Section ____________ **Name** ____

TA __________________ **Student Number** ________________________________

Math 126B **Calculus With Analytic Geometry III** **Spring 1995**

Midterm Test 2

5/8/95

- You may use two $8\frac{1}{2}''$ by $11''$ one-sided pages (or one sheet written on both sides) of your own handwritten notes (no photocopied or printed material), and a small model of an xyz-system. **No Calculators are allowed**. You may leave complex algebraic expressions such as $\sqrt{7^2 + 4.5^2}$ as they are, without simplifying. Simplify numerical expressions to the point where they could be evaluated with a simple calculator.
- There are five problems, each counting 20 points for a total of 100 points. Use the boxes for your answers when possible. Use the remaining blank space on each page for your scratch work. **Show your work**: you must justify every answer.
- Measurements must be made with a ruler, and lines and vectors must be drawn with a ruler. Lines or vectors drawn without a ruler lose 4 points.
- *Tests with wrong or incorrectly spelled TA names or section letters will lose 4 points.*
- *If you have a question during the test, raise your hand and I will come to your seat.*

For grader's use only

1:	
2:	
3:	
4:	
5:	
Total:	

1. We consider a function $g(x, y)$ of two variables. We do not know the formula for g, but we do know the formula for the linear approximation $L(x, y)$ of g at the point $Q(4, 3)$:

$$L(x, y) = 4 - 2(x - 4) + 3(y - 3).$$

(a) Do you have sufficient information to find $g(4, 3)$? If yes, do so. If no, write **??**.

(4 points) $g(4, 3) = \boxed{}$

(b) Do you have sufficient information to find $\operatorname{grad} g$ at Q? If yes, do so. If no, write **??**.

(8 points) $\operatorname{grad} g = \boxed{}$

(c) Is there a unit vector $\vec{u}$ so that the directional derivative of $g(x, y)$ at Q in the direction of $\vec{u}$ equals -13? If yes, write the components of such a vector. If there is none, write **NONE** and explain why in the space below. If there is insufficient information to answer the question, write **??**.

(8 points) $\vec{u} = \boxed{}$

2. Consider the function $F(x, y, z) = \sin xy^2 + e^z$. Let E be the set of all points (x, y, z) in space satisfying the equation $F(x, y, z) = 1$.

(a) What kinds of tangent and normal objects (lines or planes) does E have?

(5 points) Tangent objects:

(5 points) Normal objects:

(b) Write down an equation for the tangent object of E at the point $(0, 1, 0)$. If it is a tangent line, write a parameter representation. If it is a tangent plane, write its equation in the standard form $Ax + By + Cz = K$.

(10 points) Tangent object:

3. We are given that $f(x, y) = 2y^3 - 6yx$.

(a) Which of the following vectors is the gradient of $f(x, y)$? Circle one; you don't need to show work. (5 points)

$(-6y,\ 6y^2 - 6x)$	$(6y^2,\ -6y - 6x)$	$(6x^2 - 6y,\ -6x)$
$(2y^2 - 6x,\ -6x)$	$(-6y,\ 2y^2 - 6x)$	none of these

(b) Find the coordinates of a point Q in the plane for which the following is true: Among all directional derivatives of f at the point Q, the one in the direction of $\vec{\imath} = (1, 0)$ is larger than any other.

If there is more than one answer, give the coordinates of a specific point. If there is no answer, write **NONE**.

(15 points) Q : $\boxed{}$

4. We have the following information on the function $f(x,y)$: Among all directional derivatives of $f(x,y)$ at the point $P(-1,3)$ the one in the direction of $\vec{b} = (4,-3)$ is largest, and it is equal to 20.

(a) Through the point P passes a level object of f. What is this level object? (Read them all and circle one—you don't need to show work on this one). (4 points)

a surface in the xy-plane *a surface in xyz-space*

a curve in the xy-plane *a curve in xyz-space*

none of the above *can't tell (explain why)*

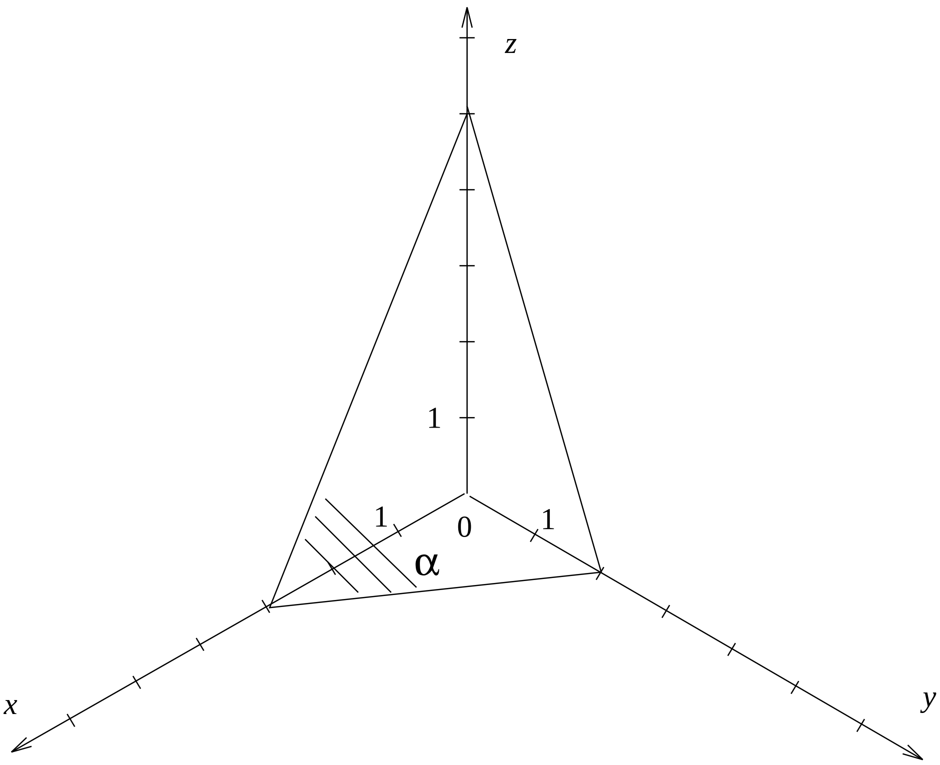
(b) Find the gradient of f at the point P.

(8 points) grad f = $\boxed{}$

(c) Find the directional derivative of f at P in the direction determined by $\vec{c} = (1,3)$.

(8 points) Directional derivative = $\boxed{}$

5. Let α be the plane $10x + 15y + 6z = 30$ shown below.



On the left of each statement below circle one of the three choices: **T** (true), **F** (false), or **??** (impossible to decide on the basis of the given information). You don't need to show work to get credit for a correct answer (4 points each).

T **F** **??** (a) α is the normal object of the line $(x, y, z) = (13 + 10t, 15 + 15t, 6 + 6t)$ at the point $(3, 0, 0)$.

T **F** **??** (b) α is a level object of the function $f(x, y, z) = 10x + 15y + 6z$.

T **F** **??** (c) α is the graph of the function $g(x, y) = -\frac{1}{6}(10x + 15y - 30)$.

T **F** **??** (d) α is the graph of the function $f(x, y, z) = 10x + 15y + 6z$.

T **F** **??** (e) α is a level object of the function $g(x, y) = \frac{1}{6}(10x + 15y - 30)$.