

CORRECTIONS TO Introduction to Topological Manifolds (Second Edition)

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- (2/25/18) **Page xii, last paragraph:** Allen Hatcher's name is misspelled. (Sorry, Allen.)
- (2/14/15) **Page 23, Exercise 2.6, first line:** Change “collection of topologies” to “nonempty collection of topologies.”
- (6/24/19) **Page 26, just above Exercise 2.12:** Replace the last sentence of the paragraph by “Symbolically, this is denoted by $x_i \rightarrow x$.”
- (6/18/19) **Page 27, paragraph before Proposition 2.19:** Just before the last sentence of that paragraph, insert “(Continuity of the restriction of a function to an open subset is understood to be with respect to the topology described in Exercise 2.5.)”
- (3/8/16) **Page 27, last line:** Change two occurrences of x to y in the displayed equation, so it reads
- $$(f|_{V_x})^{-1}(U) = \{y \in V_x : f(y) \in U\} = f^{-1}(U) \cap V_x,$$
- (9/26/19) **Page 31, second paragraph below the section heading, second sentence:** Change “two points” to “two distinct points.”
- (6/24/19) **Page 32, just above Exercise 2.38:** Insert the following sentence: “In view of the preceding proposition, in a Hausdorff space we can write $p = \lim_{i \rightarrow \infty} p_i$ as an alternative notation for $p_i \rightarrow p$.”
- (5/17/12) **Page 37, three lines from the bottom:** Change “Exercise 2.49” to “Example 2.49.”
- (11/26/15) **Page 53, part (c) of the proposition continued from the previous page:** Insert another “if” after “if and only.”
- (7/14/18) **Page 58, second display:** Replace k by $k/2$ (twice) and l by $l/2$ (twice). [The tangent and cotangent functions have period π , not 2π .]
- (9/26/19) **Page 60, last paragraph:** In this paragraph and in the first one on page 61, change all subscript k 's to n 's (a total of four times).
- (9/26/19) **Page 61, Proposition 3.31, part (c):** Change X_k to X_n .
- (3/23/12) **Page 67, Example 3.52, second sentence:** Change this sentence to read “Let \sim be the equivalence relation on X such that $a_1 \sim a_2$ for all $a_1, a_2 \in A$ and $x \sim x$ for all other $x \in X$; the partition . . .”
- (5/17/12) **Page 67, Example 3.53, last line:** Change \mathbb{B}^n to \mathbb{B}^{n+1} .
- (7/17/19) **Page 70, Example 3.66, first paragraph, next-to-last line:** Change θ to $\frac{1}{2\pi}\theta$.
- (10/7/11) **Page 74, line 5:** Change this statement to read “As a set, $X \cup_f Y$ is the disjoint union . . .” [The topology on $X \cup_f Y$ is not the disjoint union topology.]
- (6/18/19) **Page 75, proof of Theorem 3.79, second paragraph:** Starting with the sentence beginning “By shrinking . . .,” replace the rest of the paragraph with the following: “By shrinking U and V if necessary, we may assume that $h(V \cap \partial N) = U \cap \partial M$. Then we can write the coordinate maps as $\varphi(q) = (\varphi_0(q), \varphi_1(q))$ and $\psi(q) = (\psi_0(q), \psi_1(q))$ for some continuous maps $\varphi_0: U \rightarrow \mathbb{R}^{n-1}$, $\varphi_1: U \rightarrow [0, \infty)$, $\psi_0: V \rightarrow \mathbb{R}^{n-1}$, $\psi_1: V \rightarrow [0, \infty)$. Our assumption that x and y are boundary points means that $\varphi_1(x) = \psi_1(y) = 0$, and there are open subsets $U_0, V_0 \subseteq \mathbb{R}^{n-1}$ such that $\varphi_0(U \cap \partial M) = U_0$, $\psi_0(V \cap \partial M) = V_0$. (Here we are again using the theorem on invariance of the boundary.) After replacing U and V by the preimages of $U_0 \times [0, \infty)$ and $V_0 \times [0, \infty)$, respectively, we can also assume that $\hat{U} \subseteq U_0 \times [0, \infty)$ and $\hat{V} \subseteq V_0 \times [0, \infty)$.”
- (6/18/19) **Page 76, display in the middle of the page:** Change y^n to y_n (twice).
- (7/16/11) **Page 76, last paragraph of the proof of Theorem 3.79:** In the second line of the paragraph, change “embedding of N ” to “embedding of M .” In the fourth line, change “embedding of M ” to “embedding of N .”
- (8/8/17) **Page 87, Exercise 4.3:** Insert “nonempty” before “connected.”
- (8/23/11) **Page 87, Exercise 4.4:** Insert another “if” after “if and only.”

