CORRECTIONS TO
Introduction to Topological Manifolds
(Second Edition)

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UPDATED DECEMBER 26, 2018

(2/25/18) **Page xii, last paragraph:** Allen Hatcher's name is misspelled. (Sorry, Allen.)

(2/14/15) **Page 23, Exercise 2.6, first line:** Change "collection of topologies" to "nonempty collection of topologies."

(5/31/16) **Page 27, paragraph before Proposition 2.19:** Just before the last sentence of that paragraph, insert "(Continuity of the restriction of a function to an open subset is understood to be with respect to the topology described in Example 2.5.)"

(3/8/16) **Page 27, last line:** Change two occurrences of x to y in the displayed equation, so it reads

$$\left(f|_{V_x}\right)^{-1}(U) = \{y \in V_x : f(y) \in U\} = f^{-1}(U) \cap V_x,$$

(5/17/12) **Page 37, three lines from the bottom:** Change "Exercise 2.49" to "Example 2.49."

(11/26/15) **Page 53, part (c) of the proposition continued from the previous page:** Insert another "if" after "if and only."

(7/14/18) **Page 58, second display:** Replace k by $k/2$ (twice) and l by $l/2$ (twice). [The tangent and cotangent functions have period π, not 2π.]

(3/23/12) **Page 67, Example 3.52, second sentence:** Change this sentence to read "Let $\sim$ be the equivalence relation on X such that $a_1 \sim a_2$ for all $a_1, a_2 \in A$ and $x \sim x$ for all other $x \in X$; the partition ...."

(5/17/12) **Page 67, Example 3.53, last line:** Change $\mathbb{B}^n$ to $\overline{\mathbb{B}}^{n+1}$.

(10/7/11) **Page 74, line 5:** Change this statement to read "*As a set, $X \cup_f Y$ is the disjoint union* ...." [The topology on $X \cup_f Y$ is not the disjoint union topology.]

(7/16/11) **Page 76, last paragraph of the proof of Theorem 3.79:** In the second line of the paragraph, change "embedding of N" to "embedding of M." In the fourth line, change "embedding of M" to "embedding of N."

(8/8/17) **Page 87, Exercise 4.3:** Insert "nonempty" before "connected."

(8/23/11) **Page 87, Exercise 4.4:** Insert another "if" after "if and only."

(8/23/11) **Page 88, proof of Proposition 4.9, fourth paragraph:** In the first sentence of that paragraph, change "open subsets of $\bigcup_{\alpha\in A} B_\alpha$" to "open subsets of X whose union contains $\bigcup_{\alpha\in A} B_\alpha$."

(8/23/11) **Page 97, line 10:** Change $B_{n_{\max}}(a)$ to $B_{n_{\max}}(x)$.

(3/9/11) **Page 98, line 3 from bottom:** Change "this proposition" to "this lemma."

(11/11/13) **Page 104, proof of Proposition 4.60:** At the end of the first paragraph, change $B_{r(x)}(x) \subseteq \widehat{U}_i$ to $B_{2r(x)}(x) \subseteq \widehat{U}_i$. [Without this change, it might not be the case that $\mathcal{B}$ covers M.]

(8/23/11) **Page 106, line 3 from the bottom:** Change "countable union" to "countable intersection."

(8/23/11) **Page 109, statement of Lemma 4.74:** Insert another "if" after "if and only."

(7/19/15) **Page 110, next-to-last line:** Change M to X.

(8/23/11) **Page 114, proof of Corollary 4.83:** This proof is incorrect. Replace it with the following: "Given a closed subset $A \subseteq X$ and a neighborhood U of A, Lemma 4.80 shows that there is a neighborhood V of A such that $\overline{V} \subseteq U$. By Urysohn's lemma, there exists a continuous function $f: X \to [0,1]$ such that $f \equiv 1$ on A and $f \equiv 0$ on $X \smallsetminus V$. This function satisfies $\operatorname{supp} f \subseteq \overline{V} \subseteq U$, so it is the bump function we seek."

