Section ____________ **Name** ____

TA __________________ **Student Number** ________________________________

Math 126 **Calculus With Analytic Geometry III** **Fall 1995**

FINAL EXAM

12/9/95

- There are 8 problems at 25 points each, for a total of 200 points.
- You may use two $8\frac{1}{2}''$ by $11''$ one-sided pages of your own handwritten notes and a small model of the xyz-axes.
- Show your work—you must justify every answer.
- Use the boxes for your answers whenever possible. Measurements must be made with a ruler, and lines and vectors must be drawn with a ruler: lines or vectors drawn without a ruler lose 4 points.
- Calculators are allowed, but not laptop computers.
- Tests with wrong or incorrectly spelled TA names or section letters will lose 4 points.
- Simplify algebraic and numerical answers to the point where they can be evaluated by a precalculus student with a calculator, but leave exact answers in exact form.
- If you have questions, raise your hand and someone will come to your seat.

1:	
2:	
3:	
4:	
5:	
6:	
7:	
8:	

Course Grade: ☐ Total: ☐

Sign at most **one** of the following statements:

I want this test with the course grade on it left outside of my TA's office in Padelford Hall.

Date:__________Signed:______________________________Student Number:__________

I want this test with the course grade on it mailed to me in an envelope which I am providing. I have affixed \$.55 postage to the envelope.

Date:__________Signed:______________________________Student Number:__________

Good luck, and have a happy holiday!

1. We have the following information on the function $f(x, y)$:

- Among all directional derivatives of $f(x, y)$ at the point $P(2, -1)$, the one in the direction of $\vec{v} = (4, 3)$ is largest, and it is equal to 10.
- $f(2, -1) = 3$.

a) (10 points) Find the components of the gradient of $f(x, y)$ at the point $P(2, -1)$.

grad $f =$ $\boxed{}$

b) (5 points) Through the point $P(2, -1)$ passes a level curve of $f(x, y)$. The normal object to this level curve is (circle one):

a line in the plane *a line in space*

a plane in space *none of the above*

c) (10 points) Write an equation for the normal object mentioned in (b).

Normal line: $\boxed{}$

2. $f(x, y)$ is a function of two variables. We define a new function $g(u, v)$ by

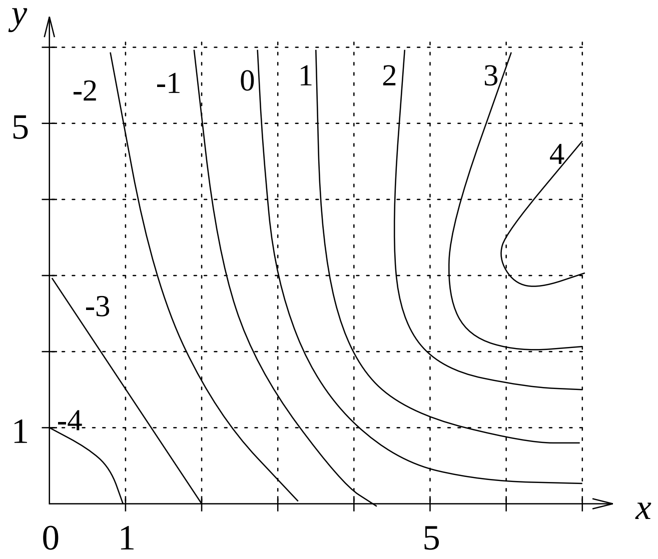
$$g(u, v) = f(u^2 - v^2, u + v).$$

a) (16 points) Express the partial derivatives g_u and g_v at an arbitrary point (u, v) in terms of u, v, and the partial derivatives f_x and f_y. Indicate where f_x and f_y are to be evaluated.

$g_u =$

$g_v =$

b) (9 points) Some level curves of $f(x, y)$ are pictured below. Give the best approximation you can for the components of $\operatorname{grad} g$ at the point $(u, v) = (2, 1)$.



$\operatorname{grad} g =$

3. $g(x, y, z)$ is a function of three variables. We do not know the formula for g, but we know the formula for the linear approximation of g at the point $B(2, 2, 2)$:

$$L(x, y, z) = 2x - 5y + z - 2.$$

If possible, find each of the following from the information given. If there is insufficient information, write IMPOSSIBLE.

a) (5 points) The gradient of g at the point $B(2, 2, 2)$.

grad g =

b) (10 points) The directional derivative $D_{\vec{u}}g(2, 2, 2)$, where $\vec{u} = (3, 4, 0)$.

$D_{\vec{u}}g(2, 2, 2) =$

c) (10 points) The normal line to the surface $g(x, y, z) = -6$ at the point $B(2, 2, 2)$.

