Math 126A & B **Calculus With Analytic Geometry III** **Spring 1995**

FINAL EXAM SOLUTIONS

1. a) **(10 points)** Find an equation for the tangent object to S at P.

$$\boxed{x + 2y - z = 2}$$

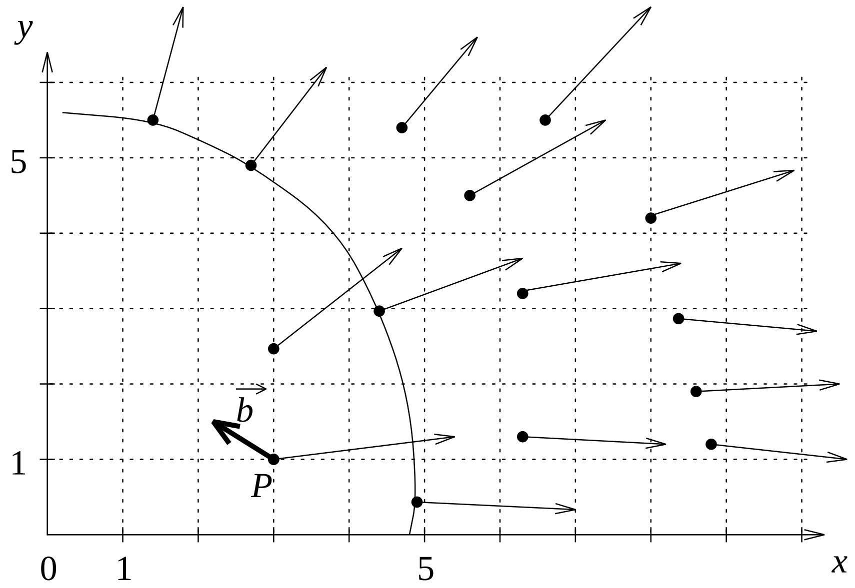
b) **(10)** Write the formula for the linear approximation to $f(x, y, z)$ at P.

$$\boxed{L(x, y, z) = 4 + (x - 1) + 2(y - 2) - (z - 3)}$$

c) **(5)** Give the best estimate you can for $f(1.01, 2.02, 2.98)$ based on the information given.

$$\boxed{4.07}$$

2. a) **(10) Draw in** a possible level curve for f which goes through at least 4 of the points at which the gradient is shown.



b) **(5)** If you move from $P(3, 1)$ to $Q(5, 0)$ in the plane, the most reasonable conclusion from the given information is that the value of $f(x, y)$ will

$\boxed{\textit{increase}}$ *decrease* *stay about the same* *can't tell*

c) **(10)** Find the directional derivative of f at the point $P(3, 1)$ in the direction of $\vec{b}$.

directional derivative = $\boxed{-1.8}$

3. a) **(15)** Compute the smallest possible directional derivative of g at the point $Q(1,3)$.

smallest possible directional derivative = $\boxed{-1}$

b) **(10)** Is there a point R in the plane and a unit vector $\vec{u}$ such that the directional derivative of g at R in the direction of $\vec{u}$ is 2?

$R:$ $\boxed{\text{NONE}}$

Note that grad g *has length* 1*, so all directional derivatives are between* 1 *and* -1.

4. a) **(5)** Do we have enough information to find $f(1,0,2)$?

$f(1,0,2) =$ $\boxed{\text{IMPOSSIBLE}}$

b) **(10)** Do we have enough information to find grad f at $(1,0,2)$?

$\operatorname{grad} f|_{(1,0,2)} =$ $\boxed{\text{IMPOSSIBLE}}$

We'd need to know the length of the gradient and which direction it points (i.e. $\pm(2,1,2)$*).*

c) **(10)** Do we have enough information to find an equation for the tangent object to T at P?

tangent object: $\boxed{2x + y + 2z = 6}$

5. Let $f(x,y) = y^2 - 2x^2y + 2x^3$.

a) **(8)** Find all critical points of $f(x,y)$ in the plane.

1. $\boxed{(0,0)}$ 2. $\boxed{(\frac{3}{2}, \frac{9}{4})}$

b) **(8)** Use the second derivative test to determine (if possible) whether each critical point is a local maximum, a local minimum, or a saddle point.

1. $\boxed{\text{CAN'T TELL}}$ 2. $\boxed{\text{SADDLE}}$

c) **(9)** Find the maximum and minimum values taken by $f(x,y)$ on the line segment C from $(0,1)$ to $(1,0)$.

maximum of $f(x,y)$ on C: $\boxed{2}$ at the point $(x,y) =$ $\boxed{(1,0)}$

minimum of $f(x,y)$ on C: $\boxed{\frac{1}{4}}$ at the point $(x,y) =$ $\boxed{(\frac{1}{2}, \frac{1}{2})}$

6. a) **(10)** Find the coordinates of all critical points of $f(x, y)$ that lie in D.

1. $\boxed{(0, -1)}$

b) **(15)** Find the coordinates of all critical points of $f(x, y)$ along C.

1. $\boxed{(0, 3)}$ 2. $\boxed{(0, -3)}$ 3. $\boxed{(\sqrt{5}, -2)}$ 4. $\boxed{(-\sqrt{5}, -2)}$

7. D is the triangle with vertices $(0, 0)$, (a, a), and $(2a, 0)$.

a) **(15)** Set up the integral $\iint_D 3xy\, dA$.

$$\int_{\boxed{y=0}}^{\boxed{y=a}} \int_{\boxed{x=y}}^{\boxed{x=2a-y}} \boxed{3xy}\, d\boxed{x}\, d\boxed{y}$$

b) **(10)** Evaluate the integral from a). Your answer may contain a.

$\boxed{a^4}$

8. D is the triangle bounded by the x-axis, the ray making an angle $\theta = \frac{2\pi}{3}$ with the x-axis, and the line $x + y = 2$:

a) **(15)** Set up in polar coordinates the integral $\iint\limits_D f(r, \theta)\, dA$, where $f(r, \theta)$ is the distance from the origin.

$$\int_{\boxed{\theta=0}}^{\boxed{\theta=\frac{2\pi}{3}}} \int_{\boxed{r=0}}^{\boxed{r=\frac{2}{\cos\theta+\sin\theta}}} \boxed{r^2}\, d\boxed{r}\, d\boxed{\theta}$$

b) **(10)** Evaluate the *inner* integral.

$$\boxed{\frac{8}{3(\cos\theta+\sin\theta)^3}}$$