CORRECTIONS TO
Introduction to Smooth Manifolds (Second Edition)
by John M. Lee
October 13, 2021

(8/16) Page 6, just below the last displayed equation: Change \( \varphi([x]) \) to \( \varphi_{n+1}[x] \), and in the next line, change \( x^i \) to \( x^{n+1} \). After “(Fig. 1.4),” insert “with similar interpretations for the other charts.”

(8/16) Page 7, Fig. 1.4: Both occurrences of \( x^i \) should be \( x^{n+1} \).

(12/19/18) Page 9, proof of Theorem 1.15: In the second line of the proof, replace “For each \( j \)” with “For each \( j \geq 0 \)” Then in the fourth-to-last line, replace “positive integers” by “nonnegative integers.”

(1/15/21) Page 13, line 1: Delete the words “and injective.”

(1/18/21) Page 20, Example 1.31: There are multiple errors in this example. Replace everything after the first two sentences by the following: For each \( i = 1, \ldots, n+1 \), let \( (U_i^\pm \cap S^n, \varphi_i^\pm) \) denote the graph coordinate charts we constructed in Example 1.4. For any distinct indices \( i \) and \( j \) and any choices of \( \pm \) signs, the transition maps \( \varphi_i^\pm \circ (\varphi_j^\pm)^{-1} \) and \( \varphi_i^\pm \circ (\varphi_j^\mp)^{-1} \) are easily computed. For example, in the case \( i < j \), we get the following formula for all \( u \) in the domain of \( \varphi_i^+ \circ (\varphi_j^+)^{-1} \):

\[
\varphi_i^+ \circ (\varphi_j^+)^{-1}(u^1, \ldots, u^n) = \left( u^1, \ldots, u^i, \sqrt{1-|u|^2}, \ldots, u^n \right)
\]

(with \( u^j \) omitted and the square root replacing \( u^j \)), and similar formulas hold in the other cases. When \( i = j \), the domains of \( \varphi_i^+ \) and \( \varphi_i^- \) are disjoint, so there is nothing to check. Thus, the collection of charts \( \{(U_i^\pm \cap S^n, \varphi_i^\pm)\} \) is a smooth atlas, and so defines a smooth structure on \( S^n \). We call this its standard smooth structure.

(6/23/13) Page 23, two lines below the first displayed equation: Change “any subspace \( S \subseteq V \)” to “any \( k \)-dimensional subspace \( S \subseteq V \).”

(9/15/19) Page 24, first full paragraph, fourth line: Change “any subspace \( S \)” to “any \( k \)-dimensional subspace \( S \).”

(12/19/18) Page 26, first line: Change \( U \cap \varphi^{-1}(\text{Int } H^n) \) to \( \varphi^{-1}(\text{Int } H^n) \).

(12/19/18) Page 27, last paragraph, sixth line: Change \( \bar{U} \cap H^n \) to \( \bar{U} \cap U \).

(2/22/15) Page 29, proof of Theorem 1.46, second paragraph, line 4: Change \( \varphi(U \cap V) \) to \( \psi(U \cap V) \).

(10/8/15) Page 30, Problem 1-6: Interpret the formula for \( F_s \) to mean \( F_s(0) = 0 \) when \( s \leq 1 \).

(1/27/18) Page 31, Fig. 1.13: Change \{\( x^n = 0 \)\} to \{\( x^{n+1} = 0 \)\}.

(3/12/18) Page 31, Problem 11-11, next-to-last line: Change \( S^n \) to \( S^n \setminus \{N\} \).

(4/25/17) Page 45, second paragraph: Replace the last sentence of that paragraph with the following: “If \( N \) has empty boundary, we say that a map \( F: A \to N \) is smooth on \( A \) if it has a smooth extension in a neighborhood of each point: that is, if for every \( p \in A \) there exist an open subset \( W \subseteq M \) containing \( p \) and a smooth map \( \bar{F}: W \to N \) whose restriction to \( W \cap A \) agrees with \( F \). When \( \partial N \neq \varnothing \), we say \( F: A \to N \) is smooth on \( A \) if for every \( p \in A \) there exist an open subset \( W \subseteq M \) containing \( p \) and a smooth chart \( (V, \psi) \) for \( N \) whose domain contains \( F(p) \), such that \( F(W \cap A) \subseteq V \) and \( \psi \circ F|_{W \cap A} \) is smooth as a map into \( \mathbb{R}^n \) in the sense defined above (i.e., it has a smooth extension in a neighborhood of each point).”

(7/23/14) Page 45, last displayed equation: The first = sign should be \( \subseteq \).

(9/15/19) Page 46, line 9: Change “on an open subset” to “on a nonempty open subset.”
Page 47, proof of Theorem 2.29, second paragraph: Replace the first sentence of the paragraph by “Let \( h : \mathbb{R}^n \to \mathbb{R} \) be a smooth bump function that is positive in \( B_1(0) \) and zero elsewhere.”

Page 56, first displayed equation: Change \( dt(v)_p \) to \( dt_p(v) \).

Page 56, just below the last displayed equation: Replace “the last two equalities follow” by “the last equality follows.”

