlewheel from the direction of the vector $\vec{n}$.



(a) We put the center of the paddlewheel at $P(1, 0, 1)$. We let its axis $\vec{n}$ point in the direction of $(1, 1, 3)$. Do you expect the paddlewheel to spin? If yes, clockwise or counterclockwise? Circle one:

no spin *clockwise* *counterclockwise*

(b) Is there a point Q at which the paddlewheel will hardly spin at all, no matter how we position $\vec{n}$? If yes, find such a point; if no, write NONE and explain in one or more complete sentences.

$Q =$ [ ]

6. For parts (a) and (b), determine whether each of the following vector fields is conservative or not. In each case, circle YES if it is definitely conservative, NO if it is definitely not conservative, or ??? if it is impossible to decide from the information given. For part (c), find the requested integral, or write IMPOSSIBLE if it is impossible to determine from the information given.

(a) The vector field in the plane is given by $\overrightarrow{F} = (6xy - 2y, 3x^2 + 6y)$ and curl $\overrightarrow{F} = 2$. Is $\overrightarrow{F}$ conservative?

Conservative? YES - NO - ???

(b) The vector field in space is given by

$$\overrightarrow{G} = \frac{(x-2, y, z+2)}{((x-2)^2 + y^2 + (z+2)^2)^{5/2}} + \frac{(x, y-3, z)}{(x^2 + (y-3)^2 + z^2)^{7/2}},$$

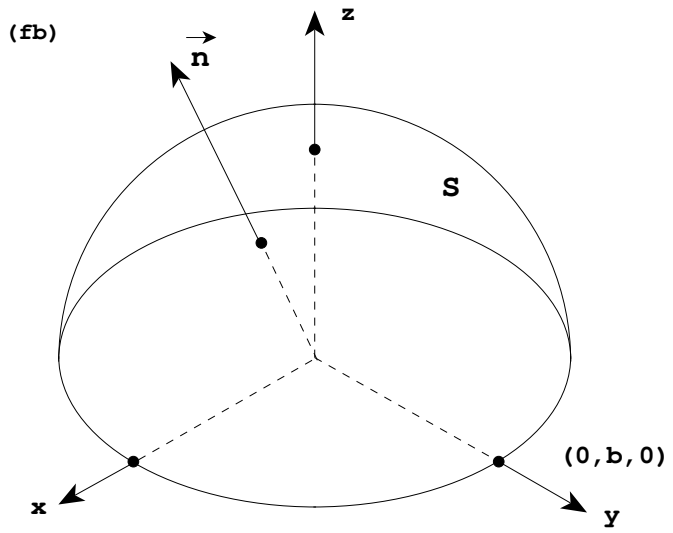
and we are told that curl $\overrightarrow{G} = (0, 0, 0)$. Is $\overrightarrow{G}$ conservative?

Conservative? YES - NO - ???

(c) The vector field in space is given by $\overrightarrow{K} = (e^x - 2xy, 3z - x^2, 3y)$. Suppose C is *any* path from $(0, 0, 0)$ to $(1, 2, 3)$. Find $\int_C \overrightarrow{K} \cdot d\vec{r}$.

$\int_C \overrightarrow{K} \cdot d\vec{r} =$

7. Suppose S is the upper half $z \geq 0$ of the spherical surface of radius b centered at the origin, as shown below. We orient S by the normal pointing away from the origin.



The vector field $\overrightarrow{F}$ is given by $\overrightarrow{F} = (z^2+1)(x,y,z)$. Give an estimate of the flux of $\overrightarrow{F}$ across S, that is, find constants c and C so that

$$c \;\leq\; \int\int_S \overrightarrow{F} \cdot d\overrightarrow{A} \;\leq\; C.$$

Hints: The volume of a sphere of radius r is $\frac{4}{3}\pi\, r^3$, and the surface area is $4\pi\, r^2$.

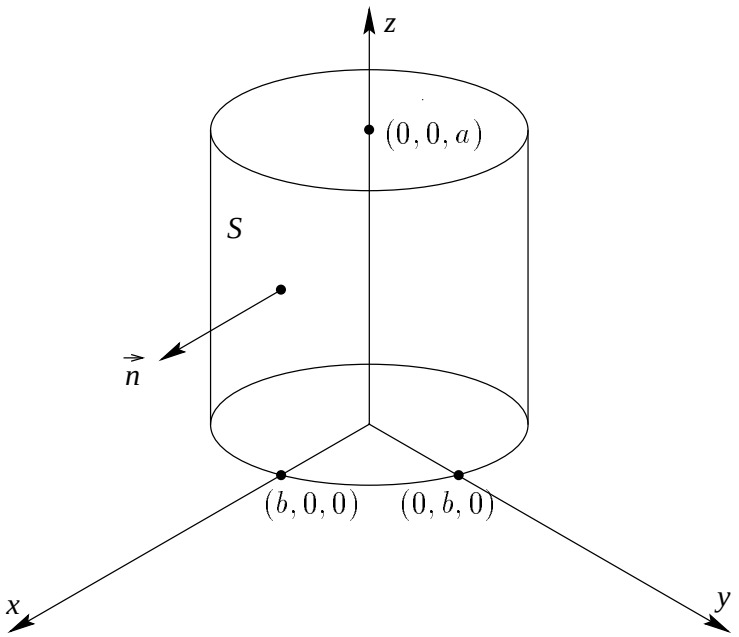
$c =$ ____

$C =$ ____

8. We are given the following surfaces:

- S is the cylindrical surface of radius b and height a, centered around the z axis (just the vertical wall of the cylinder, not its top and bottom caps), oriented by the outward-pointing normal.
- B is the disk on the bottom of the cylinder (in the plane $z = 0$), oriented by the downward-pointing normal.
- T is the disk on the top of the cylinder (in the plane $z = a$), oriented by the upward-pointing normal.

The volume of a cylinder of radius r and height h is $\pi r^2 h$.



(a) The vector field $\vec{F}$ is given by $\vec{F} = (zx, zy, e^{x^2+y^2})$. Find the flux of $\vec{F}$ across S. That is, find $\iint_S \vec{F} \cdot d\vec{A}$.

$$\iint_S \vec{F} \cdot d\vec{A} = \boxed{}$$

(b) The vector field $\vec{G}$ is given by $\vec{G} = (3x, 2x^2, x^3)$. Let E be S together with B and T; that is, E is a tin can with top and bottom caps. Find the flux of $\vec{G}$ across E. That is, find $\iint_E \vec{G} \cdot d\vec{A}$.

$$\iint_E \vec{G} \cdot d\vec{A} = \boxed{}$$