(8/23/11) **Page 121, proof of Lemma 4.94:** Replace the last two sentences of the proof with the following: "Thus x lies in the closure of $A \cap K$ in K. Because $A \cap K$ is closed in K, it follows that $x \in A \cap K \subseteq A$."

(8/23/11) **Page 123, Problem 4-15(d):** Change "every connected neighborhood" to "every neighborhood."

(9/16/11) **Page 126, Problem 4-30:** Change $\{A_\alpha\}$ to $\{X_\alpha\}_{\alpha\in A}$.

(1/20/11) **Page 133, proof of Proposition 5.7:** This should refer to Problem 5.8, not 5.7.

(5/17/12) **Page 136, four lines below the displayed equations:** Change "both X'_{n-1} and X''_{n-1} are open" to "both X'_n and X''_n are open."

(1/20/11) **Page 137, statement of Lemma 5.13:** Change "*discrete*" to "*closed and discrete*."

(1/20/11) **Page 137, proof of Lemma 5.13, first paragraph:** In line 1, change "discrete" to "closed and discrete"; and in line 2, change "discrete subset" to "closed discrete subset."

(1/20/11) **Page 137, proof of Theorem 5.14, second paragraph:** Change "infinite discrete subset" to "infinite closed discrete subset."

(9/16/11) **Page 141, line 5 from the bottom:** Change $\widetilde{U}^{n+1}_\alpha$ to $\widetilde{U}^{n+1}_{\alpha_i}$ (twice).

(2/5/13) **Page 141, line 4 from the bottom:** Change "the minimum" to "one-half the minimum."

(2/5/13) **Page 141, line 3 from the bottom:** Change "supported in $\partial D^{n+1}_\gamma(\varepsilon/2)$" to "supported in $D^{n+1}_\gamma \smallsetminus \partial D^{n+1}_\gamma(\varepsilon/2)$"

(3/24/11) **Page 156, Problem 5-4:** add the hypothesis that $\dim M > 1$.

(5/27/17) **Page 158, second sentence:** Replace this sentence by "More generally, suppose K is a finite Euclidean simplicial complex and w is a point in $\mathbb{R}^n$ such that each ray starting at w intersects $|K|$ in at most one point."

(9/16/11) **Page 172, first paragraph, next-to-last line:** Change $P'_1 \amalg Q$ to $P_1 \amalg Q$.

(9/16/11) **Page 176, Fig. 6.21:** The label b near the lower right should be c, and the label w near the middle of the right-hand side should be x.

(5/20/18) **Page 180, Proposition 6.20:** In the statement of the proposition, change "compact surface" to "connected compact surface." Then in the second sentence of the proof, change both occurrences of "surface" to "connected compact surface."

(11/5/17) **Page 181, first full paragraph:** Replace the sentence starting with "However" by "However, we will prove in Chapter 10 that a compact surface cannot have both an oriented presentation and a nonoriented one."

(2/26/18) **Page 181, Problem 6-4:** Replace the first sentence by "Suppose M is a compact 2-manifold that contains a subset $B \subseteq M$ that is homeomorphic to the Möbius band, and whose interior is homeomorphic to the Möbius band minus its boundary."

(9/16/11) **Page 190, line 3 from the bottom:** Change $\Phi_g(f)$ to $\Phi_g[f]$.

(1/20/11) **Page 193, proof of Proposition 7.16, second paragraph, line 2:** Change "$H_1 = f$" to "$H_1 = \widetilde{f}$."

(8/3/18) **Page 201, Corollary 7.38:** This corollary should be moved after the statement of Theorem 7.40.

(11/25/12) **Page 211, line 6:** Delete redundant "each."

(7/9/15) **Page 215, Problem 7-9:** Change "connected" to "path-connected."

(5/31/16) **Page 221, Theorem 8.4:** Remark: This theorem is true without the assumption that B is locally connected, and the proof is not really any more difficult; see, for example, the proof of Theorem 1.7 in [Hat02].

(1/20/11) **Page 224, two lines above the subheading:** Change $\widetilde{f}_0(1)$ to $\widetilde{f}_1(0)$.