Page 58, proof of Lemma 3.11, next-to-last line: Change \( H^n \) to \( \text{Int} H^n \).

Page 68, proof of Proposition 3.21: Insert the following sentence at the beginning of the proof: “Let \( n = \dim M \) and \( m = \dim N \).” Then in the second sentence, change (3.9) to (3.10). Finally, in the displayed equation, change \( F^n \) to \( F^m \) (twice).

Page 70, two lines above Corollary 3.25: Change “Proposition 3.23” to “Proposition 3.24.”

Page 76, Problem 3-8: Add the following remark: “(For \( p \in \partial M \), we need to allow curves with domain \([0, \varepsilon) \) or \((-\varepsilon, 0] \) and to interpret the derivatives as one-sided derivatives.)”

Page 78, proof of Prop. 4.1, third and fourth lines: Change \( m \) to \( n \) (twice).

Page 100, proof of Proposition 5.4, next-to-last line: Change “It a homeomorphism” to “It is a homeomorphism.”

Page 104, line below the proof of Theorem 5.11: Change “See Theorem 5.31” to “See Problem 5-24.” [Problem 5-24 is a new problem, described later in this list. Theorem 5.31 is not appropriate in this situation because it applies only to manifolds without boundary.]

Page 105, line 4 from the bottom: Change \( F \) to \( \Phi \).

Page 112, Fig. 5.10: Interchange the labels \( M \) and \( N \) on the figure, to be consistent with the notation in Theorem 5.29.

Page 113, line 6: Change the definition of \( \tilde{\psi} \) to \( \tilde{\psi} = \pi \circ \psi |_{V_0} \).

Page 118, Fig. 5.13: Change \( N \) to \( v \).

Page 120, proof of Proposition 5.46: At the beginning of the proof, insert this sentence: “Let \( F : D \to M \) denote the inclusion map.”

Page 121, line 5: Change \( x^m \) to \( x^n \).

Page 123, Problem 5-6: Add the assumption that \( m > 0 \).

Page 124: At the end of the page, add a new problem:
5-24. Suppose \( M \) is a smooth manifold with boundary, \( N \) is a smooth manifold, and \( F : N \to M \) is a smooth map whose image is contained in \( \partial M \). Show that \( F \) is smooth as a map into \( \partial M \), and use this to prove that \( \partial M \) has a unique smooth structure making it an embedded submanifold of \( M \).
(12/19/18) Page 129, proof of Sard’s theorem, second paragraph: Just before the last sentence of the paragraph, insert the following: “In the $\mathbb{R}^n$ case, extend $F$ to a smooth map on an open subset of $\mathbb{R}^m$, and replace $U$ by that open subset; if we can show that the set of critical values of the extended map has measure zero, then the same is true of the set of critical values of $F$.”

(3/16/19) Page 129, displayed equation near the bottom of the page: Change “ith partial derivatives” to “ith-order partial derivatives.”

(12/26/18) Page 130, just below equation (6.2): Right after the displayed equation, insert “(where the component functions $F^2, \ldots, F^m$ might be different from the ones in the original coordinate chart).”

(3/28/20) Page 131, two lines below the first displayed equation: Change $A'(R/K)^{k+1}$ to $A'(R\sqrt{m}/K)^{k+1}$.

(1/8/18) Page 131, three lines below the first displayed equation: Insert “at most” before “$K^n$ balls.”

(3/28/20) Page 131, second displayed equation: Change the left-hand side to $K^m(A')^n(R\sqrt{m}/K)^{n(k+1)}$, and in the next line, change the definition of $A''$ to $A'' = (A')^n(R\sqrt{m})^{n(k+1)}$.

(4/17/13) Page 132, proof of Lemma 6.13, second paragraph: This argument does not apply when $\partial M \neq \emptyset$, because in that case $M \times M$ is not a smooth manifold with boundary. Instead, we can consider the restrictions of $\kappa$ to $(M \times \text{Int} M) \setminus \Delta_M$ and to $(M \times \partial M) \setminus \Delta_M$ (both of which are smooth manifolds with boundary), and note that there is a point $[t] \in \mathbb{R}P^{n-1}$ that is not in the image of $\tau$ or either of these restrictions of $\kappa$. [Thanks to David Iglesias Ponte for suggesting this correction.]

(12/19/18) Page 134, proof of Theorem 6.15, just after the fourth paragraph of the proof: Insert the following: “The argument above still works when $M$ is an arbitrary compact subset of a larger manifold $\tilde{M}$ with or without boundary, by covering $M$ with finitely many coordinate balls or half-balls for $\tilde{M}$. The result is a smooth injective map $F : M \to \mathbb{R}^{nm+m}$ whose differential is injective at each point.” [This is needed in the ensuing argument for the noncompact case, because the sets $E_i$ might not be regular domains when $\partial M \neq \emptyset$.]

(7/3/15) Page 134, displayed equations two-thirds of the way down the page: In the definition of $E_i$, there’s an “$i - 1$” that should be “$i - j$.” It should read $E_i = f^{-1}([b_{i-1}, a_{i+1}])$.

