Math 126B **Calculus With Analytic Geometry III** **Spring 1995**

MIDTERM VERSION B ('MIDTERM TEST 1')

SOLUTIONS

1. The plane α shown below is parallel to the x-axis and passes through points A and B. (As always, units are centimeters.)

(a) Write an equation for α.

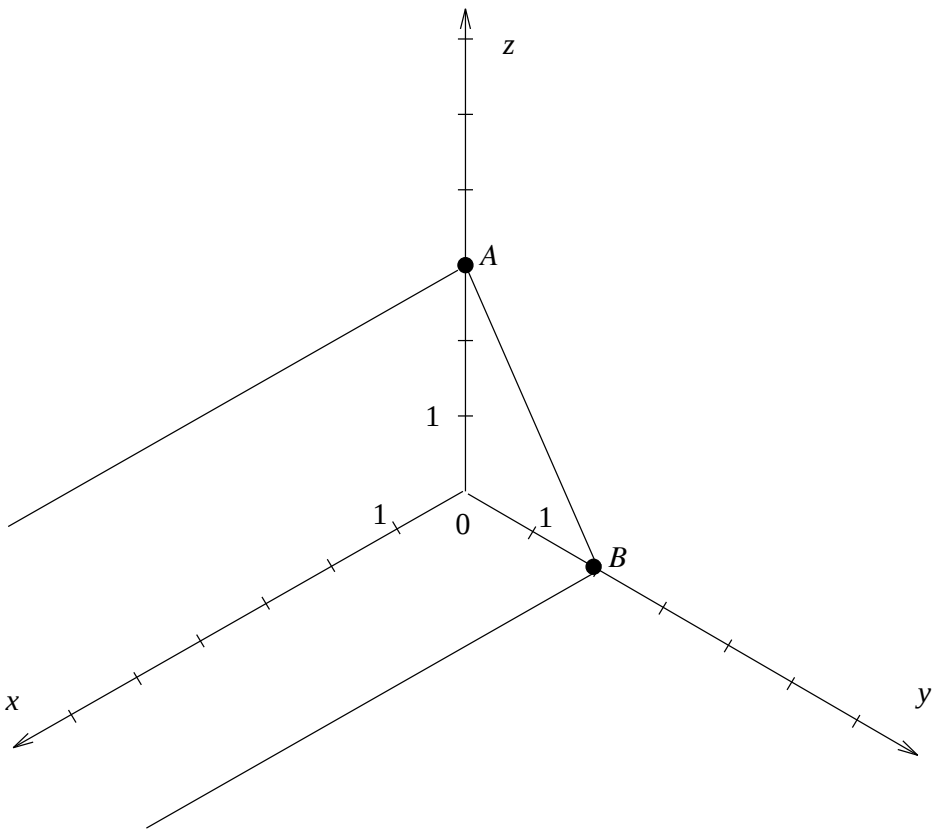
(6 points) α: $\boxed{3y + 2z = 6}$

(b) Find a normal vector $\vec{n}$ to α.

(6 points) $\vec{n} =$ $\boxed{(0, 3, 2)}$

(c) Find the distance from the point $Q(-3, 3, 1)$ to α.

(8 points) Distance $=$ $\boxed{5/\sqrt{13}}$



2. A particle P is moving at constant speed on a line ℓ in space. At time $t = 0$, its position vector is $\vec{r}(0) = (1, 2, -3)$. At time $t = 1$, its position vector is $\vec{r}(1) = (0, 1, 2)$.

At the same time, another particle Q is moving at constant speed on a different line m. At time $t = 0$, Q is at $(4, 5, -6)$, and at time $t = 1$, Q is at $(4, 5, 6)$.

(a) Do the two particles collide? Write YES or NO in the box, and, as always, show your work.

(8 points) | NO |

(b) Do the two lines intersect? (*Hint:* Here time doesn't matter any more. Try using different letters for the two parameters.) Write the point of intersection in the box, or write NONE if they don't intersect.

