CORRECTIONS TO
Introduction to Topological Manifolds
(Second Edition)
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(2/25/18) Page xii, last paragraph: Allen Hatcher’s name is misspelled. (Sorry, Allen.)
(2/14/15) Page 23, Exercise 2.6, first line: Change “collection of topologies” to “nonempty collection of topologies.”
(6/24/19) Page 26, just above Exercise 2.12: Replace the last sentence of the paragraph by “Symbolically, this is denoted by $x_i \to x$.”
(6/18/19) Page 27, paragraph before Proposition 2.19: Just before the last sentence of that paragraph, insert “(Continuity of the restriction of a function to an open subset is understood to be with respect to the topology described in Exercise 2.5.)”
(3/8/16) Page 27, last line: Change “collection of topologies” to “nonempty collection of topologies.”

(11/20/19) Page 66, after the second line: Insert the following sentence: “In view of the preceding proposition, in a Hausdorff space we can write $p = \lim_{i \to \infty} p_i$ as an alternative notation for $p_i \to p$.”
(5/17/12) Page 37, three lines from the bottom: Change Exercise 2.49 to “Example 2.49.”
(11/26/15) Page 53, part (c) of the proposition continued from the previous page: Insert another “if” after “if and only.”
(7/14/18) Page 58, second display: Replace $k$ by $k/2$ (twice) and $l$ by $l/2$ (twice). [The tangent and cotangent functions have period $\pi$, not $2\pi$.]
(9/26/19) Page 60, last paragraph: In this paragraph and in the first one on page 61, change all subscript $k$’s to $n$’s (a total of four times).

(9/26/19) Page 61, Proposition 3.31, part (c): Change $X_k$ to $X_n$.
(11/20/19) Page 66, after the second line: Add the following sentence: “We sometimes also say informally that $Y$ is a quotient space of $X$ when $Y$ is a topological space that has the quotient topology with respect to some continuous map from $X$ to $Y$.”

(3/23/12) Page 67, Example 3.52, second sentence: Change this sentence to read “Let $\sim$ be the equivalence relation on $X$ such that $a_1 \sim a_2$ for all $a_1, a_2 \in A$ and $x \sim x$ for all other $x \in X$; the partition . . . .”
(5/17/12) Page 67, Example 3.53, last line: Change $\mathbb{B}^n$ to $\mathbb{B}^{n+1}$.
(7/17/19) Page 70, Example 3.66, first paragraph, next-to-last line: Change $\theta$ to $\frac{1}{2\pi} \theta$.
(10/7/11) Page 74, line 5: Change this statement to read “As a set, $X \cup_f Y$ is the disjoint union . . . .” [The topology on $X \cup_f Y$ is not the disjoint union topology.]

(11/30/19) Page 75, proof of Theorem 3.79, second line: Change $q(\partial M \cup \partial N)$ to $q(\partial M \sqcup \partial N)$.

(6/4/21) Page 75, proof of Theorem 3.79, second paragraph: Replace that paragraph with the following: “Suppose $s \in S$, and let $y_0 \in \partial N$ and $x_0 = h(y_0) \in \partial M$ be the two points in the fiber $q^{-1}(s)$. We can choose coordinate charts $(U, \varphi)$ for $M$ and $(V, \psi)$ for $N$ such that $x_0 \in U$ and $y_0 \in V$, and let $\tilde{U} = \varphi(U)$, $\tilde{V} = \psi(V) \subseteq \mathbb{H}^n$ (Fig. 3.13). It is useful in this proof to identify $\mathbb{H}^n$ with $\mathbb{R}^{n-1} \times [0, \infty)$ and $\mathbb{R}^n$ with $\mathbb{R}^{n-1} \times \mathbb{R}$. By shrinking $U$ and $V$ if necessary, we may assume that $h(V \cap \partial N) = U \cap \partial M$. Then we can write the coordinate maps as $\varphi(p) = (\varphi_0(p), \varphi_1(p))$ and $\psi(p) = (\psi_0(p), \psi_1(p))$ for some continuous maps $\varphi_0: U \to \mathbb{R}^{n-1}$, $\varphi_1: U \to [0, \infty)$, $\psi_0: V \to \mathbb{R}^{n-1}$, $\psi_1: V \to [0, \infty)$. Our assumption that $x_0$ and $y_0$ are boundary points means that $\varphi_1(x_0) = \psi_1(y_0) = 0$, and there are open subsets $U_0, V_0 \subseteq \mathbb{R}^{n-1}$ such that $\varphi_0(U \cap \partial M) = U_0$, $\psi_0(V \cap \partial N) = V_0$. (Here we are again using the theorem on invariance of the boundary.) After replacing $U$ and $V$ by the preimages
of \( U_0 \times [0, \infty) \) and \( V_0 \times [0, \infty) \), respectively, we can also assume that \( \tilde{U} \subseteq U_0 \times [0, \infty) \) and \( \tilde{V} \subseteq V_0 \times [0, \infty) \).”

