Corrections to
Introduction to Topological Manifolds
(First edition)
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Changes or additions made in the past twelve months are dated.

- Page 29, statement of Lemma 2.11: The second sentence should be replaced by “If the open subsets of $X$ are exactly those sets that satisfy the basis criterion with respect to $B$, then $B$ is a basis for the topology of $X$.”
- Page 29, paragraph before Exercise 2.15: Instead of “the topologies of Exercise 2.1,” it should say “some of the topologies of Exercise 2.1.”
- Page 30, last sentence of the proof of Lemma 2.12: Replace $U$ by $f^{-1}(U)$ (three times).
- Page 30, first paragraph in the “Manifolds” section: Delete the sentence “Let $X$ be a topological space.”
- Page 38, Problem 2-16(b): Replace part (b) by “Show that for any space $Y$, a map $f: X \to Y$ is continuous if and only if $p_n \to p$ in $X$ implies $f(p_n) \to f(p)$ in $Y$.”
- Page 38, Problem 2-18: This problem should be moved to Chapter 3, because $\text{Int} M$ and $\partial M$ are to be interpreted as having the subspace topologies. Also, for this problem, you may use without proof the fact that $\text{Int} M$ and $\partial M$ are disjoint.
- Page 40, last line of Example 3.1: Replace “subspace topology on $B$” by “subspace topology on $C$."

Page 46, second display: Replace $k$ by $k/2$ (twice) and $l$ by $l/2$ (twice). [The tangent and cotangent functions have period $\pi/2$, not $\pi$.]

- Page 51, proof of Proposition 3.13, third line: $f_1(U_1), \ldots, f_k(U_k)$ should be replaced by $f_1^{-1}(U_1), \ldots, f_k^{-1}(U_k)$.
- Page 51, proof of Proposition 3.14, last sentence: Replace “the preceding lemma” by “the preceding proposition.”
- Page 52, first paragraph after Exercise 3.8: In the first sentence, replace the words “surjective and continuous” by “surjective.” Also, add the following sentence at the end of the paragraph: “It is immediate from the definition that every quotient map is continuous.”
- Page 52, last paragraph: Change the word “quotient” to “surjective” in the first sentence of the paragraph.

Page 53, line 1: Change the word “quotient” to “surjective” at the top of the page.
- Page 53, Lemma 3.17: Add the following sentence at the end of the statement of the lemma: (More precisely, if $U \subset X$ is a saturated open or closed set, then $\pi|_U: U \to \pi(U)$ is a quotient map.)
- Page 82, line 3 from bottom: Delete “$= U \cap Z$ . . . .”
- Page 83, Example 4.30(a): In the first sentence, change “closed” to “open” and change $B_{\varepsilon}(x)$ to $B_{\varepsilon}(x)$. 

(12/7/15)
• **Page 85, statement of Corollary 4.34:** “countable collection” should read “countable union.”

• **Page 99, Lemma 5.4:** Replace part (d) by

(d) For any topological space \( Y \), a map \( F: |X| \to Y \) is continuous if and only if its restriction to \( |\sigma| \) is continuous for each \( \sigma \in \mathcal{K} \).

• **Page 103, Proposition 5.11:** In the statement of the proposition, change “simplicial complex” to “1-dimensional simplicial complex.”

• **Page 106, line 3 from bottom:** Replace “even” by “odd.”

• **Page 111, Figure 5.12:** In \( S(SK) \), the points inside the small triangles should be at the intersections of the three medians.

• **Page 114, Problem 5-2:** Replace the statement of the problem by: “Let \( K \) be an abstract simplicial complex. For each vertex \( v \) of \( K \), let \( \text{St}_v \) (the open star of \( v \)) be the union of the open simplices \( \text{Int}|\sigma| \) as \( \sigma \) ranges over all simplices that have \( v \) as a vertex; and define a function \( t_v: |X| \to \mathbb{R} \) by letting \( t_v(x) \) be the coefficient of \( v \) in the formal linear combination representing \( x \).

(a) Show that each function \( t_v \) is continuous.

(b) Show that \( \text{St}_v \) is a neighborhood of \( v \), and the collection of open stars of all the vertices is an open cover of \( |K| \).”

• **Page 114, Problem 5-3:** Delete the phrase “and locally path connected.”

• **Page 120, Statement of Proposition 6.2(b):** Replace \( x \in \partial \mathbb{B}^2 \) by \( (x, y) \in \partial \mathbb{B}^2 \).

• **Page 126, Proposition 6.6:** Add the hypothesis that \( n \geq 2 \).

• **Page 131, Part 1 of the definition of the geometric realization:** After “sides of length 1,” insert “equal angles,”.

• **Page 136, line 8 from bottom:** Change the surface presentation in that line to \( \langle S_1, S_2, a, b, c \mid W_1c^{-1}b^{-1}a^{-1}, abcW_2 \rangle \).