Normal line:

4. We are given the function $f(x,y) = \cos x + 2\sin y$, and M is the collection of all points (x,y,z) in space that satisfy the equation $z = f(x,y)$. The point $Q(0, \pi/2, 3)$ is on M.

a) (10 points) Through Q there passes a tangent object of M. This tangent object is (read all and circle the best answer):

a curve in the plane	*a curve in space*
a line in the plane	*a line in space*
a curved surface in space	*a plane in space*

none of the above

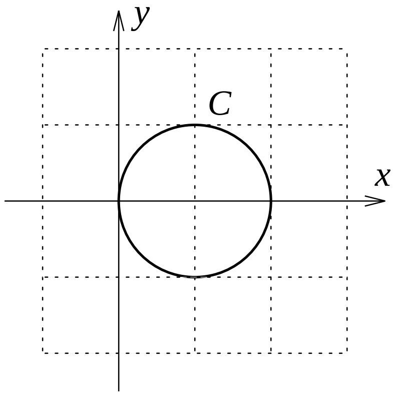
b) (15 points) Write an equation for this tangent object.

Tangent object: $\boxed{\qquad\qquad\qquad\qquad}$

5. We are given the function $f(x, y) = x^3 - 6xy + y^3$. Find the coordinates of all critical points of $f(x, y)$, and use the second-derivative test to determine if possible the type of each critical point (local maximum, local minimum, saddle point). If the second-derivative test is inconclusive, write **???** in the 'Type' box. You may not need all the boxes provided.

Critical point:		Type:	
Critical point:		Type:	
Critical point:		Type:	
Critical point:		Type:	

6. The circle C shown below is given by the equation $x^2 - 2x + y^2 = 0$.



a) (15 points) Using Lagrange multipliers, find all the critical points of the function $g(x, y) = x^2 - y^2$ **along the curve** C. You might not need all the boxes provided.

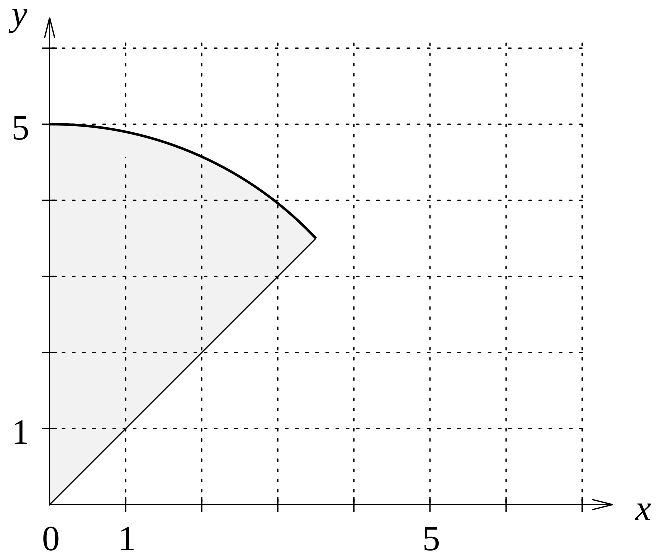
b) (10 points) Find the global maximum and minimu values of $g(x, y)$ on C, and the coordinates of the points where they occur.

Global maximum: occurs at $(x, y) =$

Global minimum: occurs at $(x, y) =$

7. D is the filled-in region shown below. The upper boundary curve of D is an arc of the circle with radius 5 and center at the origin. We are also given a function

$$f(x,y) = \frac{1}{1+2x+y}.$$



a) (10 points) Using the Basic Estimate, find constants k and K so that

$$k \le \iint_D f(x,y)\,dA \le K.$$

[Hint: use level curves to find the minimum and maximum values of the integrand on D.]

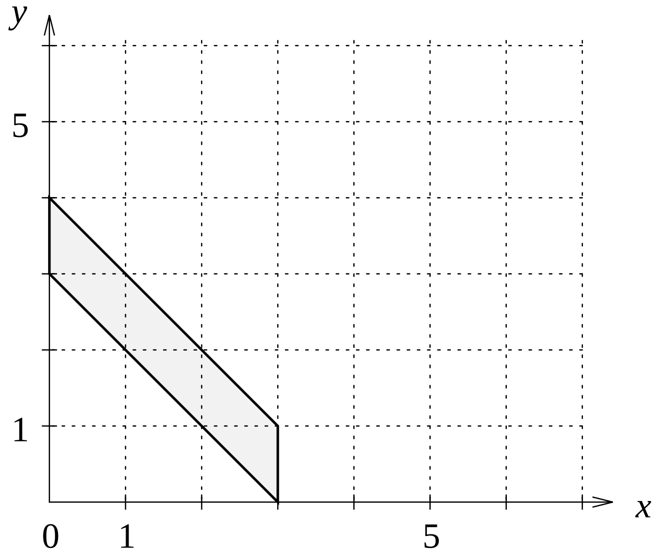
$k = \boxed{}$

$K = \boxed{}$

b) (15 points) Set up (but do not evaluate) the integral $\iint_D f(x,y)\,dA$ in polar coordinates.

$$\int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \boxed{}\, d\boxed{}\, d\boxed{}$$

8. D is the shaded parallelogram below.



a) (15 points) Decide on an order of integration, and set up the double integral $\iint_D 2y\, dA$.

$$\int_{\square}^{\square} \int_{\square}^{\square} \square \; d\square \; d\square$$

b) (10 points) Evaluate the integral that you set up in (a).

$$\iint_D 2y\, dA = \square$$