(1/20/11) **Page 228, displayed equations (8.4):** Replace these equations by

$$\begin{aligned} \deg\varphi &= \deg(\rho_\varphi \circ \varphi)_*, \\ \deg\psi &= \deg(\rho_\psi \circ \psi)_*. \end{aligned} \tag{8.4}$$

(7/13/15) **Page 230, Problem 8-5:** Replace the last sentence of the hint by the following: "Prove that $p_\varepsilon|_{\mathbb{S}^1}$ and $p_n(z) = z^n$ are homotopic as maps from $\mathbb{S}^1$ to $\mathbb{C} \smallsetminus \{0\}$. If p has no zeros, use degree theory to derive a contradiction."

(7/13/15) **Page 231, Problem 8-10(c):** Change "index of V around the loop ω" to "winding number of V around the loop ω."

(9/16/11) **Page 239, fourth line below the section heading:** Change "generated by G" to "generated by S."

(7/28/16) **Page 244, fourth line below the section heading:** Change $n \in \mathbb{Z}$ to $n \in \mathbb{N}$.

(7/28/16) **Page 247, Example 9.22, last line:** The formula for G_{tor} should be $G_{\text{tor}} = \{0\} \times \mathbb{Z}/k_1 \times \cdots \times \mathbb{Z}/k_m$.

(11/28/17) **Page 252, just above diagram (10.2):** Change “the following diagram commutes” to “the right half of the following diagram commutes.”

(9/16/11) **Page 263, line 2:** Change $\widetilde{U} \cap \widetilde{V}$ to $q(D \smallsetminus \{z\})$.

(8/2/13) **Page 268, lines 2 & 3:** Change “preceding corollary” to “preceding theorem.”

(11/5/17) **Page 268, statement of Corollary 10.24:** Change the statement to “*A compact surface cannot have both an oriented presentation and a nonoriented one.*”

(5/23/11) **Page 269, line below equation (10.7):** Insert missing comma after “surjective.”

(7/8/14) **Page 275, Problem 10-21(c):** Delete “with nonempty intersection.”

(8/27/18) **Page 278, second line below the heading:** Before “disjoint union,” insert “nonempty.”

(5/31/16) **Page 282, Theorem 11.13:** Remark: This theorem, like Theorem 8.4, is true without the assumption that B is locally connected.

(5/17/12) **Page 302, Problem 11-5, first line:** Change “dimension n” to “dimension $n \geq 2$.”

(12/10/15) **Page 303, Problem 11-12(c):** Change “$(1,0)$ or $(-1,0)$” to “1 or -1” [to be consistent with the complex notation used elsewhere for $\mathbb{S}^1$].

(5/17/12) **Page 305, Problem 11-20:** At the end of the problem, add: “For the counterexample, you may use without proof the fact that $\mathbb{S}^2$ is not contractible. (This follows, for example, from Corollary 13.11 and Theorem 13.23.)”

(2/25/18) **Page 312, last sentence of the paragraph after Exercise 12.13:** Allen Hatcher’s name is misspelled.

(7/8/14) **Page 315, paragraph above the displayed diagram:** After “Q is a normal covering map,” insert “and $\widehat{H} = \operatorname{Aut}_Q(E)$.”

(7/8/14) **Page 315, just below the displayed diagram:** Replace the last two paragraphs on page 315 and the first (partial) paragraph on page 316 with the following:

We have to show that $\widehat{q}$ is a covering map. Let $x \in X$ be arbitrary, and let U be a neighborhood of x that is evenly covered by q. We will show that U is also evenly covered by $\widehat{q}$. Given a component U_i of $q^{-1}(U)$, let $\widehat{U}_i = Q(U_i) \subseteq \widehat{E}$; then $\widehat{U}_i$ is connected, and it is open in $\widehat{E}$ because Q is an open map (Proposition 11.1). Suppose $\widehat{U}_i = Q(U_i)$ and $\widehat{U}_j = Q(U_j)$ are any two such sets. If they have a point $\widehat{e}$ in common, then $\widehat{e} = Q(e_i) = Q(e_j)$ for some $e_i \in U_i$ and $e_j \in U_j$. Since Q identifies points of E if and only if they are in the same $\widehat{H}$-orbit, there is some $\varphi \in \widehat{H}$ such that $e_j = \varphi(e_i)$. Then $\varphi(U_i) = U_j$ by Proposition 12.1(c), so $Q(U_i) = Q \circ \varphi(U_i) = Q(U_j)$. This shows that any such sets $\widehat{U}_i, \widehat{U}_j$ are either disjoint or equal. Since Q is surjective, $\widehat{q}^{-1}(U)$ is equal to the disjoint union of the connected open sets $\widehat{U}_i$ as U_i ranges over the components of $q^{-1}(U)$.