(12/19/18) Page 134, just below the displayed equations two-thirds of the way down the page: Delete the sentence “By Proposition 5.47, each $E_i$ is a compact regular domain.”

(7/2/18) Page 137, first paragraph under the subheading “Tubular Neighborhoods,” fifth line: Change $R^n$ to $\mathbb{R}^n$.

(7/27/18) Page 138, proof of Theorem 6.23, end of the first paragraph: Change “standard coordinate frame” to “standard coordinate basis.”


(12/5/16) Page 145, paragraph above Prop. 6.34: In the definition of smooth family of maps, replace “$F : M \times S \to N$” by “$F : N \times S \to M$.”

(9/28/19) Page 146, equation (6.9): Should read $dF(T_{(p,s)}W) \subseteq T_qX$. [Change the equal sign to subset.]

(9/28/19) Page 146, line below the last displayed equation: Change “$= T_qX$” to “$\subseteq T_qX$.”

(11/25/12) Page 148, Problem 6-13: Delete part (c). [This statement is simply wrong. It is true with the added hypothesis that $F'$ is an embedding, but then it’s essentially just a restatement of part (b).]

(2/10/18) Page 150, last line: Change “Theorem 20.16” to “Theorem 20.22.”

(12/30/17) Page 160, first line: Change $R_{hh}^{-1} \to R_{hh}^{-1}$. 

(33/18) Page 192, second last line: Change “Proposition 20.22” to “Proposition 20.23.”
Page 196, proof of Proposition 8.45, next-to-last line: Change \( \rho'_j(g) = \pi^j(g \cdot E_j) \).

Page 223, proof of Theorem 9.26: Replace the statement of Theorem 9.26 by “There’s a gap in this proof, because it is not necessarily the case that \( M(a) \) is a regular domain in \( \text{Int} M \). To correct the problem, we have to choose our collar neighborhood more carefully. Replace the first sentence of the proof by the following paragraph:

“Theorem 9.25 shows that \( \partial M \) has a collar neighborhood \( C_0 \) in \( M \), which is the image of a smooth embedding \( E_0 : [0,1) \times \partial M \to M \) satisfying \( E_0(0,x) = x \) for all \( x \in \partial M \). Let \( f : M \to \mathbb{R}^+ \) be a smooth positive exhaustion function. Note that \( W = \{ (t,x) : f(E_0(t,x)) > f(x) - 1 \} \) is an open subset of \( [0,1) \times \partial M \) containing \( \{0\} \times \partial M \). Using a partition of unity as in the proof of Theorem 9.20, we may construct a smooth positive function \( \delta : \partial M \to \mathbb{R}^+ \) such that \( (t,x) \in W \) whenever \( 0 \leq t < \delta(x) \). Define \( E : [0,1) \times \partial M \to M \) by \( E(t,x) = E_0(t\delta(x),x) \). Then \( E \) is a diffeomorphism onto a collar neighborhood \( C \) of \( \partial M \), and by construction \( f(E(t,x)) > f(x) - 1 \) for all \( (t,x) \in [0,1) \times \partial M \). We will show that for each \( a \in (0,1) \), the set \( E([0,a]) \times \partial M \) is closed in \( M \). Suppose \( p \) is a boundary point of \( E([0,a]) \times \partial M \) in \( M \); then there is a sequence \( \{(t_i,x_i)\} \) in \( (2/16/18) \) Page 164, just above the subheading: Replace the last line of the proof of Prop. 7.23 by “The action is smooth because each \( \varphi \) can be written locally as a composition of a smooth local section followed by \( \pi \).”

Page 169, first line: Change \( \tilde{G} \) to \( G \).

Page 170, Proof of Theorem 7.35: Replace the phrase “closed Lie subgroups such that \( N \) is normal” by “Lie subgroups such that \( N \) is normal and closed.” [In fact, using the result of Theorem 19.25 later in the book, the hypothesis that \( N \) is closed can also be omitted.]

Page 171, third line from the end of the proof: Change \( E_t \to E_j \), so the formula reads \( \rho'_j(g) = \pi^j(g \cdot E_j) \).

Page 174, Proof of Theorem 7-21: Replace the first sentence by “Prove that the groups in Problem 7-20 are isomorphic to direct products of the indicated groups in cases (a) and (c) if and only if \( n \) is odd, and in cases (b) and (d) if and only if \( n = 1 \).”

Page 178, Example 8.10(d): Change “Example 8.4” to “Example 8.5.”

Page 183, Example 8.20, next-to-last line: Change \( p = (u,v) \) to \( q = (u,v) \).

Page 184, proof of Proposition 8.22: After “Proposition 5.37,” insert “in the case \( \partial S = \emptyset \). When \( S \) has nonempty boundary, the proof of Proposition 5.37 still goes through using boundary slice coordinates for \( S \).”

Page 186, proof of Proposition 8.45, next-to-last line: Should read “\( F_* \circ (F^{-1})_* = (F \circ F^{-1})_* = \text{Id}_{\text{Lie}(H)} \) and \( (F^{-1})_* \circ F_* = \text{Id}_{\text{Lie}(G)} \).”