(6/18/19) **Page 76, display in the middle of the page:** Change \( y^n \) to \( y_n \) (twice).

(7/16/11) **Page 76, last paragraph of the proof of Theorem 3.79:** In the second line of the paragraph, change “embedding of \( N \)” to “embedding of \( M \).” In the fourth line, change “embedding of \( M \)” to “embedding of \( N \).”

(8/8/17) **Page 87, Exercise 4.3:** Insert “nonempty” before “connected.”

(8/23/11) **Page 87, Exercise 4.4:** Insert another “if” after “if and only.”

(8/23/11) **Page 88, proof of Proposition 4.9, fourth paragraph:** In the first sentence of that paragraph, change “open subsets of \( \bigcup_{\alpha \in A} B_\alpha \)” to “open subsets of \( X \) whose union contains \( \bigcup_{\alpha \in A} B_\alpha \).”

(8/23/11) **Page 97, line 10:** Change \( B_{n_{\text{max}}} (a) \) to \( B_{n_{\text{max}}} (x) \).

(3/9/11) **Page 98, line 3:** Change “this proposition” to “this lemma.”

(6/21/20) **Page 99, second paragraph:** Delete the second sentence of the paragraph. [It’s not wrong; it’s just not needed.]

(6/21/20) **Page 103, just above Exercise 4.58:** After the word “illustrates,” insert “(using the theorem on invariance of the boundary).”

(6/19/21) **Page 104, proof of Proposition 4.60:** Delete the last sentence in the first paragraph, and in the second paragraph, replace the phrase “\( r \) is any positive rational number strictly less than \( r(x) \)” by “\( r \) is any positive rational number such that \( B_{2r}(x) \subseteq \bar{U}_i \)”

(6/19/21) **Page 106, proof of Theorem 4.68, last paragraph:** After the first sentence of the paragraph, insert: “Begin by setting \( W_0 = U \).” Then in the third and fourth lines of the paragraph, replace “Choosing \( r_n < \min (\varepsilon_n, 1/n) \)” by “Choosing \( r_n < \min (\varepsilon_n, 1/n) \) and setting \( W_n = B_{r_n} (x_n) \).”

(8/23/11) **Page 106, line 3 from the bottom:** Change “countable union” to “countable intersection.”

(8/23/11) **Page 109, statement of Lemma 4.74:** Insert another “if” after “if and only.”

(7/19/15) **Page 110, next-to-last line:** Change \( M \) to \( X \).

(8/23/11) **Page 114, proof of Corollary 4.83:** This proof is incorrect. Replace it with the following: “Given a closed subset \( A \subseteq X \) and a neighborhood \( U \) of \( A \), Lemma 4.80 shows that there is a neighborhood \( V \) of \( A \) such that \( V \subseteq U \). By Urysohn’s lemma, there exists a continuous function \( f : X \to [0, 1] \) such that \( f \equiv 1 \) on \( A \) and \( f \equiv 0 \) on \( X \setminus V \). This function satisfies \( \text{supp} f \subseteq \bar{V} \subseteq \bar{U} \), so it is the bump function we seek.”

(6/17/19) **Page 119, statement of Proposition 4.93(b):** Add the hypothesis that \( Y \) is Hausdorff.

(10/24/19) **Page 119, proof of Proposition 4.93, second paragraph:** Change the first sentence to read “To prove (b), assume \( X \) is a second countable Hausdorff space and \( Y \) is Hausdorff, and suppose . . . .” Then replace the sentence beginning “Suppose on the contrary that . . . .”

