Math 310 **Introduction to Mathematical Reasoning** **Spring 2006**

Assignment #1 *(CORRECTED)*
Due 4/5/06

All problems and reading assignments are from the textbook unless otherwise noted.

Reading

- Handouts 1 and 2.
- Chapters 1–3.

Part I

A. Pages 8–9, Exercises 1.3, 1.4.

B. Pages 19–20, Exercises 2.1, 2.4, 2.5, 2.6.

Part II

A. Pages 53–57, Problems 2 and 5. *(For Problem 2, you don't have to write out a detailed proof; just construct the relevant truth tables and explain briefly why that suffices.)*

B. Consider the following sentences.

1. The number n is either prime or odd.
2. The number n is prime only if it is odd.
3. If n is prime, then it is odd.
4. The number n is prime, but it is not odd.
5. For n to be composite, a necessary condition is that it not be even.
6. If n is even, then either it is composite or it is equal to 2.
7. If n is equal to 4, then it is neither prime nor odd.
8. The number n is odd if it is prime and not equal to 2.

If we assume that n is a definite (but unknown) positive integer, then each of the above sentences is a mathematical statement. Note that "the number n" means the same thing as "n"; the circumlocution is used to avoid having to start a sentence with a mathematical symbol, which is considered bad form. For each sentence, do the following:

(a) Translate it into a symbolic statement.

(b) Write its negation in symbolic form, and simplify far enough that you don't have to use the word "not."

(c) Translate the negation back into an English statement.

For your symbolic statements, use only the following words and symbols:

and, or, not
parentheses
$\Rightarrow$
n
$=, \neq$
$P(n)$ ("n is prime")
$C(n)$ ("n is composite")
$E(n)$ ("n is even")
$O(n)$ ("n is odd").

(Note that "$O(n)$" means the same as "not $E(n)$.")

Example: The number n is both odd and prime.

Answer:

(a) Symbolic: $O(n)$ and $P(n)$.
(b) Symbolic negation: $E(n)$ or $C(n)$.
(c) English negation: Either n is even or it is composite.

C. A ***statement form*** is a symbolic expression containing variables, which becomes a statement whenever the variables are replaced by specific statements. A statement form is called a ***tautology*** if it is true regardless of the truth values of its individual statement variables, and a ***contradiction*** if it is false regardless of their truth values.

Write out the truth table for each of the following statement forms, and determine if it is a tautology, a contradiction, or neither.

(a) $P \Rightarrow \text{not}(Q \,\text{and}(\text{not}\, P))$.
(b) $P \Rightarrow ((\text{not}\, R) \,\text{or}\, Q) \,\text{and}\, R$.
(c) $(\text{not}\, P \Rightarrow (Q \,\text{and}(\text{not}\, Q))) \Rightarrow P$.