Page 196, proof of Proposition 8.45, next-to-last line: Change “\( F_* \circ (F^{-1})_* = (F \circ F^{-1})_* = \text{Id}_{\text{Lie}(H)} \) and \( (F^{-1})_* \circ F_* = \text{Id}_{\text{Lie}(G)} \)” to “\( F_* \circ (F^{-1})_* = (F \circ F^{-1})_* = \text{Id}_{\text{Lie}(H)} \) and \( (F^{-1})_* \circ F_* = \text{Id}_{\text{Lie}(G)} \).”

Page 201, Problem 8-15: At the end of the last sentence, add “provided that \( \dim S > 0 \).”

Page 208, first line: Change to “This is just the existence and smoothness statements of Theorem D.1 . . . .”

Page 213, first sentence of the last paragraph: The definition of \( t_0 \) should be \( t_0 = \sup \{ t \in \mathbb{R} : (t,p_0) \in W \} \).

Page 214, Fig. 9.6: The shaded area should be labeled \( W \), not \( \mathcal{D} \).

Page 217, Fig. 9.7: Both occurrences of \( \varphi \) should be \( \Phi \).

Page 219, second displayed equation: Change “\( V^\prime (0,p) = 0 \)” to “\( \Phi^\prime (0,p) = 0 \).”

Page 219, two lines below (9.12): Here and in the rest of the paragraph, change \( p_0 \) to \( p_1 \) (seven times) to avoid confusion with the prior unrelated use of \( p_0 \) in this proof.

Page 222, just below the section heading: Insert the following sentence: “On a manifold with boundary, the definitions of flow domain, flow, and infinitesimal generator of a flow are exactly the same as on a manifold without boundary.”

Page 223, line 2: Change \( \delta : M \to \mathbb{R}^+ \) to \( \delta : \partial M \to \mathbb{R}^+ \).

Page 223, proof of Theorem 9.26: There’s a gap in this proof, because it is not necessarily the case that \( M(a) \) is a regular domain in \( \text{Int} M \). To correct the problem, we have to choose our collar neighborhood more carefully. Replace the first sentence of the proof by the following paragraph:

“Theorem 9.25 shows that \( \partial M \) has a collar neighborhood \( C_0 \) in \( M \), which is the image of a smooth embedding \( E_0 : [0,1) \times \partial M \to M \) satisfying \( E_0(0,x) = x \) for all \( x \in \partial M \). Let \( f : M \to \mathbb{R}^+ \) be a smooth positive exhaustion function. Note that \( W = \{ (t,x) : f(E_0(t,x)) > f(x) - 1 \} \) is an open subset of \( [0,1) \times \partial M \) containing \( \{0\} \times \partial M \). Using a partition of unity as in the proof of Theorem 9.20, we may construct a smooth positive function \( \delta : \partial M \to \mathbb{R} \) such that \( (t,x) \in W \) whenever \( 0 \leq t < \delta(x) \). Define \( E : [0,1) \times \partial M \to M \) by \( E(t,x) = E_0(t\delta(x),x) \). Then \( E \) is a diffeomorphism onto a collar neighborhood \( C \) of \( \partial M \), and by construction \( f(E(t,x)) > f(x) - 1 \) for all \( (t,x) \in [0,1) \times \partial M \). We will show that for each \( a \in (0,1) \), the set \( E([0,a]) \times \partial M \) is closed in \( M \). Suppose \( p \) is a boundary point of \( E([0,a]) \times \partial M \) in \( M \); then there is a sequence \( \{(t_i,x_i)\} \) in
Page 223, proof of Theorem 9.26, last line of the first paragraph: Change $0 \leq t < a$ to $0 \leq s < a$.

Page 225, Example 9.31: At the end of the example, insert the sentence “If $n \geq 2$, then $M_1 \# M_2$ is connected.”

Page 226, Example 9.32, fifth line: Replace the sentence beginning “It is a smooth manifold without boundary …” by “It is a topological manifold without boundary, and can be given a smooth structure such that each of the natural maps $M \to D(M)$ (induced by inclusion into the left and right summands of the disjoint union) is a smooth embedding.”

Page 230, line 1 and first displayed equation: Change $\theta_t(x)$ to $\theta_t(u)$ (twice).

Page 230, second paragraph: “from Case” should be “from Case 1.”

Page 230, fourth paragraph, last line: Change $[X, Y]$ to $[V, W]$.

Page 234, proof of Theorem 9.46, second paragraph: Replace the two parenthesized sentences by the following: “(To see this, just choose $\epsilon_1 > 0$ and $U_1 \subseteq U$ such that $\theta_1$ maps $(-\epsilon_1, \epsilon_1) \times U_1$ into $U$, and then inductively choose $\epsilon_i$ and $U_i$ such that $\theta_i$ maps $(-\epsilon_i, \epsilon_i) \times U_i$ into $U_{i-1}$. Taking $\epsilon = \min \{\epsilon_i\}$ and $Y = U_k$ does the trick.)”

Page 241, Example 9.52: At the end of the example, add the sentence “Note that $u$ is smooth on the open set $\mathbb{R}^2 \setminus \{0\}$, which is a neighborhood of $S$.”

