Math 310 B & C **Introduction to Mathematical Reasoning** **Autumn 2005**

Handout #2 (10/7/05)

Notes on Writing Mathematics

Here are some observations about the writing of mathematics that I hope will be useful as you work on the writing assignments for this course.

- **Goals and audience:** As with any written piece, mathematical exposition must be written with a particular audience and specific goals in mind. Be sure you have a clear sense of what these are before you start writing.

- **The process:** Always bear in mind that writing is a *process*, just like proving a theorem. No one pours forth a well-organized, clear, and error-free exposition the first time they sit down to write, just as no one produces a complete, well-structured proof the first time they think about a problem. Most good expository prose has been thoroughly rewritten at least once before it reaches the reader, with key sections undergoing perhaps three to five major revisions.

- **Conventions:** Mathematical writing should follow the same conventions of grammar, usage, punctuation, and spelling as any other writing. This means, in particular, that you must write complete sentences organized into paragraphs.

  Of course, many ordinary English words have special technical meanings in mathematics that may be different from their usage in ordinary English—such words should only be used with their precise mathematical meaning, never in their informal English-language sense. Even when using such technical terms, you should be careful to observe the usual rules regarding parts of speech and subject-verb agreement. For example, although you will find surprisingly many mathematicians who write ugly sentences such as "f is an onto map," it is far kinder to your readers and no less clear to follow the usual rules of grammar and write something like "f maps A onto B."

  If you are not a native English speaker, it would be a good idea to cultivate the habit of asking a native speaker to look over your writing before you submit it.

- **Precision:** In mathematical writing more than any other kind, precision is of paramount importance. Every mathematical statement you make must have a precise mathematical meaning. Specifically, every term you use must be well defined, and used properly according to its definition; every mathematical conclusion you reach must be justified; and every symbol you mention must be either previously defined or quantified in some appropriate way. If you write $f(a) > 0$, do you mean that this is true for every $a \in X$, or that there exists some $a \in X$ for which it's true, or that it's true for a particular a that you introduced earlier in the proof?

  Ask yourself these two key questions about each sentence you write:

  - *What exactly does this mean?*
  - *Why exactly is this true?*

- **Clarity:** Just as important as mathematical precision is making sure your writing is easily comprehensible to your intended audience. Don't be stingy with intuitive explanations of what's going on and why. For any argument that's longer than a sentence or two, it's good to begin by describing informally what you're going to do, then do it, then say what you've done.

  It's all too easy to write a sequence of mathematical statements that are entirely precise and mathematically correct, and yet that are incomprehensible to a human being. If you have to write a long series of equations, intersperse them at carefully chosen places with some words about of what you're doing and why, or reasons why one step follows from another.

- **The first person:** Most authors avoid using the word "I" in mathematical writing. It is standard practice to use "we" whenever it can reasonably be interpreted as referring to "the writer and the reader." Thus: "We will prove the theorem by induction on n," and "Because f is injective, we see that $x_1 = x_2$." But if you're really referring only to yourself, it's better to go ahead and use "I": "I learned this technique from Richard Melrose."

- **Abbreviations:** There are a host of abbreviations that we use frequently in informal mathematical communication: "iff" (if and only if), "s.t." (such that), "w.r.t." (with respect to), and "WLOG" (without loss of generality) are some of the most common. These are indispensable for writing on the blackboard and taking notes, but should never be used in written mathematical exposition. The only exceptions are abbreviations that would be acceptable in any formal writing, such as "i.e." (that is) or "e.g." (for example); but if you use these, be sure you know the difference between them!

- **Mathematical symbols:** The feature that most clearly distinguishes mathematical writing from other kinds is the extensive use of symbols and formulas. If used appropriately, these can make mathematical writing dramatically easier to read. The sentence "Let f be the function whose value at a particular number is equal to the square of the number added to the number itself" is far less clear than "Let f be the function defined by $f(x) = x^2 + x$." On the other hand, symbols must be used judiciously, because their excessive use can lead to writing that is just as obscure as writing with no symbols.

  Here are some guidelines for using mathematical symbols in your writing:

  - Single symbols and short, simple formulas should usually be included right in your paragraphs; but a formula that is large or especially important should be centered on a line by itself (this is called a "displayed formula").
  - Every mathematical symbol or formula, whether included in the text or displayed, must have a definite grammatical function in a sentence, almost invariably as a noun or a clause. Consider the sentence "If $x > 2$, we see that $x^2 + x$ must be greater than 6." Here the formula "$x > 2$" is a clause whose verb is "$>$," while "$x^2 + x$" and "6" function as nouns.
  - If a displayed formula ends a sentence, it must be followed by a period.
  - The best way to ensure that your formulas function grammatically correctly is to read each sentence aloud. When you do so, bear in mind that many symbols can be read in several different ways—for example, the symbol "=" can be read as "equals," "equal to," "be equal to," "is equal to," or "which is equal to," depending on context.
  - Symbols representing mathematical relations (like $=$, $>$, $\in$, or $\subseteq$) or operators (like $+$, $-$, or $\cap$) should be used only to connect other mathematical symbols, not to connect words with symbols or with each other. For example, do *not* write "If x is a real number that is > 2, then $x^2 + x$ must be > 6."
  - Fractions and fractional expressions included in the text should be written with a slash, as in "$x/(y+2)$". If a fraction is so large or complicated that it needs to be written using a horizontal bar, it should be displayed. The only exception is small numerical fractions such as $\frac{1}{2}$, which can be included in text as long as they are written small enough to fit naturally on a line.
  - It's almost always bad form to begin a sentence with a mathematical symbol or variable name. For example, if you find yourself wanting to write a sentence that begins "f is a continuous function," you could write instead "The function f is continuous."
  - Avoid writing two formulas separated only by a comma or other punctuation mark, because they will look like one long formula. For example, the sentence "If $x \neq 0$, $x^2 > 0$" can be confusing; it would be easier to read if a word were interposed between the two formulas, as in "If $x \neq 0$, then $x^2 > 0$."
  - The use of symbols for logical terms, such as the ones below, is considered bad style in formal mathematical writing, unless you are writing about symbolic logic and the symbols occur as part of logical formulas:

$\exists$	(there exists)	$\Rightarrow$ or $\rightarrow$	(implies)
$\forall$	(for all)	$\Leftarrow$ or $\leftarrow$	(is implied by)
$\wedge$	(and)	$\Leftrightarrow$ or $\leftrightarrow$	(if and only if)
$\vee$	(or)	$\neg$ or $\sim$	(not)
$\ni$	(such that)	$\therefore$	(therefore).

    Write out the words instead. The only common exception is the use of the symbol $\Rightarrow$ in contexts like this: "We will show that (a) and (b) are equivalent by first proving (a) $\Rightarrow$ (b) and then proving (b) $\Rightarrow$ (a)."