- (8/23/11) **Page 88, proof of Proposition 4.9, fourth paragraph:** In the first sentence of that paragraph, change “open subsets of $\bigcup_{\alpha \in A} B_\alpha$ ” to “open subsets of X whose union contains $\bigcup_{\alpha \in A} B_\alpha$.”
- (8/23/11) **Page 97, line 10:** Change $B_{n_{\max}}(a)$ to $B_{n_{\max}}(x)$.
- (3/9/11) **Page 98, line 3 from bottom:** Change “this proposition” to “this lemma.”
- (11/11/13) **Page 104, proof of Proposition 4.60:** At the end of the first paragraph, change $B_{r(x)}(x) \subseteq \hat{U}_i$ to $B_{2r(x)}(x) \subseteq \hat{U}_i$. [Without this change, it might not be the case that \mathcal{B} covers M .]
- (8/23/11) **Page 106, line 3 from the bottom:** Change “countable union” to “countable intersection.”
- (8/23/11) **Page 109, statement of Lemma 4.74:** Insert another “if” after “if and only.”
- (7/19/15) **Page 110, next-to-last line:** Change M to X .
- (8/23/11) **Page 114, proof of Corollary 4.83:** This proof is incorrect. Replace it with the following: “Given a closed subset $A \subseteq X$ and a neighborhood U of A , Lemma 4.80 shows that there is a neighborhood V of A such that $\bar{V} \subseteq U$. By Urysohn’s lemma, there exists a continuous function $f: X \rightarrow [0, 1]$ such that $f \equiv 1$ on A and $f \equiv 0$ on $X \setminus V$. This function satisfies $\text{supp } f \subseteq \bar{V} \subseteq U$, so it is the bump function we seek.”
- (6/17/19) **Page 119, statement of Proposition 4.93(b):** Add the hypothesis that Y is Hausdorff.
- (10/24/19) **Page 119, proof of Proposition 4.93, second paragraph:** Change the first sentence to read “To prove (b), assume X is a second countable Hausdorff space and Y is Hausdorff, and suppose” Then replace the sentence beginning “Suppose on the contrary” by the following: “Suppose on the contrary that (x_i) is a sequence in L with no convergent subsequence in L . Because Y is Hausdorff, K is closed and therefore so is L , which means that (x_i) has no convergent subsequence in X .”
- (8/23/11) **Page 121, proof of Lemma 4.94:** Replace the last two sentences of the proof with the following: “Thus x lies in the closure of $A \cap K$ in K . Because $A \cap K$ is closed in K , it follows that $x \in A \cap K \subseteq A$.”
- (8/23/11) **Page 123, Problem 4-15(d):** Change “every connected neighborhood” to “every neighborhood.”
- (9/16/11) **Page 126, Problem 4-30:** Change $\{A_\alpha\}$ to $\{X_\alpha\}_{\alpha \in A}$.
- (1/20/11) **Page 133, proof of Proposition 5.7:** This should refer to Problem 5.8, not 5.7.
- (5/17/12) **Page 136, four lines below the displayed equations:** Change “both X'_{n-1} and X''_{n-1} are open” to “both X'_n and X''_n are open.”
- (1/20/11) **Page 137, statement of Lemma 5.13:** Change “discrete” to “closed and discrete.”
- (1/20/11) **Page 137, proof of Lemma 5.13, first paragraph:** In line 1, change “discrete” to “closed and discrete”; and in line 2, change “discrete subset” to “closed discrete subset.”
- (1/20/11) **Page 137, proof of Theorem 5.14, second paragraph:** Change “infinite discrete subset” to “infinite closed discrete subset.”
- (7/17/19) **Page 140, displayed formulas:** In both displayed formulas, change \mathbb{R} to $[0, 1]$.
- (9/16/11) **Page 141, line 5 from the bottom:** Change \tilde{U}_α^{n+1} to $\tilde{U}_{\alpha_i}^{n+1}$ (twice).
- (2/5/13) **Page 141, line 4 from the bottom:** Change “the minimum” to “one-half the minimum.”
- (2/5/13) **Page 141, line 3 from the bottom:** Change “supported in $\partial D_\gamma^{n+1}(\varepsilon/2)$ ” to “supported in $D_\gamma^{n+1} \setminus \partial D_\gamma^{n+1}(\varepsilon/2)$ ”
- (7/17/19) **Page 143, proof of Proposition 5.24, last paragraph:** Change $U \cap e_0$ to $U \cap \bar{e}_0$.
- (7/24/19) **Page 145, second paragraph:** Change e_n to e_k twice (once in the first line, and once in (5.1)).
- (7/22/19) **Page 146, Case 1, second paragraph:** Change Y_n to Y_{v_n} (twice).
- (3/24/11) **Page 156, Problem 5-4:** add the hypothesis that $\dim M > 1$.
- (5/27/17) **Page 158, second sentence:** Replace this sentence by “More generally, suppose K is a finite Euclidean simplicial complex and w is a point in \mathbb{R}^n such that each ray starting at w intersects $|K|$ in at most one point.”
- (11/7/19) **Page 165, Example 6.7:** After the second sentence, add “(The disks should be chosen so that their closures are disjoint.)”
- (9/16/11) **Page 172, first paragraph, next-to-last line:** Change $P'_1 \amalg Q$ to $P_1 \amalg Q$.

