Name: ________________________________ **Student Number:** ______________

Math 324A **Advanced Multivariable Calculus I** **Spring 1997**

FINAL EXAM: 6/11/97

- There are 8 problems at 20 points each, for a total of 160 points.
- You may use two $8\frac{1}{2}'' \times 11''$ one-sided pages (or one sheet written on both sides) of your own handwritten notes (no photocopied or printed material).
- *Show your work:* If your work demonstrates that you had the right idea, you might get partial credit even if your answer is wrong. On the other hand, if your answer is not justified by your work, you might lose points even if your answer is correct. If you need more room, continue your work on the back of the page.
- Use the boxes for answers when possible. Measurements must be made with a ruler, and lines and vectors must be drawn with a ruler.
- Simplify algebraic and numerical expressions in answers to the point where they can be evaluated by a student in a precalculus course with a calculator, but you do not need to take them any further. Leave answers such as $\sqrt{7+(6.8)^2}$ as they are, but work out vector expressions such as dot products and cross products. Calculators are allowed, but you won't need one; it is preferable to leave numerical answers in unsimplified form. Laptop computers are not allowed.
- If you have a question, raise your hand and I will come to your seat.

Graded exams and course grades will be available at 6:00 PM Friday, June 13. If you don't choose any of the options below for getting your test back, you may pick it up personally from me (in Padelford C-546) after July 7.

☐ I want my exam and course grade left outside Professor Lee's office (Padelford C-546).

☐ I want my exam and course grade mailed to me. I am providing a large, self-addressed envelope with 55 cents postage.

☐ I want my exam score and course grade posted on the Web (by student number only).

Signed:________________________________

Good luck, and have a happy summer!

1:		5:	
2:		6:	
3:		7:	
4:		8:	

Exam score:	
Course grade:	

1. For parts (a) and (b), determine whether each of the following vector fields is conservative or not. In each case, circle YES if it is definitely conservative, NO if it is definitely not conservative, or ??? if it is impossible to decide from the information given. For part (c), find the requested integral, or write IMPOSSIBLE if it is impossible to determine from the information given.

(a) The vector field in the plane is given by

$$\overrightarrow{G} = \frac{(x-2, y)}{((x-2)^2 + y^2)^{5/2}} + \frac{(x, y-3)}{(x^2 + (y-3)^2)^{7/2}},$$

and we are told that curl $\overrightarrow{G} = 0$. Is $\overrightarrow{G}$ conservative?

Conservative? $\boxed{\text{YES - NO - ???}}$

(b) The vector field in space is given by $\overrightarrow{F} = (6xy - 2z, 3x^2 + 6y, z^2)$ and curl $\overrightarrow{F} = (0, -2, 0)$. Is $\overrightarrow{F}$ conservative?

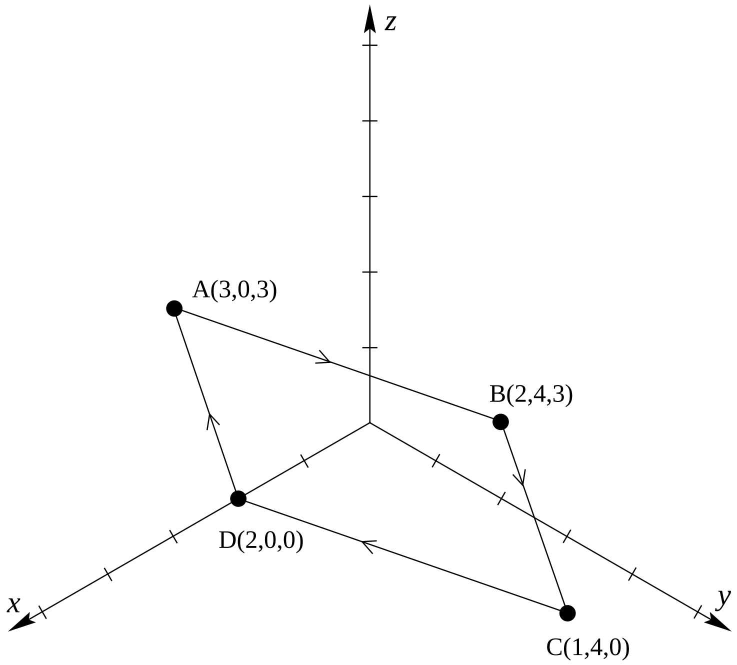
Conservative? $\boxed{\text{YES - NO - ???}}$

(c) The vector field in space is given by $\overrightarrow{K} = (\cos x, 2y - z^2, -2yz)$. Suppose C is *any* path from $(0,0,0)$ to $(\pi, 2\pi^2, 3\pi)$. Find $\int_C \overrightarrow{K} \cdot d\vec{r}$.

$$\int_C \overrightarrow{K} \cdot d\vec{r} = \boxed{}$$

2. We are given a parallelogram $ABCDA$ in space, shown below. $\overrightarrow{F}$ is the vector field $\overrightarrow{F} = (-z, (z-1)e^z, yze^z)$. You may or may not need part or all of the following information:

$$\text{div}\,\overrightarrow{F} = (z+1)ye^z; \qquad \text{curl}\,\overrightarrow{F} = (0, -1, 0).$$



(a) Compute $\displaystyle\int_M \overrightarrow{F} \cdot d\vec{r}$, where M is the line segment DA oriented as shown.