Page 246, Problem 9-11: Delete the second sentence of the hint. [Because $N$ is inward-pointing along $\partial M$, no integral curve that starts on $\partial M$ can hit the boundary again, because the vector field would have to be tangent to $\partial M$ or outward-pointing at the first such point.]

Page 248, first displayed equation: Should read

$$V(t, p) = \frac{\partial}{\partial t} H(H_t^{-1}(p), t).$$

Page 248, Problem 9-22(c): Replace the problem statement by

(c) $\frac{\partial u}{\partial x} + u \frac{\partial u}{\partial y} = -y$, \hspace{1cm} $u(0, y) = 0$.

[Without this sign change, the third claim in Problem 9-23 is not true.]

Page 254, paragraph beginning “With respect to,” third line: Replace $V_p \times \mathbb{R}^k$ with $U_a \times \mathbb{R}^k$.

Page 255, Example 10.8: Replace the sentence beginning with “If $E$ is a smooth vector bundle” by the following: “If $E$ is a smooth vector bundle and $S \subseteq M$ is an embedded submanifold, it follows easily from the chart lemma that $E|_S$ is a smooth vector bundle. If $S$ is merely immersed, we give $E|_S$ a topology and smooth structure making it into a smooth rank-$k$ vector bundle over $S$ as follows: For each $p \in S$, choose a neighborhood $U$ of $p$ in $M$ over which there is a local trivialization $\Phi$ of $E$, and a neighborhood $V$ of $p$ in $S$ that is embedded in $M$ and contained in $U$. Then the restriction of $\Phi$ to $\pi^{-1}(V)$ is a bijection from $\pi^{-1}(V)$ to $V \times \mathbb{R}^k$, and we can apply the chart lemma to these bijections to yield the desired structure.”
Page 260, two lines above Proposition 10.22: Change \( \tau^n(p) \) to \( \tau^k(p) \).

Page 261, statement of Proposition 10.25, first line: Change \( \pi' : E \to M' \) to \( \pi' : E' \to M' \).

Page 264, paragraph above the subheading, first sentence: “homomorphism” should be “homomorphisms.”

Page 271, Problem 10-18: Change “a properly embedded” to “an embedded.”

Page 271, Problem 10-19(d): Add the following: [Hint: For the “only if” direction, to show that \( F \) is compact, use a finite number of local trivializations to construct a closed set over which \( E \) is trivial.]

Page 301, Problem 11-10(c): Change “closed forms” to “closed covector fields” (twice).

Page 301, Problem 11-10(c): Change \( S^2 \) to \( S^2 \).

Page 303, Problem 11-13: Add the assumption that \( n > 0 \).

Page 303, just below the commutative diagram: Insert this sentence: “A natural transformation is called a natural isomorphism if each map \( \lambda_X \) is an isomorphism in \( D \).”

Page 303, Problem 11-18(b) and (c): Change “natural transformation” to “natural isomorphism” in both parts.

Page 317, paragraph beginning “Any one”: At the end of the paragraph, add this sentence: “If \( A \) and \( B \) are tensor fields, then \( A \otimes B \) denotes the tensor field defined by \( (A \otimes B)_p = A_p \otimes B_p \).”

Page 320, displayed equation just below the middle of the page: Change \( A^{i_1 \ldots i_k}_{j_1 \ldots j_l} \) to \( A^{i_1 \ldots i_k}_{j_1 \ldots j_l} \) on the third line of the display, and again on the line below the display. [The last lower index should be \( j_l \), not \( i_l \).]

Page 320, statement of Proposition 12.25: Change the domain and codomain of \( G \): It should read \( G : P \to M \).

Page 320, Proposition 12.25(e): Should read \((F \circ G)^* B = G^*(F^* B)\).

Page 333, first line: Change \( U \subseteq M \) to \( V \subseteq M \).

Page 345, Problem 13-10: In the last line of the problem statement, change \( L_g(\vec{y}) > L_g(\gamma) \) to \( L_g(\vec{y}) \geq L_g(\gamma) \), and delete the phrase “unless \( \vec{y} \) is a reparametrization of \( \gamma \).” [Because the definition of reparametrization that I’m using requires a diffeomorphism of the parameter domain, the original problem statement was not true.]

Page 355, proof of Lemma 14.10: At the beginning of the proof, insert “Let \( (E_1, \ldots, E_n) \) be the basis for \( V \) dual to \( (e^j) \).”

Page 356, Case 4, second line: Should read “brings us back to Case 3.”

Page 368, second paragraph: At the end of the first sentence of the paragraph, insert “(see pp. 341–343).”

Page 368, paragraph below equation (14.25): Change \( TM \) to \( T\mathbb{R}^3 \) (twice).

Page 371, three lines above (14.31): Change that sentence to “The only terms in this sum that can possibly be nonzero are those for which \( J \) has no repeated indices and \( m \) is equal to one of the indices in \( J \), say \( m = j_p \).”

