Section ____________ **Name** ____

TA__________________ **Student Number**______________________________

Math 126A & B **Calculus With Analytic Geometry III** **Spring 1995**

FINAL EXAM

3 JUNE 1995

- You may use two 8.5 by 11 pages of your own handwritten notes (no printed or copied material).
- You must **show your work** — justify every answer.
- Use the boxes for the answers. **In some problems, you may not need all the boxes provided.**
- In some of the questions you may have to measure and draw: Measurements must be made with a ruler, and lines, vectors, etc. must be drawn with a ruler; tests with lines etc. drawn by hand lose 4 points.
- No calculators are allowed.
- There are 8 problems for a total of 200 points.
- *Tests with wrong or incorrectly spelled TA names or section letters will lose 4 points.*
- Don't worry about algebraically simplifying numbers such as $\sqrt{7^2+4^2}$; leave them as they are.
- Units are centimeters throughout.

1:	
2:	
3:	
4:	
5:	
6:	
7:	
8:	

Course Grade: ______ Total: ______

Sign at most **one** of the following statements:

I want this test with the course grade on it left outside of my TA's office in Padelford Hall.

Date:__________Signed:______________________________Student Number:__________

I want this test with the course grade on it mailed to me in an envelope which I am providing. I have affixed $.55 postage to the envelope.

Date:__________Signed:______________________________Student Number:__________

1. We do not know the formula for $f(x,y,z)$, but we are given the following information:

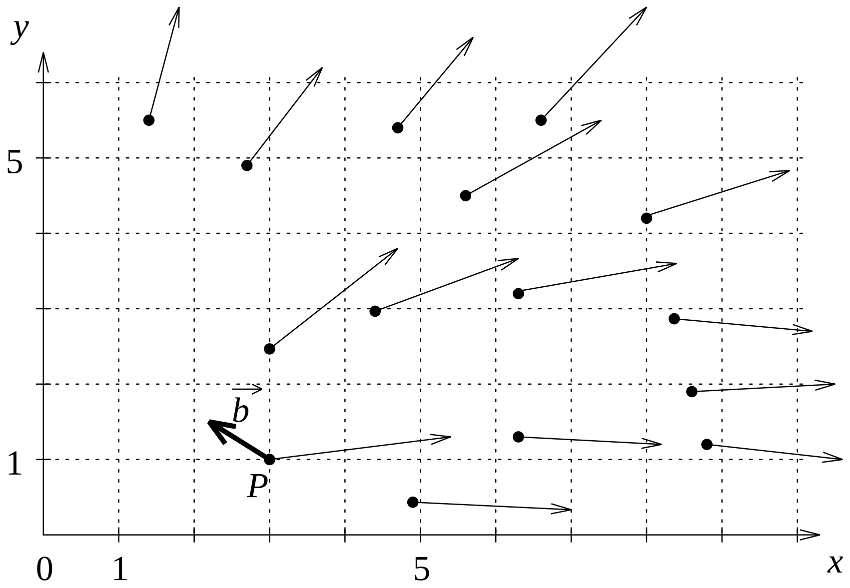
- $f(1,2,3) = 4$;
- $\operatorname{grad} f = (1,2,-1)$ at $P(1,2,3)$ (i.e. $\operatorname{grad} f|_{(1,2,3)} = (1,2,-1)$).

a) **(10 points)** S is the level surface of $f(x,y,z)$ passing through the point $P(1,2,3)$. Find an equation for the tangent object to S at P.

b) **(10)** Write the formula for the linear approximation to $f(x,y,z)$ at P.

c) **(5)** Give the best estimate you can for $f(1.01, 2.02, 2.98)$ based on the information given.

2. $f(x, y)$ is a function of two variables. Below are shown the values of grad f for some points in the plane. Recall that grad $f|_{(x,y)}$ is always drawn with its initial point at (x, y). At P, the vector shown in bold is *not* grad f.



a) **(10) Draw in** a possible level curve for f which goes through at least 4 of the points at which the gradient is shown.

b) **(5)** If you move from $P(3, 1)$ to $Q(5, 0)$ in the plane, the most reasonable conclusion from the given information is that the value of $f(x, y)$ will (circle one; you don't need to show work):

increase *decrease* *stay about the same* *can't tell*

c) **(10)** Find the directional derivative of f at the point $P(3, 1)$ in the direction of $\vec{b}$, where $\vec{b}$ is the vector shown in bold in the picture.

directional derivative = $\boxed{}$

3. We define the function $g(x, y)$ by letting $g(x, y)$ be the scalar projection of the vector (x, y) onto the vector $(4, 1)$.