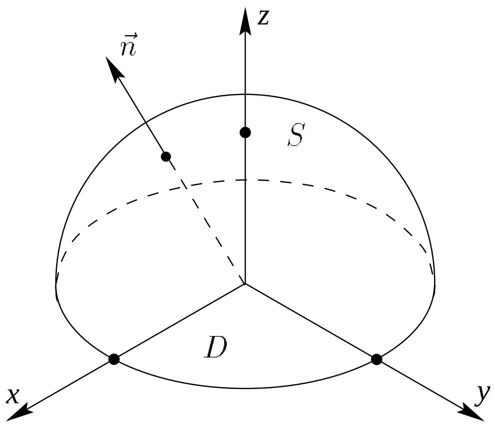
$$\int_M \overrightarrow{F} \cdot d\vec{r} = \boxed{}$$

(b) Compute $\displaystyle\int_E \overrightarrow{F} \cdot d\vec{r}$, where E is the closed parallelogram $ABCDA$, oriented as shown.

$$\int_E \overrightarrow{F} \cdot d\vec{r} = \boxed{}$$

3. We are given the following surfaces:

- S is the upper half $z \geq 0$ of the spherical surface of radius 2 centered at the origin, as shown below. (S includes only the upper hemisphere; it does *not* include the horizontal disk.) We orient S by the normal pointing away from the origin.
- D is the disk of radius 2 in the xy-plane, centered at the origin, oriented by the downward-pointing normal.
- T is S together with D; that is, T is an upside-down bowl with a lid.



The vector field $\overrightarrow{F}$ is given by $\overrightarrow{F} = (0, 0, 1 + 2z)$. The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$, and its surface area is $4\pi r^2$.

(a) Using the normal component of $\overrightarrow{F}$, give an estimate of the flux of $\overrightarrow{F}$ across S, that is, find constants c and C so that

$$c \leq \iint_S \overrightarrow{F} \cdot d\vec{A} \leq C.$$

$c =$ ______

$C =$ ______

(b) Find the flux of $\overrightarrow{F}$ across T, that is, compute the surface integral $\iint_T \overrightarrow{F} \cdot d\vec{A}$.

$\iint_T \overrightarrow{F} \cdot d\vec{A} =$ ______

4. g is the function of two variables defined by $g(x, y) = x^2 - 2y^2$, and $\overrightarrow{F} = \operatorname{grad} g$.

(a) If $\vec{r}(t) = (x(t), y(t))$ is a flow line of $\overrightarrow{F}$, write down the system of ordinary differential equations that $x(t)$ and $y(t)$ must satisfy.

ODE:

(b) Find a parameter representation of the flow line that passes through the point $(2, 1)$ at time $t = 0$.

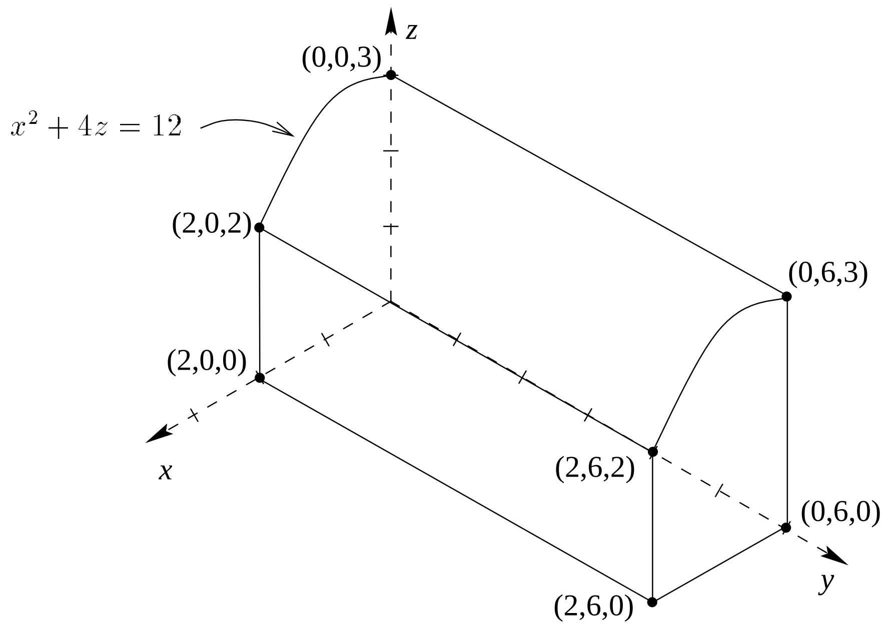
$x(t) =$

$y(t) =$

(c) Is there a time t when this flow line is tangent to a level curve of g? If there is such a time, write it in the box below. If not, write NONE and **explain why in the space below**.

$t =$

5. K is the solid shown below. Its front, back, and side walls are vertical, and its top surface is described by the equation $x^2 + 4z = 12$.



