Math 300A&B Introduction to Mathematical Reasoning Fall 2009

Assignment #9: Due Friday, 12/11/09

Part II:

For the proofs below, if you're being asked to prove one of the theorems from the text, you may use any of the theorems stated in class, any theorem stated in the text ***before*** *the one you're proving, and any theorems proved in previous homework problems. If the theorem you're being asked to prove is not one of the theorems from the text, then in addition you may use any theorems from Chapters 1–11 and 14.*

Exception: *The following theorem from class should* ***not*** *be used in the proof of Exercise 11.1, because that would create a circular argument:*

Thoerem: *If X is a finite set and there exists a surjection $X \to S$, then S is also finite and $|S| \leq |X|$.*

4. Eccles, page 143, Exercise 11.2.

5. Eccles, page 184, Problem 12.

6. If X is a set that contains an uncountable subset, prove that X is uncountable.

7. If X is an uncountable set and A is a countable subset of X, prove that $X - A$ is uncountable.

8. Determine whether each of the following sets is empty, finite but nonempty, denumerable, or uncountable. No proofs necessary.

   (a) $\{1/n : n \in \mathbb{Z}^+\}$.

   (b) $\mathbb{R} - \mathbb{Q}$.

   (c) $\mathbb{Z} \times \mathbb{R}$.

   (d) $[0, \infty)$.

   (e) $\{x \in \mathbb{R} : x^2 \in \mathbb{Z}\}$.

   (f) $\{x \in \mathbb{Z} : x^2 \in \mathbb{R} - \mathbb{Z}\}$.