- (9/16/11) **Page 176, Fig. 6.21:** The label b near the lower right should be c , and the label w near the middle of the right-hand side should be x .
- (5/20/18) **Page 180, Proposition 6.20:** In the statement of the proposition, change “compact surface” to “connected compact surface.” Then in the second sentence of the proof, change both occurrences of “surface” to “connected compact surface.”
- (11/5/17) **Page 181, first full paragraph:** Replace the sentence starting with “However” by “However, we will prove in Chapter 10 that a compact surface cannot have both an oriented presentation and a nonoriented one.”
- (2/26/18) **Page 181, Problem 6-4:** Replace the first sentence by “Suppose M is a compact 2-manifold that contains a subset $B \subseteq M$ that is homeomorphic to the Möbius band, and whose interior is homeomorphic to the Möbius band minus its boundary.”
- (9/16/11) **Page 190, line 3 from the bottom:** Change $\Phi_g(f)$ to $\Phi_g[f]$.
- (1/20/11) **Page 193, proof of Proposition 7.16, second paragraph, line 2:** Change “ $H_1 = f$ ” to “ $H_1 = \tilde{f}$.”
- (8/3/18) **Page 201, Corollary 7.38:** This corollary should be moved after the statement of Theorem 7.40.
- (11/25/12) **Page 211, line 6:** Delete redundant “each.”
- (7/9/15) **Page 215, Problem 7-9:** Change “connected” to “path-connected.”
- (5/31/16) **Page 221, Theorem 8.4:** Remark: This theorem is true without the assumption that B is locally connected, and the proof is not really any more difficult; see, for example, the proof of Theorem 1.7 in [Hat02].
- (7/22/19) **Page 222, first paragraph:** Change $\{J_1, \dots, J_k\}$ to $\{J_1, \dots, J_m\}$.
- (1/20/11) **Page 224, two lines above the subheading:** Change $\tilde{f}_0(1)$ to $\tilde{f}_1(0)$.
- (1/20/11) **Page 228, displayed equations (8.4):** Replace these equations by
- $$\begin{aligned}\deg \varphi &= \deg(\rho_\varphi \circ \varphi)_*, \\ \deg \psi &= \deg(\rho_\psi \circ \psi)_*.\end{aligned}\tag{8.4}$$
- (7/13/15) **Page 230, Problem 8-5:** Replace the last sentence of the hint by the following: “Prove that $p_\varepsilon|_{\mathbb{S}^1}$ and $p_n(z) = z^n$ are homotopic as maps from \mathbb{S}^1 to $\mathbb{C} \setminus \{0\}$. If p has no zeros, use degree theory to derive a contradiction.”
- (7/13/15) **Page 231, Problem 8-10(c):** Change “index of V around the loop ω ” to “winding number of V around the loop ω .”
- (9/16/11) **Page 239, fourth line below the section heading:** Change “generated by G ” to “generated by S .”
- (7/28/16) **Page 244, fourth line below the section heading:** Change $n \in \mathbb{Z}$ to $n \in \mathbb{N}$.
- (7/28/16) **Page 247, Example 9.22, last line:** The formula for G_{tor} should be $G_{\text{tor}} = \{0\} \times \mathbb{Z}/k_1 \times \dots \times \mathbb{Z}/k_m$.
- (11/28/17) **Page 252, just above diagram (10.2):** Change “the following diagram commutes” to “the right half of the following diagram commutes.”
- (7/29/19) **Page 256, statement of Theorem 10.7:** Change “spaces” to “path-connected spaces.”
- (7/29/19) **Page 257, last paragraph, second sentence:** Change that sentence to read “If two or more edges are incident with the same two vertices, or if two or more self-loops are incident with the same vertex, they are called *multiple edges*.”
- (9/16/11) **Page 263, line 2:** Change $\tilde{U} \cap \tilde{V}$ to $q(D \setminus \{z\})$.
- (8/2/13) **Page 268, lines 2 & 3:** Change “preceding corollary” to “preceding theorem.”
- (11/5/17) **Page 268, statement of Corollary 10.24:** Change the statement to “A compact surface cannot have both an oriented presentation and a nonoriented one.”
- (5/23/11) **Page 269, line below equation (10.7):** Insert missing comma after “surjective.”
- (7/8/14) **Page 275, Problem 10-21(c):** Delete “with nonempty intersection.”
- (8/27/18) **Page 278, second line below the heading:** Before “disjoint union,” insert “nonempty.”
- (7/29/19) **Page 279, second line:** Change “Theorem 4.15” to “Proposition 4.13.”
- (5/31/16) **Page 282, Theorem 11.13:** Remark: This theorem, like Theorem 8.4, is true without the assumption that B is locally connected.
- (5/17/12) **Page 302, Problem 11-5, first line:** Change “dimension n ” to “dimension $n \geq 2$.”

