Math 300A&B **Introduction to Mathematical Reasoning** **Fall 2009**

Assignment #4: Due Wednesday, 10/28/09

Part III:

3. In addition to the two-column proofs you were asked to do in Part I of this assignment, write paragraph-style proofs of the following two theorems, making sure to follow the guidelines in the handout *Conventions for Writing Mathematical Proofs*.

   - Theorem 7(c).
   - Theorem 9(b).

As a sample, here's my two-column proof of one of the theorems I proved in class today, together with a paragraph-style version of the same proof. Of course, this is just one way to write it up; there are many, many others.

Theorem. *If $a < b$ and $ac \leq bc$, then $c \leq 0$.*

Statement	Reason
1. Let $a, b, c \in \mathbb{R}$	hypothesis
2. Assume $a < b$ and $ac \leq bc$	hypothesis
3. Assume $c > 0$	for contradiction
4. $a < b$	Step 2, logic
5. $ac > bc$	multiplying step 4 by c, Axiom 13
6. $ac \not> bc$	trichotomy, Step 2
7. Contradiction	Steps 5 and 6

Theorem. *If $a < b$ and $ac \leq bc$, then $c \leq 0$.*

Proof. We will prove this theorem by contradiction. Thus suppose a, b, and c are real numbers such that $a < b$ and $ac \leq bc$, and assume for the sake of contradiction that $c > 0$. When we multiply both sides of the inequality $a < b$ by the negative number c, the multiplicative law for inequalities yields

$$ac > bc.$$

But this contradicts our assumption that $ac \leq bc$. Thus the assumption must have been false, and we must have $c \geq 0$ as claimed. □