(12 points) Intersection: | $(4, 5, -18)$ |

3. A particle P moves through the xy-plane on a curve C. The position vector $\vec{r}(t)$ of P at time t is given by

$$\vec{r}(t) = (x(t), y(t)),$$

where the functions $x(t)$ and $y(t)$ are given by their graphs below (the curve C itself is *not* shown on the graph).

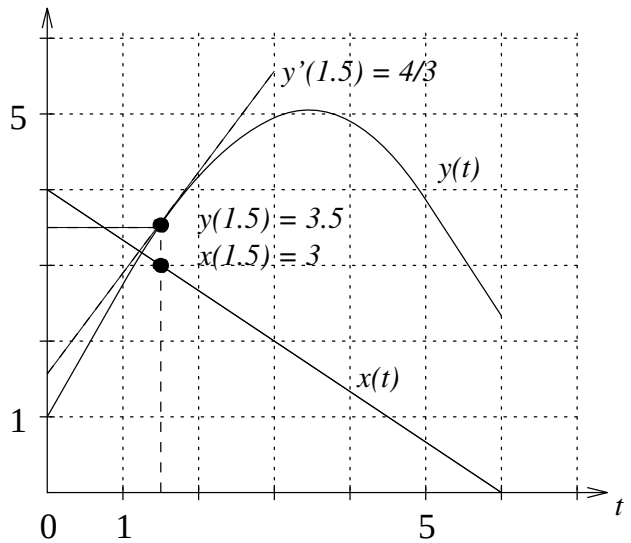
(a) Write down a parameter representation for the tangent line to C at the point where $t = 1.5$.

(14 points) Tangent line: $\boxed{(x, y) = (3, 3.5) + s(-\frac{2}{3}, \frac{4}{3})}$

(b) The speed of P at time t is the length of the velocity vector: $|\vec{r}\,'(t)|$. We are interested in how the speed changes between the times $t = 0$ and $t = 3.5$. As t increases from $t = 0$ to $t = 3.5$, the speed of P (circle one):

always increases	first increases then decreases	**always decreases**	first decreases then increases	impossible to tell

(6 points)



4. We define a function of one variable by $F(t) = |(1,2,t) \times (t,2,1)|$.

(a) Find $F(-2)$.

(6 points) $F(-2) =$ | 9 |

(b) Find a value of t such that $F(t) = 0$. (*Hint:* what vectors have length 0?)

(6 points) $t =$ | 1 |

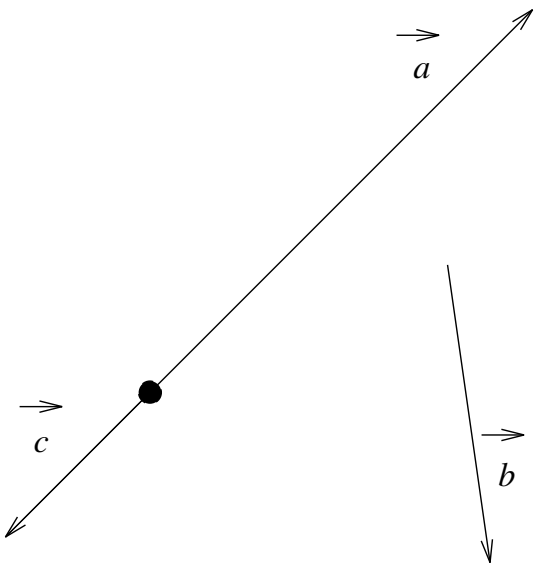
(c) Find an equation for the plane α that passes through the origin and to which both $\vec{a} = (1,2,t)$ and $\vec{b} = (t,2,1)$ are parallel (your answer may contain t).

(8 points) α: | $(2-2t)x + (t^2-1)y + (2-2t)z = 0$ |

5. The vectors $\vec{a}$ and $\vec{b}$ lie in the plane as shown below.

(a) Into the diagram below, draw the vector projection of $\vec{b}$ onto $\vec{a}$, with the same initial point as $\vec{a}$, and label it $\vec{c}$.

(10 points) SEE BELOW

(b) Use your ruler to evaluate $\vec{a} \cdot \vec{b}$.

(10 points) $\vec{a} \cdot \vec{b} =$ $\boxed{(-2.4)(7.1) \approx -17}$