Page 374, Problem 14-2: Add “[Hint: One way to approach this is to prove first that a \( k \)-covector \( \omega \) is decomposable if and only if the map from \( \mathbb{R}^n \) to \( \Lambda^{k-1}(\mathbb{R}^n^*) \) given by \( v \mapsto v \cdot \omega \) has \( (n-k) \)-dimensional kernel.]”

Page 377, line 4: Change “is a simply” to “is simply.”
Page 386, just above Proposition 15.24: After “determines an orientation on $\partial M$,” insert “if $M$ is oriented.”

Page 389, Exercise 15.30: Change “a local isometry” to “an orientation-preserving local isometry.”

Page 397, Problem 15-1: At the end of the last sentence, add “when $n > 1$.”

Page 397, Problem 15-3: Change $x_B^n$ to $x_B^{n+1}$ (twice).

Page 397, Problem 15-4: Change the first sentence to “Let $\theta$ be the flow of a smooth vector field on a smooth manifold.” [The stated result is true also for manifolds with boundary and for nonmaximal flows, but to prove it, one must first do a little work to generalize some of the results of Theorem 9.12 to more general flows.]

Page 402, lines 2–3: There should not be a paragraph break before “and.”

Page 403, just after the last displayed equation: Add “(In the $H_n$ case, apply Theorem C.26 to the interiors of $D$ and $E$ considered as subsets of $\mathbb{R}^n$.)”

Page 409, line 2: Change $'i$ to $'i$.

Page 415, paragraph above Example 16.19: Change “interior charts and charts with corners” to “interior charts, boundary charts, and charts with corners.”

Page 416, line 3 from the bottom: Change “$0/D_p$” to “$\partial M_0/D_p$.”

Page 418, statement of Proposition 16.21: Delete “compact,” and change “$n$-manifold” to “$n$-manifold.”

Page 419, proof of Theorem 16.25, first paragraph: Replace the second and third sentences of the paragraph by the following: “By means of smooth charts and a partition of unity, we may reduce the theorem to the cases in which $M = \mathbb{R}^n$, $M = \mathbb{H}^n$, or $M = \mathbb{R}^n_+$. The $\mathbb{R}^n$ and $\mathbb{H}^n$ cases are treated just as before.”

Page 424, second displayed equation: Change $S.\beta(X)$ to $\partial M_0.\beta(X)$.

Page 426, three lines below the section heading: “cam” should be “can.”

Page 430, Proposition 16.38(c): This statement is wrong. Change it to “If $F$ is smooth, then $F^* \mu$ is a continuous density on $M$; and if $F$ is a local diffeomorphism, $F^* \mu$ is smooth.”

Page 439, Problem 16-23: The formula for $g$ should be

$$g = \frac{dx^2 + dy^2}{(1-x^2-y^2)^2}.$$
Let $Y_1 = \bigcup_{i=2}^{\infty} W_i$. Because $M$ is connected, each component of $Y_1$ meets $W_1$, and by local finiteness of $\{ W_j \}$, there are only finitely many such components. Such a component is precompact in $M$ if and only if it is a union of finitely many $W_i$’s. Let $V_1$ be the union of $W_1$ together with all of the precompact components of $Y_1$, and let $X_1$ be the union of all $W_i$’s not contained in $V_1$. Then $V_1$ is connected and precompact, and $X_1$ has no precompact components. Proceeding by induction, suppose we have defined connected, precompact open sets $V_1, \ldots, V_m$ whose union contains $W_1 \cup \cdots \cup W_m$, and such that the union $X_m$ of all the $W_i$’s not contained in $V_1 \cup \cdots \cup V_m$ has no precompact components. Let $j_m$ be the smallest index such that $W_{j_m}$ is not contained in $V_1 \cup \cdots \cup V_m$, and let $Y_{m+1}$ be the union of all $W_i$’s other than $W_{j_m}$ not contained in $V_1 \cup \cdots \cup V_m$. Any precompact component of $Y_{m+1}$ must meet $W_{j_m}$, because otherwise it would be a precompact component of $X_m$. Let $V_{m+1}$ be the union of $W_{j_m}$ with all of the precompact components of $Y_{m+1}$. As before, $V_{m+1}$ is precompact and connected, and the union $X_{m+1}$ of the $W_i$’s not contained in $V_1 \cup \cdots \cup V_{m+1}$ has no precompact components. Then by construction, for each $j$, the set $X_j = \bigcup_{j \geq j} V_i$ has no precompact components. If some $V_j$ does not meet $V_k$ for any $k > j$, then $V_j$ itself is a precompact component of $X_{j-1}$, which is a contradiction. Thus for each $j$, there is some $k > j$ such that $V_j \cap V_k \neq \emptyset$.

**Proof of Theorem 17.32.** Choose an orientation on $M$. Let $\{ V_j \}_{j=1}^{\infty}$ be an open cover of $M$ satisfying the conclusions of the preceding lemma. For each $j$, let $K(j)$ denote the least integer $k > j$ such that $V_j \cap V_k \neq \emptyset$, and let $\theta_j$ be an $n$-form compactly supported in $V_j \cap V_{K(j)}$ whose integral is 1. Let $\{ \psi_j \}_{j=1}^{\infty}$ be a smooth partition of unity subordinate to $\{ V_j \}_{j=1}^{\infty}$.

