Math 300A&B **Introduction to Mathematical Reasoning** **Fall 2009**

Assignment #1: Due Wednesday, 10/7/09

1. Eccles, p. 9, problems 1.2, 1.3.

2. A *statement form* is a sentence in which variables are used to represent propositions, such that it becomes a proposition when actual propositions are substituted for the variables. For example, if P and Q represent propositions, then $P \wedge Q$ is a statement form. The truth value of a statement form depends on the truth values of the individual statement variables, and can be determined by a truth table. For example, the usual truth table for "and" determines the truth value of $P \wedge Q$.

   Two statement forms are said to be ***equivalent*** if they give the same truth value for all possible truth values of the statement variables that appear in them. For example, the following truth tables show that the statement forms $P \wedge Q$ and $Q \wedge P$ are equivalent:

P	Q	$P \wedge Q$	$Q \wedge P$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

   Because the entries in the last two columns are identical, the two statement forms $P \wedge Q$ and $Q \wedge P$ are equivalent.

   For each of the following pairs of statement forms, write out the truth tables and determine whether they are equivalent or not.

   (a) $P \wedge (Q \wedge R)$ and $(P \wedge Q) \wedge R$.
   (b) $P \wedge (Q \vee R)$ and $(P \wedge Q) \vee R$.
   (c) $P \vee (Q \wedge R)$ and $(P \vee Q) \wedge R$.
   (d) $P \wedge (Q \vee R)$ and $(P \wedge Q) \vee (P \wedge R)$.
   (e) $P \vee (Q \wedge R)$ and $(P \vee Q) \wedge (P \vee R)$.
   (f) $\sim(P \wedge Q)$ and $(\sim P) \vee (\sim Q)$.
   (g) $\sim(P \vee Q)$ and $(\sim P) \wedge (\sim Q)$.

3. If P represents "George washed his hands" and Q represents "George got swine flu," write natural-sounding English sentences for each of the following statement forms:

   (a) $P \wedge Q$.
   (b) $P \wedge (\sim Q)$.
   (c) $(\sim P) \wedge Q$.

(d) $(\sim P) \wedge (\sim Q)$.

(e) $P \vee Q$.

(f) $P \vee (\sim Q)$.

(g) $(\sim P) \vee Q$.

(h) $(\sim P) \vee (\sim Q)$.

4. Consider the following predicates, in which n represents an unknown positive integer:

   (a) n is either prime or odd.

   (b) n is an odd prime.

   (c) n is prime, but it is not odd.

   (d) n is either prime and odd, or composite and even.

   For each sentence, do the following:

   (i) Translate it into a symbolic statement.

   (ii) Write its negation in symbolic form, and simplify far enough that you don't have to use the symbol $\sim$ for "not."

   (iii) Translate the negation back into an English statement.

   For your symbolic statements, use only the following symbols:

   $\wedge, \vee, \sim, =$
   parentheses
   $P(n)$ ("n is prime")
   $C(n)$ ("n is composite")
   $E(n)$ ("n is even")
   $O(n)$ ("n is odd").

   (Hint: look at parts (f) and (g) of Problem 2 above. Note that $O(n)$ means the same as $\sim E(n)$, and $C(n)$ means the same as $\sim P(n)$.)

   Example: n is both even and prime.

   Answer:

   (a) Symbolic: $E(n) \wedge P(n)$.

   (b) Symbolic negation: $O(n) \vee C(n)$.

   (c) English negation: Either n is odd or it is composite.