Math 300A&B **Introduction to Mathematical Reasoning** **Fall 2009**

Assignment #3: Due Wednesday, 10/21/09

Part I:

1. For each of the following statements, do the following things:

   - Translate it into symbols. (Be sure that your symbolic statement explicitly includes implied universals and domains of quantifiers.)
   - Negate the symbolic statement and simplify. (In particular, this means to remove parentheses in expressions of the form $\sim(\ldots)$.)
   - Translate the negated statement back into a clear and precise English sentence, without using the word "no" or "not."

   (a) For every real number x, there is an integer that is greater than x.

   (b) There is an integer that is greater than every real number.

   (c) There is a largest integer.

   (d) There exist integers p and q such that $q > 0$ and $p^2/q^2 = 2$.

   (e) Between any integer and any larger integer, there is a real number.

   (f) There is an integer that is not the square of any integer.

   (g) Every real number has a unique cube root.

   (h) For every $\varepsilon \in \mathbb{R}^+$, there is a positive real number δ such that $|x - 2| < \delta$ implies $|x^2 - 4| < \varepsilon$ whenever $x \in \mathbb{R}$.

2. Find predicates $P(x)$ and $Q(x)$ such that one of the following statements is true and the other is false:

$$\exists x \in \mathbb{R}, P(x) \wedge Q(x) \qquad \text{and} \qquad (\exists x \in \mathbb{R}, P(x)) \wedge (\exists x \in \mathbb{R}, Q(x)).$$