• **Page 139, proof of the classification theorem:** Replace the first sentence of the proof with “Let \( M \) be the compact surface determined by the given presentation.”

• **Page 140, line 14:** Change “Step 3” to “Step 2.”

• **Page 149, Example 7.3:** The first line should read “Define maps \( f, g: \mathbb{R} \to \mathbb{R}^2 \) by . . . .”

• **Page 155, line 3:** Change \( \Phi_g(f) \) to \( \Phi_g[f] \).

• **Page 156, Figure 7.7:** The labels \( I \times I, F, \) and \( X \) should all be in math italics.

• **Page 156, Exercise 7.2:** Change the first sentence to “Let \( X \) be a path connected topological space.”

• **Page 159, second line from bottom:** “induced homeomorphism” should read “induced homomorphism.”

• **Page 160, Proposition 7.18:** In the statement and proof of the proposition, change \( (\iota_A)^* \) to \( (\iota_A)_* \) three times (the asterisk should be a subscript).

• **Page 174, proof of Lemma 7.35:** In the second-to-last line of the proof, change “Theorem 3.10” to “Theorem 3.11.”
• Page 176, Problem 7-5: Change “compact surface” to “connected compact surface.”

• Page 188, proof of Theorem 8.7: Replace the third sentence of the proof by “If \( f : I \to \mathbb{S}^n \) is any loop based at a point in \( U \cap V \), by the Lebesgue number lemma there is an integer \( m \) such that on each subinterval \([k/m, (k + 1)/m]\), \( f \) takes its values either in \( U \) or in \( V \). If \( f(k/m) = N \) for some \( k \), then the two subintervals \([(k - 1)/m, k/m]\) and \([k/m, (k + 1)/m]\) must be both be mapped into \( V \). Thus, letting \( 0 = a_0 < \cdots < a_i = 1 \) be the points of the form \( k/m \) for which \( f(a_i) \neq N \), we obtain a sequence of curve segments \( f[a_{i-1}, a_i] \) whose images lie either in \( U \) or in \( V \), and for which \( f(a_i) \neq N \).” Also, in the last line of the proof, replace “\( f \) is homotopic to a path” by “\( f \) is path homotopic to a loop.”

• Page 189, proof of Proposition 8.9: In the last sentence of the proof, change the domain of \( H \) to \( I \times I \), and change the definition of \( H \) to

\[
H(s, t) = (H_1(s, t), \ldots, H_n(s, t)).
\]

• Page 191, Problem 8-7: In the third line of the problem, change \( \varphi(\gamma) \) to \( \varphi_s(\gamma) \).

• Page 192, line 4: Change the definition of \( \varphi \) to \( \varphi(x) = (x - f(x))/|x - f(x)| \).

• Page 199, second-to-last paragraph: In the second sentence, after “a product of elements of \( S \),” insert “or their inverses.”

• Page 208, Problem 9-4(b): Change the first phrase to “Show that \( \text{Ker} f_1 \ast f_2 \) is equal to the normal closure of \( \text{Im} j_1 \ast j_2 \), . . . .” Add the following hint: “[Hint: Let \( N \) denote the normal closure of \( \text{Im} j_1 \ast j_2 \), so it suffices to show that \( f_1 \ast f_2 \) descends to an isomorphism from \((G_1 \ast G_2)/N \) to \( H_1 \ast H_2 \). Construct an inverse by showing that each composite map \( G_2 \to G_1 \ast G_2 \to (G_1 \ast G_2)/N \) passes to the quotient yielding a map \( H_j \to (G_1 \ast G_2)/N \), and then invoking the characteristic property of the free product.]”

• Page 213, proof of Proposition 10.5: In the second sentence of the proof, change \( \{q\} \) to \( \{*\} \).

• Page 218, Figure 10.4: In the upper diagram, one of the arrows labeled \( a_i \) should be reversed.

• Page 227, line 8: Replace \( \overline{R} \ast \overline{S} \) by \( R \ast S \).

• Page 233, last line: Change the last sentence to “This brings us to the next-to-last major subject in the book: . . . .”

• Page 238, proof of Proposition 11.10, second line: Change “\( p \) maps . . . .” to “\( f \) maps . . . .”

• Page 248, Example 11.26: Change \( C_\pi(\mathbb{P}^n) \) to \( C_\pi(\mathbb{S}^n) \).

• Page 249, line 5: Change the formula to “\( p(\varphi(q)) = p(q) = q \)” (not \( p \)).

• Page 253, Problem 11-9: Change “path connected” to “locally path connected.”

• Page 265, Step 4: In the second line of Step 4, replace “as in Step 3” by “as in Step 2.”

• Page 268, proof of Theorem 12.11: The first and last paragraphs of this proof can be simplified considerably by using the result of Problem 3-15.