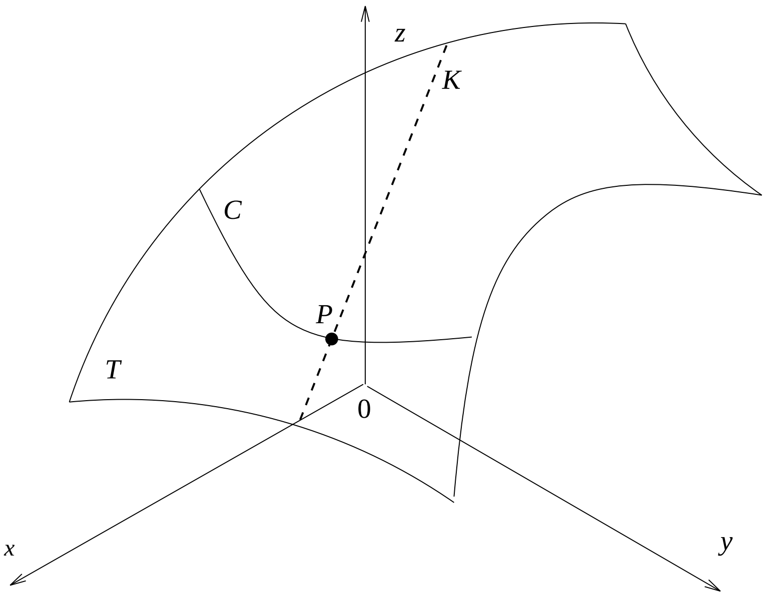
a) **(15)** Compute the smallest possible directional derivative of g at the point $Q(1, 3)$.

smallest possible directional derivative = $\boxed{}$

b) **(10)** Is there a point R in the plane and a unit vector $\vec{u}$ such that the directional derivative of g at R in the direction of $\vec{u}$ is 2? If so, find R; if not, write NONE and explain.

R : $\boxed{}$

4. $f(x, y, z)$ is a function of three variables, and T is the level surface of $f(x, y, z)$ passing through the point $P(1, 0, 2)$. We don't know the formula for $f(x, y, z)$, but we know that the curves $C : (x, y, z) = (t, -t^2 - 2t + 3, t + 1)$ and $K : (x, y, z) = (1, 2u, 2 - u)$ both lie in T. (See the diagram below, which is not to scale. It is easy to verify that C and K intersect at P.)



a) **(5)** Do we have enough information to find $f(1, 0, 2)$? If yes, find it; if not, write IMPOSSIBLE.

$f(1, 0, 2) =$ $\boxed{}$

b) **(10)** Do we have enough information to find grad f at $(1, 0, 2)$? If yes, find it; if not, write IMPOSSIBLE and explain what additional information we would need.

grad $f|_{(1,0,2)} =$ $\boxed{}$

c) **(10)** Do we have enough information to find an equation for the tangent object to T at P? If yes, find the equation; if not, write IMPOSSIBLE and explain what additional information we would need.

tangent object: $\boxed{}$

5. Let $f(x,y) = y^2 - 2x^2y + 2x^3$.

a) **(8)** Find all critical points of $f(x,y)$ in the plane. You may not need all the boxes provided.

1. $\boxed{}$ 2. $\boxed{}$ 3. $\boxed{}$

b) **(8)** Use the second derivative test to determine (if possible) whether each critical point is a local maximum, a local minimum, or a saddle point. Write MAX, MIN, SADDLE, or CAN'T TELL in the box corresponding to each of the critical points you found above.