Now suppose $\omega$ is any $n$-form on $M$, and let $\omega_j = \psi_j \omega$ for each $j$. Let $c_1 = \int_{V_1} \omega_1$, so that $\omega_1 - c_1 \theta_1$ is compactly supported in $V_1$ and has zero integral. It follows from Theorem 17.30 that there exists $\eta_1 \in \Omega_c^{n-1}(V_1)$ such that $d\eta_1 = \omega_1 - c_1 \theta_1$. Suppose by induction that we have found $\eta_1, \ldots, \eta_m$ and constants $c_1, \ldots, c_m$ such that for each $j = 1, \ldots, m$, $\eta_j \in \Omega_c^{n-1}(V_j)$ and

$$d\eta_j = \left( \omega_j + \sum_{i : K(i) = j} c_i \theta_i \right) - c_j \theta_j.$$  

(1)  

Let

$$c_{j+1} = \int_{V_{j+1}} \left( \omega_{j+1} + \sum_{i : K(i) = j+1} c_i \theta_i \right).$$

Then by Theorem 17.30, there exists $\eta_{j+1} \in \Omega_c^{n-1}(V_{j+1})$ satisfying the analog of (1) with $j$ replaced by $j+1$. Set $\eta = \sum_{j=1}^{\infty} \eta_j$, with each $\eta_j$ extended to be zero on $M \sim V_j$. By local finiteness, this is a smooth $(n-1)$-form on $M$. It satisfies

$$d\eta = \omega + \sum_{j=1}^{\infty} \left( \sum_{i : K(i) = j} c_i \theta_i \right) - \sum_{j=1}^{\infty} c_j \theta_j.$$  

Each term $c_i \theta_i$ appears exactly once in the first sum above, so the two sums cancel each other.

(7/27/16) **Page 457, line below the second displayed equation:** Change “Theorem 17.31” to “Theorem 17.30.”

(7/12/16) **Page 463, line above equation (17.15):** Insert missing space before “Similarly.”

(7/13/16) **Page 464, end of proof of Corollary 17.42:** Insert “Note that this construction produces a form $\sigma$ whose support is contained in $U \cap V$. [This might be useful for solving Problem 18-6.]”

(7/12/16) **Page 471, last paragraph:** Replace the sentence starting “The hardest part …” with “The hardest part is showing that the singular chain complex of $M$ can be replaced by a chain complex built out of simplices whose images lie in either $U$ or $V$, without changing the homology.”

(9/12/17) **Page 487, Problem 18-1, first line:** Change “an oriented smooth manifold” to “a smooth manifold.”
(8/8/18) **Page 489, Problem 18-7(b):** Add to the hint: “In order to use Lemma 17.27, you’ll need to prove the following fact: _Every bounded convex open subset of \( \mathbb{R}^n \) is diffeomorphic to \( \mathbb{R}^n \)._ To prove this, let \( U \) be such a subset, and without loss of generality assume \( 0 \in U \). First show that there exists a smooth nonnegative function \( f \in C^\infty(U) \) such that \( f(0) = 0 \) and \( f(x) \geq 1/d(x) \) away from a small neighborhood of \( 0 \), where \( d(x) \) is the distance from \( x \) to \( \partial U \). Next, show that \( g(x) = 1 + \int_0^1 f(tx) dt \) is a smooth positive exhaustion function on \( U \) that is nondecreasing along each ray starting at \( 0 \). Finally, show that the map \( F: U \to \mathbb{R}^n \) given by \( F(x) = g(x)x \) is a bijective local diffeomorphism. Also, you may use the fact that the conclusion of the five lemma is still true even if the appropriate diagram commutes only up to sign.”

(1/15/13) **Page 491, Example 19.1(c):** Delete the word “unit.”

(5/22/15) **Page 492, line above Proposition 19.2:** Change “lie” to “Lie.”

(12/17/15) **Page 492, proof of Proposition 19.2, fourth line:** Change “Given \( p \in M’’ \) to “Given \( p \in U \).”

(9/12/16) **Page 506, Lemma 19.24, last line:** Before “left-invariant,” insert “smooth.”

(6/1/20) **Page 512, Problem 19-4:** In the first line of the problem, change “all three coordinates are positive” to “\( z \) is positive.” Then replace the last sentence by “Find an explicit global chart on \( U \) in which \( D^3 \) is spanned by the first two coordinate vector fields.” [Technically it might not be a flat chart because its image need not be a cube in \( \mathbb{R}^3 \).]

(10/4/17) **Page 518, sentence before Prop. 20.3:** Change “one-parameter subgroups of GL(\( n, \mathbb{R} \))” to “one-parameter subgroups of subgroups of GL(\( n, \mathbb{R} \)).”

(5/23/16) **Page 521, first displayed equation:** Change \( d \Phi_0 \) to \( d \Phi_e \) (twice).

(6/9/19) **Page 528, line 9:** Change two instances of \( (g, p) \) in subscripts to \( (g, q) \).

