Math 126B **Calculus With Analytic Geometry III** **Spring 1995**

MIDTERM TEST 2 ANSWERS

1. We consider a function $g(x, y)$ of two variables. We do not know the formula for g, but we do know the formula for the linear approximation $L(x, y)$ of g at the point $Q(4, 3)$:

$$L(x, y) = 4 - 2(x - 4) + 3(y - 3).$$

(a) Do you have sufficient information to find $g(4, 3)$? If yes, do so. If no, write **??**.

(4 points) $g(4, 3) = \boxed{4}$

(b) Do you have sufficient information to find $\operatorname{grad} g$ at Q? If yes, do so. If no, write **??**.

(8 points) $\operatorname{grad} g = \boxed{(-2, 3)}$

(c) Is there a unit vector $\vec{u}$ so that the directional derivative of $g(x, y)$ at Q in the direction of $\vec{u}$ equals -13? If yes, write the components of such a vector. If there is none, write **NONE** and explain why in the space below. If there is insufficient information to answer the question, write **??**.

(8 points) $\vec{u} = \boxed{\text{NONE}}$

2. Consider the function $F(x, y, z) = \sin xy^2 + e^z$. Let E be the set of all points (x, y, z) in space satisfying the equation $F(x, y, z) = 1$.

(a) What kinds of tangent and normal objects (lines or planes) does E have?

(5 points) Tangent objects: $\boxed{\text{Planes}}$

(5 points) Normal objects: $\boxed{\text{Lines}}$

(b) Write down an equation for the tangent object of E at the point $(0, 1, 0)$. If it is a tangent line, write a parameter representation. If it is a tangent plane, write its equation in the standard form $Ax + By + Cz = K$.

(10 points) Tangent object: $\boxed{x + z = 0}$

3. We are given that $f(x, y) = 2y^3 - 6yx$.

(a) Which of the following vectors is the gradient of $f(x, y)$? Circle one; you don't need to show work. (5 points)

$\boxed{(-6y,\ 6y^2 - 6x)}$ $\qquad (6y^2,\ -6y - 6x)$ $\qquad (6x^2 - 6y,\ -6x)$

$(2y^2 - 6x,\ -6x)$ $\qquad (-6y,\ 2y^2 - 6x)$ $\qquad$ none of these

(b) Find the coordinates of a point Q in the plane for which the following is true: Among all directional derivatives of f at the point Q, the one in the direction of $\vec{\imath} = (1, 0)$ is larger than any other.

If there is more than one answer, give the coordinates of a specific point. If there is no answer, write **NONE**.

(15 points) $\qquad Q:$ $\boxed{(a^2, -a) \text{ for any } a > 0}$

4. We have the following information on the function $f(x, y)$: Among all directional derivatives of $f(x, y)$ at the point $P(-1, 3)$ the one in the direction of $\vec{b} = (4, -3)$ is largest, and it is equal to 20.

(a) Through the point P passes a level object of f. What is this level object? (Read them all and circle one—you don't need to show work on this one). (4 points)

a surface in the xy-plane $\qquad$ *a surface in xyz-space*

$\boxed{\textit{a curve in the xy-plane}}$ $\qquad$ *a curve in xyz-space*

none of the above $\qquad$ *can't tell (explain why)*

(b) Find the gradient of f at the point P.

(8 points) $\qquad$ grad $f =$ $\boxed{(16, -12)}$

(c) Find the directional derivative of f at P in the direction determined by $\vec{c} = (1, 3)$.

(8 points) $\qquad$ Directional derivative $=$ $\boxed{-20/\sqrt{10}}$

5. Let α be the plane $10x + 15y + 6z = 30$ shown in the graph.

$\boxed{\textbf{T}}$ **F** ?? (a) α is the normal object of the line $(x, y, z) = (13+10t, 15+15t, 6+6t)$ at the point $(3, 0, 0)$.

$\boxed{\textbf{T}}$ **F** ?? (b) α is a level object of the function $f(x, y, z) = 10x + 15y + 6z$.

$\boxed{\textbf{T}}$ **F** ?? (c) α is the graph of the function $g(x, y) = -\frac{1}{6}(10x + 15y - 30)$.

T $\boxed{\textbf{F}}$?? (d) α is the graph of the function $f(x, y, z) = 10x + 15y + 6z$.

T $\boxed{\textbf{F}}$?? (e) α is a level object of the function $g(x, y) = \frac{1}{6}(10x + 15y - 30)$.