

First, explain as clearly as possible the definitions of injective, surjective, and bijective functions; and for each property, give an example of a function that satisfies the property and an example of a function that does not (no proofs necessary for this part). Write the definition in your own words rather than copying from the textbook; and don't use any examples that appear explicitly in the textbook or in the homework or in the list below.

Then for each of the following functions, do the following, and prove all of your answers correct:

- Decide whether it is injective, surjective, neither, or both.
- For those that are not surjective, determine the range.
- For those that are bijective, find a formula for the inverse function.

(a)  $f_1: \mathbb{R} - \{1\} \rightarrow \mathbb{R} - \{1\}; \quad f_1(x) = \frac{x+1}{x-1}.$

(b)  $f_2: \mathbb{R}^2 \rightarrow \mathbb{R}^2; \quad f_2(x, y) = (2y - 3x, 3x - 2).$

(c)  $f_3: \mathbb{R}^2 \rightarrow \mathbb{R}; \quad f_3(x, y) = xy.$

(d)  $f_4: \mathbb{R} \rightarrow \mathbb{R}^2; \quad f_4(x) = (x, x^2).$