(8/23/11) **Page 121, proof of Lemma 4.94:** Replace the last two sentences of the proof with the following: “Thus \( x \) lies in the closure of \( A \cap K \) in \( K \). Because \( A \cap K \) is closed in \( K \), it follows that \( x \in A \cap K \subseteq A \).”

(8/23/11) **Page 123, Problem 4-15(d):** Change “every connected neighborhood” to “every neighborhood.”

(9/16/11) **Page 126, Problem 4-30:** Change \( \{A_\alpha\} \) to \( \{X_\alpha\}_{\alpha \in A} \).

(4/12/20) **Page 126, Problem 4-31(c):** In the last sentence, change “every element of \( U \)” to “every nonempty element of \( U \).”

(4/12/20) **Page 128, proof of Prop. 5.1:** Insert before the first sentence of the proof: “The proposition is true by definition when \( n = 0 \), so assume that \( n > 0 \).”

(3/20/21) **Page 133, line above Theorem 5.6:** Change “an \( n \)-dimensional subcomplex” to “a subcomplex of dimension at most \( n \).”

(1/20/11) **Page 133, proof of Proposition 5.7:** This should refer to Problem 5-8, not 5-7.

(5/17/12) **Page 136, four lines below the displayed equations:** Change “both \( X'_{n-1} \) and \( X''_{n-1} \) are open” to “both \( X'_n \) and \( X''_n \) are open.”
(1/20/11) Page 137, statement of Lemma 5.13: Change “discrete” to “closed and discrete.”

(1/20/11) Page 137, proof of Lemma 5.13, first paragraph: In line 1, change “discrete” to “closed and discrete”; and in line 2, change “discrete subset” to “closed discrete subset.”

(1/20/11) Page 137, proof of Theorem 5.14, second paragraph: Change “infinite discrete subset” to “infinite closed discrete subset.”

(7/17/19) Page 140, displayed formulas: In both displayed formulas, change \( \mathbb{R} \) to \([0, 1]\).

(4/12/20) Page 141, just above the displayed equation: In the line above the display and in the display itself, change \( A \) to \( B \) (four times), to avoid conflict with the use of \( A \) as the index set for the open cover.

(4/12/20) Page 141, displayed equation: Change \( D^n_{\alpha} \) to \( D^n_{\alpha} - \{0\} \).

(9/16/11) Page 141, line 5 from the bottom: Change \( U_{\alpha} \) to \( U_{\alpha_i} \) (twice).

(2/5/13) Page 141, line 4 from the bottom: Change “the minimum” to “one-half the minimum.”

(2/5/13) Page 141, line 3 from the bottom: Change “supported in \( \partial D^n_{\alpha}(\epsilon/2) \)” to “supported in \( D^n_{\alpha} \setminus \partial D^n_{\alpha}(\epsilon/2) \)”

(7/17/19) Page 143, proof of Proposition 5.24, last paragraph: Change \( U \cap e_0 \) to \( U \cap \bar{e}_0 \).

(10/16/20) Page 144, three lines above Lemma 5.26: Change “the finite subcomplex \( \mathcal{E}_n \)” to “the finite subcomplex \( \mathcal{E}_n \).”

(7/24/19) Page 145, second paragraph: Change \( e_n \) to \( e_k \) twice (once in the first line, and once in (5.1)).

(7/22/19) Page 146, Case 1, second paragraph: Change \( Y_n \) to \( Y_{n_i} \) twice.

(4/12/20) Page 152, sentence after the proof of Prop. 5.38: Change \( i = 1, \ldots, k \) to \( i = 0, \ldots, k \).

(3/24/18) Page 156, Problem 5-4: replace the first sentence by “Suppose \( M \) contains a subset \( \tilde{M} \) homeomorphic to the M"obius band, and whose interior is homeomorphic to the M"obius band minus its boundary.”

(2/26/18) Page 181, Problem 6-4: Replace the first sentence by “Suppose \( M \) is a compact 2-manifold that contains a subset \( B \subseteq M \) that is homeomorphic to the M"obius band, and whose interior is homeomorphic to the M"obius band minus its boundary.”

(9/16/11) Page 190, line 3 from the bottom: Change \( \Phi_g(f) \) to \( \Phi_g[f] \).

