Math 300C Introduction to Mathematical Reasoning

1. Each of the following formulas purports to define a function $f: \mathbb{R} \rightarrow \mathbb{R}$, but some of them do not. In each case, explain whether the function is
(a) everywhere defined;
(b) uniquely defined;
(c) well defined.

If the answer to any of the above questions is "no," give a brief reason why not. For those formulas that do define a function, determine the range of that function.
(a) $f(x)=\frac{x^{3}+3}{x+5}$.
(b) $f(x)= \begin{cases}x^{2} & \text { if } x \geq 1, \\ x^{3} & \text { if } x \leq 0 .\end{cases}$
(c) $f(x)= \begin{cases}x^{2}+1 & \text { if } x \geq 1, \\ x & \text { if } x<1 .\end{cases}$
(d) $f(x)= \begin{cases}\frac{x^{2}-2 x+1}{x-1} & \text { if } x<1, \\ x-1 & \text { if } x>0 .\end{cases}$
(e) $f(x)= \begin{cases}\frac{1}{x+1} & \text { if } x \geq 0, \\ x-1 & \text { if } x<1 .\end{cases}$