(5/19/18) **Page 528, just below the displayed equation in the middle of the page:** The smoothness of the map \( \sigma_q \) is not quite immediate from the definition. Replace the three sentences beginning “It follows” with this: “Because \( S_p \) is a weakly embedded submanifold by Theorem 19.17, to show that \( \sigma_q \) is a smooth local section of \( S_p \), it suffices to show that it is smooth into \( G \times M \) and takes its values in \( S_p \). The first component function is smooth as a map into \( G \) by smoothness of group multiplication. To show that the second component is smooth into \( M \) as a function of \( \tilde{X} \) (and therefore of \( \exp X \)), you need to use the argument sketched out just below equation (20.10): as in the proof of Prop. 20.8, apply the fundamental theorem on flows to the vector field \( \tilde{X}(p, X) = (\tilde{X}_p, 0) \) on \( M \times q \). A straightforward computation shows that \( y(t) = (\exp tX, \eta(t, \tilde{X}(p, X)) \) is an integral curve of \( \tilde{X} \) starting at \( (g, q) \), from which it follows easily that \( \sigma_q(\exp X) = y(1) \in S_p \).”

(1/10/17) **Page 537, Problem 20-6(a):** Change \( B \in \mathfrak{sl}(n, \mathbb{R}) \) to \( B \in \mathfrak{sl}(n, \mathbb{R}) \).

(5/31/16) **Page 538, Problem 20-11(b):** Here’s a better hint, which doesn’t require proving part (a) first: “[Hint: Consider the graph of \( F \) as a subgroup of \( G \times H \).]”

(10/18/17) **Page 542, middle of the paragraph before Example 21.3:** Change “the action of \( \mathbb{R}^k \) on \( \mathbb{R}^n \)” to “the action of \( \mathbb{R}^k \) on \( \mathbb{R}^k \times \mathbb{R}^n \).”

(2/25/18) **Page 548, last two lines:** Allen Hatcher’s name is misspelled. (Sorry, Allen.)

(5/23/16) **Page 549, proof of Proposition 21.12, last sentence:** Change the first phrase of that sentence to “Second, if \( p, p' \in E \) are in different orbits and \( \pi(p) \neq \pi(p') \), …” Then add the following sentences at the end of the proof: “If \( p \) and \( p' \) are in different orbits and \( \pi(p) = \pi(p') \), let \( W \) be an evenly covered neighborhood of \( \pi(p) \), and let \( V, V' \) be the components of \( \pi^{-1}(W) \) containing \( p \) and \( p' \), respectively. For any \( g \in \text{Aut}_g(E) \), a simple connectedness argument shows that \( g \cdot V \) is a component of \( \pi^{-1}(W) \); if it had nontrivial intersection with \( V \) it would have to be equal to \( V \), which would imply \( g \cdot p = p' \), a contradiction.”
Page 567, two lines above Proposition 22.8: Insert “a” before “2-covector.”

Page 568, Example 22.9(a), first line: The coordinates should be \((x^1, \ldots, x^n, y^1, \ldots, y^n)\). (The last coordinate is \(y^n\), not \(x^n\).)

Page 572, middle of the page: Replace the sentence starting “On the other hand” by this: “On the other hand, the left-hand side is just the ordinary \(t\)-derivative of a time-dependent tensor on a fixed vector space, and expanding in terms of a basis shows that it satisfies a similar product rule.”

Page 573, statement of Proposition 22.15, second line: Change “\(V W J M\)” to “\(V W J M! TM\)” and change \(\) to \(\).
(1/18/21) **Page 664, statement of Theorem D.1(b):** After the phrase “Any two differentiable solutions to (D.3)–(D.4),” insert “defined on intervals containing \( t_0 \).”

(12/2/15) **Page 666, just below the fifth display:** After the sentence ending “by our choice of \( \delta \) and \( \epsilon \),” insert “(If \( t < t_0 \), interchange \( t \) and \( t_0 \) in the second line above.)”

(1/18/21) **Page 664, statement of Theorem D.4:** After the phrase “any two differentiable solutions to (D.3)–(D.4),” insert “defined on intervals containing \( t_0 \).”

(1/18/21) **Page 668, paragraph below equation (D.10):** In the fourth line of the paragraph, change \( \overline{W} \) to \( W \).

(1/18/21) **Page 670, displayed inequality between (D.17) and (D.18):** Change \( n \) to \( n^2 \).

(1/18/21) **Page 670, last line:** Change \( n \) to \( n^2 \) in the definition of \( B \).

(1/18/21) **Page 671, inequality (D.19):** Change \( n \) to \( n^2 \) (twice).

(12/15/20) **Page 671, just below (D.19):** Replace the sentence “Since the expression on the right can be made as small as desired by choosing \( h \) and \( \tilde{h} \) sufficiently small, this shows . . . ” by the following: “Thus the expression on the left can be made as small as desired by choosing \( h \) and \( \tilde{h} \) sufficiently small. This shows . . . ”

(6/11/19) **Page 692:** Under the entry for “Form,” delete the references to page 294 for “closed” and page 292 for “exact.”

(2/25/18) **Page 693:** The index entry for “Hatcher, Allen” is misspelled.