Math 310 **Introduction to Mathematical Reasoning** **Spring 2006**

Assignment #5
Due 5/3/06

Midterm: *Friday, May 5, in class. It will cover everything we've done in Chapters 1–8.*

Reading

- Handout 4.
- Chapter 7 again (but skip Examples 7.7.3–7.6.4); Chapter 8 (but skip sections 8.3 and 8.4).

Part I

A. For each of the following statements, do the following things:

- Translate it into symbols.
- Negate the symbolic statement and simplify.
- Translate the negated statement back into a clear and precise English sentence, without using the word "not."

(a) The square of every real number is positive.
(b) The square of some real number is positive.
(c) There is an integer that is smaller than its square.
(d) Every integer is smaller than its square.
(e) For every real number x, there is an integer that is greater than x.
(f) There is an integer that is greater than every real number.
(g) There exist integers p and q such that $q > 0$ and $p^2/q^2 = 2$.
(h) For every real number x, if x^2 is positive then x is positive.
(i) For every $\varepsilon \in \mathbb{R}^+$, there is a positive real number δ such that $|x - 2| < \delta$ implies $|x^2 - 4| < \varepsilon$ whenever $x \in \mathbb{R}$.

B. Page 117, #12.

C. Pages 99–100, Exercises 8.1, 8.2 (see note), 8.5. (*Note: in Exercise 8.2, the notations $fg(x)$ and $gf(x)$ should be replaced by $f \circ g(x)$ and $g \circ f(x)$, respectively.*)

D. Decide whether each of the following formulas describes a well-defined function $f\colon \mathbb{R} \to \mathbb{R}$. For those that do not, give a brief reason why not.

(a) $f(x) = \dfrac{x^2+3}{x+5}$.

(b) $f(x) = \begin{cases} x^2 & \text{if } x \geq 1, \\ x^3 & \text{if } x \leq 0. \end{cases}$

(c) $f(x) = \begin{cases} x^3 + 2 & \text{if } x \geq 1, \\ x & \text{if } x < 1. \end{cases}$

(d) $f(x) = \begin{cases} \dfrac{1}{x+1} & \text{if } x \geq 0, \\ x - 1 & \text{if } x < 1. \end{cases}$

Part II

E. Write a complete proof or disproof of each of the following statements. (To "disprove" a statement, you prove its negation.)

(a) For each positive integer m, there is an integer n such that $n > m$ and $m|n$.

(b) There is a positive integer m such that for each integer $n > m$, it is the case that $m|n$.

F. Let $h\colon \mathbb{R} \to \mathbb{R}$ be the function defined by $h(x) = x+2$. Suppose $f : \mathbb{R} \to \mathbb{R}$ and $g\colon \mathbb{R} \to \mathbb{R}$ are functions such that $g \circ f = h$. Show that there is at most one solution x to the equation $f(x) = 0$.