(1/20/11) Page 193, proof of Proposition 7.16, second paragraph, line 2: Change “\( H_1 = f \)” to “\( H_1 = \tilde{f} \)”.

(12/8/21) Page 195, displayed equation: Replace the last “\( \leq \)” sign by “\( \leq \)”.

(8/3/18) Page 201, Corollary 7.38: This corollary should be moved after the statement of Theorem 7.40.

(11/25/12) Page 211, line 6: Delete redundant “each.”

(7/9/15) Page 215, Problem 7-9: Change “connected” to “path-connected.”

(5/31/16) Page 221, Theorem 8.4: Remark: This theorem is true without the assumption that \( B \) is locally connected, and the proof is not really any more difficult; see, for example, the proof of Theorem 1.7 in [Hat02].

(7/22/19) Page 222, first paragraph: Change \( \{J_1, \ldots, J_k\} \) to \( \{J_1, \ldots, J_m\} \).
(1/20/11) **Page 224, two lines above the subheading:** Change \( f_0(1) \) to \( f_1(0) \).

(1/20/11) **Page 228, displayed equations (8.4):** Replace these equations by

\[
\deg \varphi = \deg(\rho_\varphi \circ \varphi),
\]

\[
\deg \psi = \deg(\rho_\psi \circ \psi),
\]

(8.4)

(7/13/15) **Page 230, Problem 8-5:** Replace the last sentence of the hint by the following: “Prove that \( p_n(z) = z^n \) are homotopic as maps from \( S^1 \) to \( \mathbb{C} \setminus \{0\} \). If \( p \) has no zeros, use degree theory to derive a contradiction.”

(7/13/15) **Page 231, Problem 8-10(c):** Change “index of \( V \) around the loop \( \omega \)” to “winding number of \( V \) around the loop \( \omega \).”

(9/16/11) **Page 239, fourth line below the section heading:** Change “generated by \( G \)” to “generated by \( S \).”

(11/29/19) **Page 241, middle of the page:** Change the definition of group presentation as follows: “We define a group presentation to be an ordered pair, denoted by \( (S|R) \), where \( S \) is an arbitrary set and \( R \) is a set of words formed from the elements of \( S \).”

(11/29/19) **Page 241, just below the last displayed equation:** Replace “where \( \bar{R} \) is the normal closure of \( R \) in \( F(S) \)” by “where now we interpret \( R \) as a set of elements of the free group \( F(S) \), and \( \bar{R} \) is the normal closure of \( R \) in \( F(S) \)”.

(7/28/16) **Page 244, fourth line below the section heading:** Change \( n \in \mathbb{Z} \) to \( n \in \mathbb{N} \).

(7/28/16) **Page 247, Example 9.22, last line:** The formula for \( G_{\text{tor}} \) should be \( G_{\text{tor}} = \{0\} \times \mathbb{Z} / k_1 \times \cdots \times \mathbb{Z} / k_m \).

(12/3/19) **Page 249, Problem 9-4(b):** Change “a subset of the free group \( F(S_i) \)” to “a set of words in the elements of \( S_i \).”

(12/3/19) **Page 249, Problem 9-5:** Change “subsets of the free group \( F(S) \)” to “sets of words in the elements of \( S \).”

(11/28/17) **Page 252, just above diagram (10.2):** Change “the following diagram commutes” to “the right half of the following diagram commutes.”

(7/29/19) **Page 256, statement of Theorem 10.7:** Change “spaces” to “path-connected spaces.”

(12/1/20) **Page 257, Example 10.8:** In the first line of the example, and in the three lines immediately above it, change “proposition” to “theorem” (three times).

(7/29/19) **Page 257, last paragraph, second sentence:** Change that sentence to read “If two or more edges are incident with the same two vertices, or if two or more self-loops are incident with the same vertex, they are called multiple edges.”

(12/7/20) **Page 260, proof of Theorem 10.12, second paragraph, first line:** Change \( \Gamma \) to \( \pi_1(\Gamma, v) \).

(12/7/20) **Page 261, last paragraph of the proof, fourth line from the bottom:** Change two equalities to isomorphisms: \( \pi_1(V,v) \cong F([f_1], \ldots, [f_n]) \) and \( \pi_1(U,v) \cong F([f_1 + 1]) \).

