CORRECTIONS TO
Introduction to Riemannian Manifolds (Second Edition)

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(7/31/19) Page ix, near the middle of the page: “Preissmann” is misspelled.

(6/16/21) Page 15, line 3: After “connected if and only if $M$ is connected,” insert “(when $\dim M > 1$).”

(11/17/21) Page 16, line after Example 2.13: Change “next lemma” to “next proposition.”

(7/29/19) Page 20, Exercise 2.23: Change “Exercise 2.21” to “Example 2.21.”

(1/25/21) Page 27, third-to-last displayed equation: In the line below that equation, change $\alpha_k$ to $\alpha_k$. [The numeral 1 should be a letter l.] Then in the second and third lines below the equation, change $g_k^{ij}$ to $g_{pq}$ twice [to avoid conflict with the notation $(k, l)$ for the type of $F$].

(1/25/21) Page 27, third line from the bottom: Change $g_k^{ij}$ to $g_{pq}$.

(9/4/22) Page 33, second paragraph under “Lengths and Distances,” next-to-last sentence: Replace that sentence by “(If $\partial M \neq \emptyset$, this is still true provided we embed $M$ in a smooth manifold $\tilde{M}$ without boundary such as the double of $M$ [LeeSM, Example 9.32], and consider an extension of $y$ as a map into $\tilde{M}$.)”

(10/19/21) Page 41, lines 4 & 3 from the bottom: Change “inner products of pairs of elements of $S$” to “scalar products of pairs of elements of $S^\perp$.”

(1/23/21) Page 56, third line from the bottom: Change “Chapter 1” to “Chapter 2.”

(6/24/22) Page 63, Fig. 3.3: The map $\kappa$ should be going in the opposite direction.

(5/3/20) Page 82, Problem 3-21: In the third line, change $M$ to $\tilde{M}$.

(7/8/20) Page 87, last two lines: The second occurrence of $\frac{\partial}{\partial x}$ in each line should be $\frac{\partial}{\partial y}$.

(9/4/22) Page 97, second line: After the last sentence of the proof, add the following: “It is then a straightforward computational exercise to show that the resulting connection satisfies conditions (i)–(iii). To prove (iv), first observe that every $(k, l)$-tensor field can be written locally as a sum of tensor fields of the form $Z_1 \otimes \cdots \otimes Z_k \otimes \xi^1 \otimes \cdots \otimes \xi^l$, and for such a tensor field the trace on the $i$th contravariant index and the $j$th covariant one satisfies

$$\text{tr} (Z_1 \otimes \cdots \otimes Z_k \otimes \xi^1 \otimes \cdots \otimes \xi^l) = \xi^j (Z_1) Z_1 \otimes \cdots \otimes \tilde{Z}_i \otimes \cdots \otimes Z_k \otimes \xi^1 \otimes \cdots \otimes \tilde{\xi}^i \otimes \cdots \otimes \xi^l.$$

Then (iv) follows by applying (4.12) and (4.13) to this formula.”
(4/22/19) **Page 100, first displayed equation:** Change the last expression on the right to \( X(Yu) - (\nabla_X Y)u \).

(9/4/22) **Page 102, proof of Theorem 4.24, second paragraph, last sentence:** Replace that sentence by “(If \( t_0 \) is an endpoint of \( I \), extend \( y \) to a slightly bigger open interval, prove the lemma there, and then restrict back to \( I \). If \( M \) has nonempty boundary, we can do this after first embedding \( M \) into a smooth manifold \( \tilde{M} \) without boundary and extending \( V \) arbitrarily to a connection on \( \tilde{M} \).)”

(11/7/19) **Page 105, first paragraph under the section heading:** Delete the sentence beginning “As we did with geodesics . . . .”

(7/8/20) **Page 106, last line:** Change \( .t/ \) to \( 0.t/ \).

(7/5/20) **Page 109, Corollary 4.33, third and fourth lines:** Replace \( T.t/_{a} \) and \( T.t/_{M} \) by \( T.a/_{M} \).