- (11/6/19) **Page 303, Problem 11-9:** Add the hypothesis that the spaces are nonempty.
- (12/10/15) **Page 303, Problem 11-12(c):** Change “ $(1, 0)$ or $(-1, 0)$ ” to “ 1 or -1 ” [to be consistent with the complex notation used elsewhere for \mathbb{S}^1].
- (5/17/12) **Page 305, Problem 11-20:** At the end of the problem, add: “For the counterexample, you may use without proof the fact that \mathbb{S}^2 is not contractible. (This follows, for example, from Corollary 13.11 and Theorem 13.23.)”
- (2/25/18) **Page 312, last sentence of the paragraph after Exercise 12.13:** Allen Hatcher’s name is misspelled.
- (7/8/14) **Page 315, paragraph above the displayed diagram:** After “ Q is a normal covering map,” insert “and $\hat{H} = \text{Aut}_Q(E)$.”
- (7/8/14) **Page 315, just below the displayed diagram:** Replace the last two paragraphs on page 315 and the first (partial) paragraph on page 316 with the following:

We have to show that \hat{q} is a covering map. Let $x \in X$ be arbitrary, and let U be a neighborhood of x that is evenly covered by q . We will show that U is also evenly covered by \hat{q} . Given a component U_i of $q^{-1}(U)$, let $\hat{U}_i = Q(U_i) \subseteq \hat{E}$; then \hat{U}_i is connected, and it is open in \hat{E} because Q is an open map (Proposition 11.1). Suppose $\hat{U}_i = Q(U_i)$ and $\hat{U}_j = Q(U_j)$ are any two such sets. If they have a point \hat{e} in common, then $\hat{e} = Q(e_i) = Q(e_j)$ for some $e_i \in U_i$ and $e_j \in U_j$. Since Q identifies points of E if and only if they are in the same \hat{H} -orbit, there is some $\varphi \in \hat{H}$ such that $e_j = \varphi(e_i)$. Then $\varphi(U_i) = U_j$ by Proposition 12.1(c), so $Q(U_i) = Q \circ \varphi(U_i) = Q(U_j)$. This shows that any such sets \hat{U}_i, \hat{U}_j are either disjoint or equal. Since Q is surjective, $\hat{q}^{-1}(U)$ is equal to the disjoint union of the connected open sets \hat{U}_i as U_i ranges over the components of $q^{-1}(U)$.