It remains only to show that for any such set $\widehat{U}_i$, the restricted map $\widehat{q}\colon \widehat{U}_i \to U$ is a homeomorphism. The following diagram commutes:

$$\begin{tikzcd} U_i \arrow[dd, "q"'] \arrow[dr, "Q"] & \\ & \widehat{U}_i \arrow[dl, "\widehat{q}"] \\ U. & \end{tikzcd} \tag{12.3}$$

Since $q = \widehat{q} \circ Q$ is injective on U_i, so is Q; and $Q\colon U_i \to \widehat{U}_i$ is surjective by definition. Because Q is an open map, it follows that $Q\colon U_i \to \widehat{U}_i$ is a homeomorphism. Since q and Q are homeomorphisms in (12.3), so is $\widehat{q}$.

(9/27/11) **Page 318, statement of Proposition 12.21, second line:** Insert “*on*” after “*acting*.”

(9/23/14) **Page 321, line 4:** Change $E \times E$ to E.

(9/27/11) **Page 329, paragraph just below the diagram:** Change every occurrence of $\widetilde{p}$ to $\widetilde{q}$ (five times).

(9/27/11) **Page 330, just below the bulleted list:** Change $\widetilde{p}$ to $\widetilde{q}$.

(9/27/11) **Page 332, first full paragraph, second line:** Change $\widetilde{p}$ to $\widetilde{q}$.

(9/27/11) **Page 332, second full paragraph, lines 6 and 7:** Change $\widetilde{p}$ to $\widetilde{q}$ (twice).

(9/16/14) **Page 335, Problem 12-10:** Interchange the definitions of G and H in the sixth and seventh lines. (Otherwise, part (c) is false as stated.)

(10/12/14) **Page 337, Problem 12-19:** Replace the first sentence of the problem with the following: "Suppose we are given a continuous action of a metrizable topological group (e.g., a discrete group) G on a first countable Hausdorff space E."

(9/27/11) **Page 352, lines 3 and 4:** Change c_p to c_q (twice), and change p to q (twice).

(12/15/17) **Page 354, paragraph above the last display:** Insert "of some reparametrization" after "extension of the circle representative."

(10/8/15) **Page 370, line 5 from the bottom:** Change "It follows . . . " to "Assuming X is path-connected, it follows . . . ."

(10/8/15) **Page 371, at the end of the first (partial) paragraph:** Insert "If X is not path-connected, just apply this argument to the path component containing the image of φ, and use Proposition 13.5."

(9/26/17) **Page 371, statement of Theorem 13.34(e):** Change "dimension n" to "dimension $n \geq 2$," and change "the zero map" to "not injective."

(9/26/17) **Page 372, proof of Theorem 13.34, last paragraph:** Change "if $\varphi_* = 0$" to "if φ_* is injective."

(9/26/17) **Page 372, Example 13.35(b), last line:** Change "the zero map" to "noninjective."

(9/29/17) **Page 372, Example 13.35(c):** Replace the last sentence by "The image of φ_* is the infinite cyclic group generated by $\gamma\left(\alpha_1^2 \dots \alpha_n^2\right)$, so φ_* is injective and $H_2(M) = 0$."

(12/26/18) **Page 401, line 4 from the bottom:** Change "subset" to "nonempty subset

(6/6/18) **Page 411, near the middle of the page:** The index entry for $\overline{R}$ should read "(normal closure of a subset)."

(2/25/18) **Page 422:** The index entry for "Hatcher, Allen" is misspelled.