Math 126B **Calculus With Analytic Geometry III** **Spring 1995**

MIDTERM TEST #2 ANSWERS

1. We consider a function $g(x, y)$ of two variables. We do not know the formula for g, but we do know the formula for the linear approximation $L(x, y)$ of g at the point $Q(4, 3)$:

$$L(x, y) = 2 - 3(x - 4) + 2(y - 3).$$

(a) Do you have sufficient information to find $g(4, 3)$? If yes, do so. If no, write **??**.

(4 points) $g(4, 3) =$ | 2 |

(b) Do you have sufficient information to find grad g at Q? If yes, do so. If no, write **??**.

(8 points) grad $g =$ | $(-3, 2)$ |

(c) Is there a unit vector $\vec{u}$ so that the directional derivative of $g(x, y)$ at Q in the direction of $\vec{u}$ equals $-\sqrt{13}$? If yes, write the components of such a vector. If there is none, write **NONE** and explain why in the space below. If there is insufficient information to answer the question, write **??**.

(8 points) $\vec{u} =$ | $(3/\sqrt{13}, -2/\sqrt{13})$ |

2. Consider the function $F(x, y, z) = \sin yx^2 + e^z$. Let E be the set of all points (x, y, z) in space satisfying the equation $F(x, y, z) = 1$.

(a) What kinds of tangent and normal objects (lines or planes) does E have?

(5 points) Tangent objects: | Planes |

(5 points) Normal objects: | Lines |

(b) Write down an equation for the tangent object of E at the point $(1, 0, 0)$. If it is a tangent line, write a parameter representation. If it is a tangent plane, write its equation in the standard form $Ax + By + Cz = K$.

(10 points) Tangent object: | $y + z = 0$ |

3. We are given that $f(x,y) = 2x^3 - 6yx$.

(a) Which of the following vectors is the gradient of $f(x,y)$? Circle one; you don't need to show work. (5 points)

$(-6y,\ 6y^2 - 6x)$ $\quad$ $(6x^2,\ -6x - 6y)$ $\quad$ $\boxed{(6x^2 - 6y,\ -6x)}$

$(2x^2 - 6y,\ -6x)$ $\quad$ $(6x,\ 2x^2 - 6y)$ $\quad$ none of these

(b) Find the coordinates of a point Q in the plane for which the following is true: Among all directional derivatives of f at the point Q, the one in the direction of $\vec{j} = (0,1)$ is larger than any other.

If there is more than one answer, give the coordinates of a specific point. If there is no answer, write **NONE**.

(15 points) $\quad$ Q : $\boxed{(-a, a^2) \text{ for any } a > 0}$

4. We have the following information on the function $f(x,y)$: Among all directional derivatives of $f(x,y)$ at the point $P(-1,3)$ the one in the direction of $\vec{b} = (-3,4)$ is largest, and it is equal to 10.

(a) Through the point P passes a level object of f. What is this level object? (Read them all and circle one—you don't need to show work on this one). (4 points)

a surface in xyz-space $\quad$ *a surface in the xy-plane*

a curve in xyz-space $\quad$ $\boxed{\textit{a curve in the xy-plane}}$

none of the above $\quad$ *can't tell (explain why)*

(b) Find the gradient of f at the point P.

(8 points) $\quad$ grad $f =$ $\boxed{(-6, 8)}$

(c) Find the directional derivative of f at P in the direction determined by $\vec{c} = (2,3)$.

(8 points) $\quad$ Directional derivative = $\boxed{12/\sqrt{13}}$

5. Let α be the plane $10x + 15y + 6z = 30$ shown in the graph.

[T]	**F**	??	(a) α is the normal object of the line $(x,y,z) = (13+10t, 15+15t, 6+6t)$ at the point $(3,0,0)$.
T	**[F]**	??	(b) α is a level object of the function $g(x,y) = \frac{1}{6}(10x + 15y - 30)$.
T	**[F]**	??	(c) α is the graph of the function $f(x,y,z) = 10x + 15y + 6z$.
[T]	**F**	??	(d) α is the graph of the function $g(x,y) = -\frac{1}{6}(10x + 15y - 30)$.
[T]	**F**	??	(e) α is a level object of the function $f(x,y,z) = 10x + 15y + 6z$.