It remains only to show that for any such set \hat{U}_i , the restricted map $\hat{q}: \hat{U}_i \rightarrow U$ is a homeomorphism. The following diagram commutes:

$$\begin{array}{ccc} U_i & & \\ & \searrow Q & \\ q \downarrow & & \hat{U}_i \\ & \nearrow \hat{q} & \\ U & & \end{array} \quad (12.3)$$

Since $q = \hat{q} \circ Q$ is injective on U_i , so is Q ; and $Q: U_i \rightarrow \hat{U}_i$ is surjective by definition. Because Q is an open map, it follows that $Q: U_i \rightarrow \hat{U}_i$ is a homeomorphism. Since q and Q are homeomorphisms in (12.3), so is \hat{q} .

- (9/27/11) **Page 318, statement of Proposition 12.21, second line:** Insert “on” after “acting.”
- (9/23/14) **Page 321, line 4:** Change $E \times E$ to E .
- (9/27/11) **Page 329, paragraph just below the diagram:** Change every occurrence of \tilde{p} to \tilde{q} (five times).
- (9/27/11) **Page 330, just below the bulleted list:** Change \tilde{p} to \tilde{q} .
- (9/27/11) **Page 332, first full paragraph, second line:** Change \tilde{p} to \tilde{q} .
- (9/27/11) **Page 332, second full paragraph, lines 6 and 7:** Change \tilde{p} to \tilde{q} (twice).
- (9/16/14) **Page 335, Problem 12-10:** Interchange the definitions of G and H in the sixth and seventh lines. (Otherwise, part (c) is false as stated.)
- (10/12/14) **Page 337, Problem 12-19:** Replace the first sentence of the problem with the following: “Suppose we are given a continuous action of a metrizable topological group (e.g., a discrete group) G on a first countable Hausdorff space E .”
- (7/22/19) **Page 349, line 3:** Change Δ_p to Δ_{p+1} .
- (9/27/11) **Page 352, lines 3 and 4:** Change c_p to c_q (twice), and change p to q (twice).
- (7/22/19) **Page 352, next-to-last line:** Change c_p to c_q (twice), and change p to q (once).
- (12/15/17) **Page 354, paragraph above the last display:** Insert “of some reparametrization” after “extension of the circle representative.”

- (7/22/19) **Page 360, proof of Lemma 13.20:** In the second line of the displayed equation, change $F_{i,p}$ to $F_{i,p+1}$.
- (7/22/19) **Page 361, first line of text:** Change “ $\in \mathbb{R}^n$ ” to “ $\subseteq \mathbb{R}^n$.”
- (10/8/15) **Page 370, line 5 from the bottom:** Change “It follows . . .” to “Assuming X is path-connected, it follows . . .”
- (10/8/15) **Page 371, at the end of the first (partial) paragraph:** Insert “If X is not path-connected, just apply this argument to the path component containing the image of φ , and use Proposition 13.5.”
- (9/26/17) **Page 371, statement of Theorem 13.34(e):** Change “dimension n ” to “dimension $n \geq 2$,” and change “the zero map” to “not injective.”
- (9/26/17) **Page 372, proof of Theorem 13.34, last paragraph:** Change “if $\varphi_* = 0$ ” to “if φ_* is injective.”
- (9/26/17) **Page 372, Example 13.35(b), last line:** Change “the zero map” to “noninjective.”
- (9/29/17) **Page 372, Example 13.35(c):** Replace the last sentence by “The image of φ_* is the infinite cyclic group generated by $\gamma(\alpha_1^2 \dots \alpha_n^2)$, so φ_* is injective and $H_2(M) = 0$.”
- (9/26/19) **Page 399, next-to-last line:** Change $x \in X$ to $x \in M_1$.
- (12/26/18) **Page 401, line 4 from the bottom:** Change “subset” to “nonempty subset.”
- (10/7/19) **Page 402, Exercise C.1:** Change “any subset” to “any nonempty subset.”
- (6/6/18) **Page 411, near the middle of the page:** The index entry for \bar{R} should read “(normal closure of a subset).”
- (2/25/18) **Page 422:** The index entry for “Hatcher, Allen” is misspelled.