(9/16/11) **Page 263, line 2:** Change \( \bar{U} \cap \bar{V} \) to \( q(D \setminus \{z\}) \).

(8/2/13) **Page 268, lines 2 & 3:** Change “preceding corollary” to “preceding theorem.”

(11/5/17) **Page 268, statement of Corollary 10.24:** Change the statement to “A compact surface cannot have both an oriented presentation and a nonoriented one.”

(5/23/11) **Page 269, line below equation (10.7):** Insert missing comma after “surjective.”

(10/3/20) **Page 271, line 3:** Replace the phrase “the endpoints of the paths \( a_i \) in this product are of the form \( i/n \)” by “the paths \( a_i \) in this product are defined on subintervals whose endpoints are integral multiples of \( 1/n \).”

(7/8/14) **Page 275, Problem 10-21(c):** Delete “with nonempty intersection.”

(8/27/18) **Page 278, second line below the heading:** Before “disjoint union,” insert “nonempty.”

(7/29/19) **Page 279, second line:** Change “Theorem 4.15” to “Proposition 4.13.”

(5/31/16) **Page 282, Theorem 11.13:** Remark: This theorem, like Theorem 8.4, is true without the assumption that \( B \) is locally connected.

(5/17/12) **Page 302, Problem 11-5, first line:** Change “dimension \( n \)” to “dimension \( n \geq 2 \).”

(11/6/19) **Page 303, Problem 11-9:** Add the hypothesis that the spaces are nonempty.
Page 303, Problem 11-12(c): Change “(1, 0) or (−1, 0)” to “1 or −1” [to be consistent with the complex notation used elsewhere for $S^1$].

Page 305, Problem 11-20: At the end of the problem, add: “For the counterexample, you may use without proof the fact that $S^2$ is not contractible. (This follows, for example, from Corollary 13.11 and Theorem 13.23.)”

Page 312, last sentence of the paragraph after Exercise 12.13: Allen Hatcher’s name is misspelled.

Page 315, paragraph above the displayed diagram: After “$Q$ is a normal covering map,” insert “and $\tilde{H} = \text{Aut}_Q(E)$.”

Page 315, just below the displayed diagram: Replace the last two paragraphs on page 315 and the first (partial) paragraph on page 316 with the following:

We have to show that $\tilde{q}$ is a covering map. Let $x \in X$ be arbitrary, and let $U$ be a neighborhood of $x$ that is evenly covered by $q$. We will show that $U$ is also evenly covered by $\tilde{q}$. Given a component $U_i$ of $q^{-1}(U)$, let $\tilde{U}_i = Q(U_i) \subseteq \tilde{E}$; then $\tilde{U}_i$ is connected, and it is open in $\tilde{E}$ because $Q$ is an open map (Proposition 11.1). Suppose $\tilde{U}_i = Q(U_i) = \tilde{U}_j = Q(U_j)$ are any two such sets. If they have a point $\tilde{e}$ in common, then $\tilde{e} = Q(e_i) = Q(e_j)$ for some $e_i \in U_i$ and $e_j \in U_j$. Since $Q$ identifies points of $E$ if and only if they are in the same $\tilde{H}$-orbit, there is some $\phi \in \tilde{H}$ such that $e_j = \phi(e_i)$. Then $\phi(U_i) = U_j$ by Proposition 12.1(c), so $Q(U_i) = Q \circ \phi(U_i) = Q(U_j)$. This shows that any such sets $\tilde{U}_i, \tilde{U}_j$ are either disjoint or equal. Since $Q$ is surjective, $\tilde{q}^{-1}(U)$ is equal to the disjoint union of the connected open sets $\tilde{U}_i$ as $U_i$ ranges over the components of $q^{-1}(U)$.

It remains only to show that for any such set $\tilde{U}_i$, the restricted map $\tilde{q}: \tilde{U}_i \to U$ is a homeomorphism. The following diagram commutes:

$$
\begin{array}{c}
U_i \\
q \downarrow \\
\tilde{u}_i \\
\tilde{q}
\end{array}
\begin{array}{c}
Q \\
\downarrow \\
\tilde{U}_i
\end{array}
\begin{array}{c}
U
\end{array}
\tag{12.3}
$$

Since $q = \tilde{q} \circ Q$ is injective on $U_i$, so is $Q$; and $Q: U_i \to \tilde{U}_i$ is surjective by definition. Because $Q$ is an open map, it follows that $Q: U_i \to \tilde{U}_i$ is a homeomorphism. Since $q$ and $Q$ are homeomorphisms in (12.3), so is $\tilde{q}$.