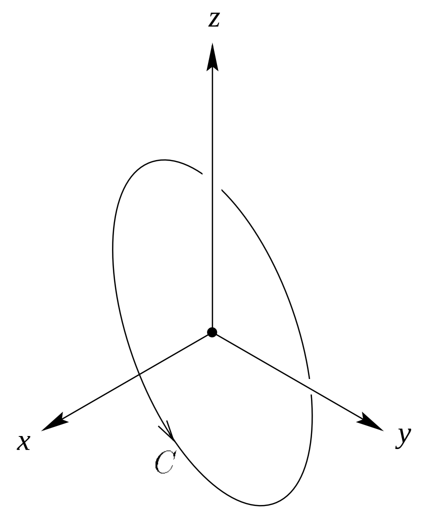
Set up (but do not evaluate) the triple integral $\iiint_K g(x,y,z)\,dV$, where $g(x,y,z)$ is the function $g(x,y,z) = x^2 + 4z$. You may use any order of integration. In the limits, you must name the variables.

$$\int_{\square}^{\square}\int_{\square}^{\square}\int_{\square}^{\square} \square \; d\square \; d\square \; d\square$$

6. The vector field $\overrightarrow{F}$ is defined by $\overrightarrow{F} = (8y + 12z, 2x^2 - y^2z, -yz^2 - x^2z)$. You may or may not need part or all of the following information:

$$\text{div } \overrightarrow{F} = -4yz - x^2, \qquad \text{curl } \overrightarrow{F} = (y^2 - z^2, 2xz + 12, 4x - 8).$$

We are interested in the circulation of $\overrightarrow{F}$ around circles C of radius $b = 0.001$ centered at the origin, as shown below. Each of these circles lies in a plane α that passes through the origin.



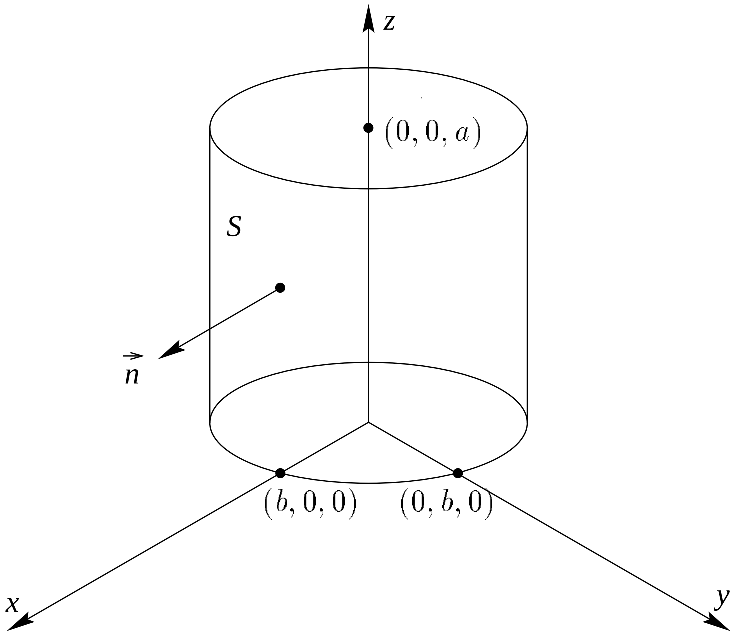
(a) We position the circle in the xy-plane, and orient it in the counterclockwise direction as viewed from above. Do we expect the circulation $\int_C \overrightarrow{F} \cdot d\vec{r}$ to be positive, negative, or zero? Circle one:

positive *negative* *zero*

(b) Give the best estimate you can for the plane α (i.e., for its equation $Ax+By+Cz = D$) so that the absolute value of the circulation is the maximum possible.

α : $\boxed{}$

7. S is the cylindrical surface of radius b and height a, centered around the z axis (just the vertical wall of the cylinder, not including its top or bottom caps), oriented by the outward-pointing normal.



(a) Write a parameter representation for S, using u and v as parameters.

$\vec{r}(u,v) = \boxed{}$

(b) The vector field $\overrightarrow{F}$ is given by $\overrightarrow{F} = (xz^2, yz^2, e^{x^2})$. Find the flux of $\overrightarrow{F}$ across S. That is, compute $\iint_S \overrightarrow{F} \cdot d\vec{A}$.

$\iint_S \overrightarrow{F} \cdot d\vec{A} = \boxed{}$

8. S is the surface in space described by $z = e^x + y - xy^2$.

(a) Write a parameter representation for S using u and v as parameters.

$\vec{r}(u, v) =$ $\boxed{}$

(b) The point $P(0, 1, 2)$ lies on S. Write the components of a normal vector $\vec{n}$ to S at P.

$\vec{n} =$ $\boxed{}$

(c) Through $P(0, 1, 2)$ passes a u-coordinate curve (that is, a coordinate curve defined by setting $v =$ constant). Write a parameter representation for this curve.

$x =$ $\boxed{}$

$y =$ $\boxed{}$

$z =$ $\boxed{}$