(6/27/19) **Page 113, Problem 4-11(b):** Replace “\( G \) is abelian” by “the identity component of \( G \) is abelian.”

(1/23/21) **Page 119, just above the last display:** Change \( V_i; W_j W_i ! R \) to \( V_i; W_j W_i .t/ \); \( t_0 C / ! R \).

(8/26/20) **Page 132, proof of Proposition 5.23, fourth and fifth lines:** Change \( d '.p / \) to \( d '.p / (twice). \)

(11/23/20) **Page 147, Problem 5-8(c):** Change “geodesics of \( g \)” to “maximal geodesics of \( g \).”

(2/11/21) **Page 155, just below equation (6.1):** Change “admissible partition for \( V \)” to “admissible partition for \( \Gamma \).”

(8/26/20) **Page 160, line 4:** Change \( @i j p \) to \( @i j p \).

(5/29/20) **Page 163, second paragraph, third line:** Delete the repeated word “metric.”

(8/13/21) **Page 164, line 8 from the bottom:** Change \( =c \) to \( c \) (twice).

(12/5/20) **Page 173, proof of Proposition 6.25, last line:** Change “no longer than” to “no shorter than” (twice).

(1/25/21) **Page 175, two lines above Theorem 6.31:** Change “it it” to “it is.”

(1/25/21) **Page 177, line 7 from the bottom:** Change \( B_{e}(p) \) to \( B_{e,p}(p) \).

(2/27/21) **Page 179, second paragraph, first line:** Change \( NS \) to \( NP \).

(3/3/21) **Page 197, Proposition 7.5, fourth line of the proof:** Change “Formula (4.15)” to “The product rule for covariant derivatives along curves.” Then two lines below that, change “(4.15)” to “the product rule.”

(10/19/19) **Page 199, Proof of Lemma 7.8, second paragraph, last sentence:** Replace the phrase “an inductive application of the theorem concerning smooth dependence of solutions to linear ODEs on initial conditions (Thm. 4.31)” by “an inductive application of Theorem A.42 to vector fields of the form \( W_k|_{(x,v)} = \partial/\partial x^k - v^i \Gamma^j_{ki}(x) \partial/\partial v^j \) on \( C_x \times \mathbb{R}^n \).”

(6/15/20) **Page 201, third line from the bottom:** Change “application of Theorem 4.31” to “application of Theorem A.42 as in the proof of Lemma 7.8.”

(11/1/19) **Page 214, middle displayed equation:** The indices on the left-hand side should be \( jk \) (not \( jk \)).

(11/1/19) **Page 217, last line:** Change \( \langle df,df \rangle_g^2 \) to \( \langle df,df \rangle_g \).
Page 354, Corollary 12.14: Change the statement of the corollary to “If $M$ is a connected smooth manifold that admits a complete metric of nonpositive sectional curvature, then $M$ is aspherical.” Then in the first sentence of the proof, change “a nonpositively curved metric” to “a complete nonpositively curved metric.”

Page 357, subheading and first three paragraphs: “Preissmann” is misspelled (four times).

Page 357, statement of Lemma 12.21: Replace “every covering automorphism” by “every nontrivial covering automorphism.”

Page 360, near the top and near the bottom: “Preissmann” is misspelled (twice).

Page 374, paragraph above Prop. A.10, first line: Change “every coordinate chart” to “every coordinate chart $(U, \varphi)$.”

Page 383, third paragraph, last sentence: Change “component functions of $a$” to “component functions of $\tau$.”

Page 397, three lines above equation (B.8): Change $T^{(k,j)}(T^*M)$ to $T^{(k,j)}TM$.

Page 415, reference [BBBMP]: “Zürich” is misspelled.

Page 415, reference [Cha06]: The phrase “A modern introduction” should be part of the title: Riemannian Geometry: A Modern Introduction.

Page 418, reference [Pre43]: “Preissmann” is misspelled.

Page 433: “Preissmann” is misspelled (twice).