• Page 272, first paragraph: The last sentence should read “It can be identified with a quotient of the group of matrices of the form \( \left( \begin{array}{cc} a & 0 \\ \lambda & 1 \end{array} \right) \) (identifying two matrices if they differ by a scalar multiple), and so is a topological group acting continuously on \( \mathbb{B}^2 \).”
• **Page 284, just below the first displayed equation**: Replace everything on that page below the first displayed equation with the following:

We have to show that \( p' \) is a covering map. Let \( q_1 \in X \) be arbitrary, and let \( U \) be a neighborhood of \( q_1 \) that is evenly covered by \( p \). We will show that \( U \) is also evenly covered by \( p' \). Given a component \( \bar{U} \) of \( p^{-1}(U) \), let \( \bar{U}' = \pi(\bar{U}) \subset X' \); since \( \pi \) is an open map (Problem 3-15), \( \bar{U}' \) is open in \( X' \). Suppose \( \bar{U}'_1 = \pi(\bar{U}_1) \) and \( \bar{U}'_2 = \pi(\bar{U}_2) \) are any two such sets. If they have a point \( q' \) in common, then \( q' = \pi(q_1) = \pi(q_2) \) for some \( \tilde{q}_1, \tilde{q}_2 \in \bar{U}_1, \tilde{q}_2 \in \bar{U}_2 \). Since \( \pi \) identifies points of \( \bar{X} \) if and only if they are in the same \( \bar{H} \)-orbit, there is some \( \varphi \in \bar{H} \) such that \( \tilde{q}_2 = \varphi(\tilde{q}_1) \). Then \( \varphi \) maps \( \bar{U}_1 \) homeomorphically onto \( \bar{U}_2 \), so \( \pi(\bar{U}_2) = \pi \circ \varphi(\bar{U}_1) = \pi(\bar{U}_1) \). This shows that any such sets \( \bar{U}'_1, \bar{U}'_2 \) are either disjoint or equal. Since \( \pi \) is surjective, \( p'^{-1}(U) \) is equal to the disjoint union of the sets \( \pi(\bar{U}) \) as \( \bar{U} \) ranges over the components of \( p^{-1}(U) \).

It remains only to show that for any such set \( U' = \pi(\bar{U}) \), \( p': U' \to U \) is a homeomorphism. The following diagram commutes:

\[
\begin{array}{ccc}
\bar{U} & \xrightarrow{\pi} & U' \\
p \downarrow & & \downarrow p' \\
U & \xrightarrow{p} & U \end{array}
\]  

(12.8)

Since \( p = p' \circ \pi \) is injective on \( \bar{U} \), so is \( \pi \); and \( \pi: \bar{U} \to U' \) is surjective by definition. Because \( \pi \) is an open map, it follows that \( \pi: \bar{U} \to U' \) is a homeomorphism. Since \( p \) and \( \pi \) are homeomorphisms in (12.8), so is \( p' \).

• **Page 287, line 10**: The sentence “Thus (i) corresponds to the rank 1 case” should read “Thus (ii) corresponds to the rank 1 case.”

• **Page 289, Problem 12-5**: Replace the statement of the problem by “Find a group \( \Gamma \) acting freely and properly on the plane such that \( \mathbb{R}^2/\Gamma \) is homeomorphic to the Klein bottle.”

• **Page 290, Problem 12-9**: Replace the second sentence by “For any element \( e \) in the fiber over the identity element of \( G \), show that \( \bar{G} \) has a unique group structure such that \( e \) is the identity, \( \bar{G} \) is a topological group, and the covering map \( p: \bar{G} \to G \) is a homomorphism with discrete kernel.”

• **Page 301, just above the third displayed equation**: In the last sentence of the paragraph, replace \( G_{i,p}: \Delta_p \to \Delta_p \times I \) by \( G_{i,p}: \Delta_{p+1} \to \Delta_p \times I \).

• **Page 316, first paragraph**: Change the fourth sentence to: “For \( p > 0 \), if \( \alpha: \Delta_p \to \mathbb{R}^n \) is an affine \( p \)-simplex, set

\[
\sigma_\alpha = \alpha(b_p) \ast s \partial \alpha
\]

(where \( b_p \) is the barycenter of \( \Delta_p \)), and extend linearly to affine chains.”

• **Page 319, statement of Lemma 13.21**: \( H^{n-1} \) should be \( H_{n-1} \).

• **Page 320, first paragraph**: In the last two lines, \( H^{n-1} \) should be \( H_{n-1} \) (twice).

• **Page 325, second to last displayed equation**: Change \( H_p(\mathcal{X}) \) to \( H_p^\Delta(\mathcal{X}) \).

• **Page 330, paragraph after Exercise 13.4**: Replace [Mun75] by [Mun84].

• **Page 332, line 1**: The first word on the page should be “subgroups” instead of “spaces.”

• **Page 333, line 7**: Change “coboundary” to “cocycle.”
• **Page 335, Problem 13-12:** Add the hypothesis that $U \cup V = X$.

• **Page 344, Exercise A.7(a):** Since this exercise requires the axiom of choice, it should be moved after exercise A.9.