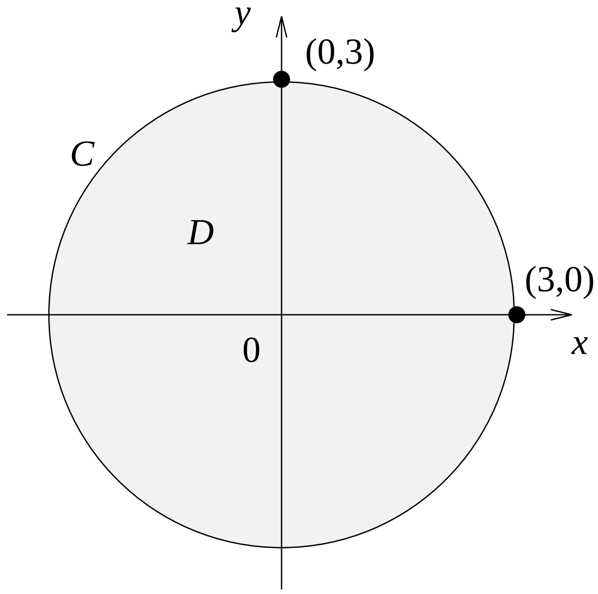
1. $\boxed{}$ 2. $\boxed{}$ 3. $\boxed{}$

c) **(9)** Find the maximum and minimum values taken by $f(x,y)$ on the line segment C from $(0,1)$ to $(1,0)$.

maximum of $f(x,y)$ on C: $\boxed{}$ at the point $(x,y) = \boxed{}$

minimum of $f(x,y)$ on C: $\boxed{}$ at the point $(x,y) = \boxed{}$

6. $f(x,y)$ is the function defined by $f(x,y) = x^2 + \frac{2}{3}y^3 + 5y^2 + 8y + 6$. In all parts of this problem, NONE may or may not be the correct answer.



a) **(10)** D is the filled-in circle shown above. Find the coordinates of all critical points of $f(x,y)$ that lie in D.

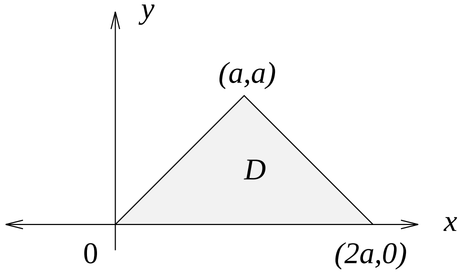
1. [ ] 2. [ ] 3. [ ]

b) **(15)** C is the circle (curve) shown above (the boundary of D). Find the coordinates of all critical points of $f(x,y)$ along C.

1. [ ] 2. [ ] 3. [ ]

4. [ ] 5. [ ] 6. [ ]

7. D is the region pictured below. The picture is **not** to scale.

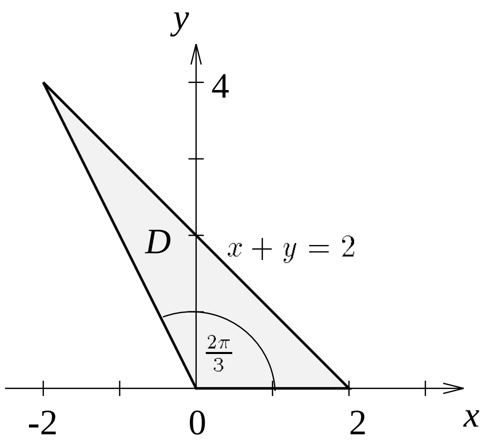


a) **(15)** Set up the integral $\iint_D 3xy\, dA$. You may use any coordinate system and any order of integration. As always, the integration limits must include the variables to which they refer.

$$\int \quad \int \quad d \quad d$$

b) **(10)** Evaluate the integral from a). Your answer may contain a.

8. D is the triangle shown below, bounded by the x-axis, the ray making an angle $\theta = \frac{2\pi}{3}$ with the x-axis, and the line $x + y = 2$:



a) **(15)** Set up in polar coordinates the integral $\iint_D f(r,\theta)\,dA$, where $f(r,\theta)$ is the distance from the origin.

$$\int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \boxed{}\; d\,\boxed{}\; d\,\boxed{}$$

b) **(10)** Evaluate the *inner* integral.

$$\boxed{}$$