Page 318, statement of Proposition 12.21, second line: Insert “on” after “acting.”

Page 320, paragraph after the proof of Prop. 12.24, first line: Before “locally,” insert “nonempty.”

Page 321, line 4: Change $E \times E$ to $E$.

Page 329, paragraph just below the diagram: Change every occurrence of $\tilde{p}$ to $\tilde{q}$ (five times).

Page 330, just below the bulleted list: Change $\tilde{p}$ to $\tilde{q}$.

Page 332, first full paragraph, second line: Change $\tilde{p}$ to $\tilde{q}$.

Page 332, second full paragraph, lines 6 and 7: Change $\tilde{p}$ to $\tilde{q}$ (twice).

Page 335, Problem 12-10: Interchange the definitions of $G$ and $H$ in the sixth and seventh lines. (Otherwise, part (c) is false as stated.)

Page 337, Problem 12-19: Replace the first sentence of the problem with the following: “Suppose we are given a continuous action of a metrizable topological group (e.g., a discrete group) $G$ on a first countable Hausdorff space $E$.”

Page 349, line 3: Change $\Delta_p$ to $\Delta_{p+1}$.

Page 352, lines 3 and 4: Change $c_p$ to $c_q$ (twice), and change $p$ to $q$ (twice).

Page 352, next-to-last line: Change $c_p$ to $c_q$ (twice), and change $p$ to $q$ (once).

Page 354, paragraph above the last display: Insert “of some reparametrization” after “extension of the circle representative.”
(3/16/21) **Page 355, commutative diagram near the bottom of the page:** Change the period after \( X \) to a comma.

(7/22/19) **Page 360, proof of Lemma 13.20:** In the second line of the displayed equation, change \( F_{i,p} \) to \( F_{i,p+1} \).

(7/22/19) **Page 361, first line of text:** Change “\( \in \mathbb{R}^n \)” to “\( \subseteq \mathbb{R}^n \)”.

(4/1/21) **Page 369, line above Proposition 13.33:** Delete spurious “and.”

(10/8/15) **Page 370, line 5 from the bottom:** Change “It follows . . .” to “Assuming \( X \) is path-connected, it follows . . .”

(10/8/15) **Page 371, at the end of the first (partial) paragraph:** Insert “If \( X \) is not path-connected, just apply this argument to the path component containing the image of \( \varphi \), and use Proposition 13.5.”

(9/26/17) **Page 371, statement of Theorem 13.33(e):** Change “dimension \( n \)” to “dimension \( n \geq 2 \)” and change “the zero map” to “not injective.”

(4/1/21) **Pages 371–372, proof of Theorem 13.34:** Change “Theorem 13.33” to “Proposition 3.33” (five times).

(9/26/17) **Page 372, proof of Theorem 13.34, last paragraph:** Change “if \( \varphi_* = 0 \)” to “if \( \varphi_* \) is injective.”

(9/26/17) **Page 372, Example 13.35(b), last line:** Change “the zero map” to “noninjective.”

(9/29/17) **Page 372, Example 13.35(c):** Replace the last sentence by “The image of \( \varphi_* \) is the infinite cyclic group generated by \( \gamma(\alpha_1^2 \ldots \alpha_n^2) \), so \( \varphi_* \) is injective and \( H_2(M) = 0 \).”

(9/26/19) **Page 399, next-to-last line:** Change \( x \in X \) to \( x \in M_1 \).

(12/26/18) **Page 401, line 4 from the bottom:** Change “subset” to “nonempty subset.”

(10/7/19) **Page 402, Exercise C.1:** Change “any subset” to “any nonempty subset.”

(6/6/18) **Page 411, near the middle of the page:** The index entry for \( \bar{R} \) should read “(normal closure of a subset).”

(2/25/18) **Page 422:** The index entry for “Hatcher, Allen” is misspelled.