CORRECTIONS TO
Introduction to Smooth Manifolds (Second Edition)
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(8/16) Page 6, just below the last displayed equation: Change \( \varphi([x]) \) to \( \varphi_{n+1}[x] \), and in the next line, change \( x^i \) to \( x^{n+1} \). After “(Fig. 1.4),” insert “with similar interpretations for the other charts.”

(8/16) Page 7, Fig. 1.4: Both occurrences of \( x^i \) should be \( x^{n+1} \).

(12/19/18) Page 9, proof of Theorem 1.15: In the second line of the proof, replace “For each \( j \)” with “For each \( j \geq 0 \).” Then in the fourth-to-last line, replace “positive integers” by “nonnegative integers.”

(1/15/21) Page 13, line 1: Delete the words “and injective.”

(1/18/21) Page 20, Example 1.31: There are multiple errors in this example. Replace everything after the first two sentences by the following: For each \( i = 1, \ldots, n+1 \), let \( (U_i^\pm \cap S^n, \varphi_i^\pm) \) denote the graph coordinate charts we constructed in Example 1.4. For any distinct indices \( i \) and \( j \) and any choices of \( \pm \) signs, the transition maps \( \varphi_i^+ \circ (\varphi_j^-)^{-1} \) and \( \varphi_i^- \circ (\varphi_j^+)^{-1} \) are easily computed. For example, in the case \( i < j \), we get the following formula for all \( u \) in the domain of \( \varphi_i^+ \circ (\varphi_j^+)^{-1} \):

\[
\varphi_i^+ \circ (\varphi_j^+)^{-1}(u^1, \ldots, u^n) = (u^1, \ldots, \hat{u}^i, \ldots, u^n, \sqrt{1 - |u^i|^2}, \ldots, u^n)
\]

(with \( u^i \) omitted and the square root replacing \( u^i \)), and similar formulas hold in the other cases. When \( i = j \), the domains of \( \varphi_i^+ \) and \( \varphi_i^- \) are disjoint, so there is nothing to check. Thus, the collection of charts \( \{(U_i^\pm \cap S^n, \varphi_i^\pm)\} \) is a smooth atlas, and so defines a smooth structure on \( S^n \). We call this its standard smooth structure.

(6/23/13) Page 23, two lines below the first displayed equation: Change “any subspace \( S \subseteq V \)” to “any \( k \)-dimensional subspace \( S \subseteq V \)”.

(9/15/19) Page 24, first full paragraph, fourth line: Change “any subspace \( S \)” to “any \( k \)-dimensional subspace \( S \).”

(12/19/18) Page 26, first line: Change \( U \cap \varphi^{-1}(\text{Int } \mathbb{H}^n) \) to \( \varphi^{-1}(\text{Int } \mathbb{H}^n) \).

(12/19/18) Page 27, last paragraph, sixth line: Change \( \bar{U} \cap \mathbb{H}^n \) to \( \bar{U} \cap U \).

(2/22/15) Page 29, proof of Theorem 1.46, second paragraph, line 4: Change \( \varphi(U \cap V) \to \psi(U \cap V) \).

(10/8/15) Page 30, Problem 1-6: Interpret the formula for \( F_s \) to mean \( F_s(0) = 0 \) when \( s \leq 1 \).

(1/27/18) Page 31, Fig. 1.13: Change \( \{x^n = 0\} \) to \( \{x^{n+1} = 0\} \).

(3/12/18) Page 31, Problem 11, next-to-last line: Change \( S^n \) to \( S^n \setminus \{N\} \).

(4/25/17) Page 45, second paragraph: Replace the last sentence of that paragraph with the following: “If \( N \) has empty boundary, we say that a map \( F : A \to N \) is smooth on \( A \) if it has a smooth extension in a neighborhood of each point: that is, if for every \( p \in A \) there exist an open subset \( W \subseteq M \) containing \( p \) and a smooth map \( \tilde{F} : W \to N \) whose restriction to \( W \cap A \) agrees with \( F \). When \( \partial N \neq \emptyset \), we say \( F : A \to N \) is smooth on \( A \) if for every \( p \in A \) there exist an open subset \( W \subseteq M \) containing \( p \) and a smooth chart \( (V, \psi) \) for \( N \) whose domain contains \( F(p) \), such that \( F(W \cap A) \subseteq V \) and \( \psi \circ F|_{W \cap A} \) is smooth as a map into \( \mathbb{R}^n \) in the sense defined above (i.e., it has a smooth extension in a neighborhood of each point).”

(7/23/14) Page 45, last displayed equation: The first = sign should be \( \subseteq \).

(9/15/19) Page 46, line 9: Change “on an open subset” to “on a nonempty open subset.”
Page 47, proof of Theorem 2.29, second paragraph: Replace the first sentence of the paragraph by “Let $h: \mathbb{R}^n \rightarrow \mathbb{R}$ be a smooth bump function that is positive in $B_1(0)$ and zero elsewhere.”

Page 49, Problem 2-10(c): Change “an isomorphism” to “a bijection.”

Page 54, just after the first sentence: Insert “(The integral is a smooth function of $x$ by iterative application of Theorem C.14.)”

Page 56, first displayed equation: Change $d(v)_p$ to $d_p(v)$.

Page 56, just below the last displayed equation: Replace “the last two equalities follow” by “the last equality follows.”

Page 58, proof of Lemma 3.11, next-to-last line: Change $\mathbb{H}^n$ to $\text{Int} \mathbb{H}^n$.

Page 68, proof of Proposition 3.21: Insert the following sentence at the beginning of the proof: “Let $n = \dim M$ and $m = \dim N$.” Then in the second sentence, change (3.9) to (3.10). Finally, in the displayed equation, change $F^n$ to $F^m$ (twice).

Page 70, two lines above Corollary 3.25: Change “Proposition 3.23” to “Proposition 3.24.”

Page 76, Problem 3-8: Add the following remark: “(For $p \in \partial M$, we need to allow curves with domain $[0, \varepsilon)$ or $(-\varepsilon, 0]$ and to interpret the derivatives as one-sided derivatives.)”

Page 78, proof of Prop. 4.1, third and fourth lines: Change $m \times n$ to $n \times m$ (twice).

Page 79, proof of Theorem 4.5, fourth line: Change $\tilde{F}(p)$ to $\tilde{F}(0)$.

Page 82, line 4 from the bottom: Change “This is a diffeomorphism onto its image” to “This is an open map and a diffeomorphism onto its image.”

Page 83, proof of Theorem 4.14, line 8: Change “no open subset” to “no nonempty open subset.”

Page 96, Problem 4-3: This problem probably needs a better hint. First, to get a good result, you’ll have to add the assumption that $\ker dF_p \subsetneq T_p \partial M$. After choosing smooth coordinates, you can assume $M \subset \mathbb{H}^m$ and $N \subset \mathbb{R}^n$, and extend $F$ to a smooth function $\tilde{F}$ on an open subset of $\mathbb{R}^m$. If $\text{rank } F = r$, show that there is a coordinate projection $\pi: \mathbb{R}^n \rightarrow \mathbb{R}^r$ such that $\pi \circ \tilde{F}$ is a submersion, and apply the rank theorem to $\pi \circ \tilde{F}$ to find new coordinates in which $\tilde{F}$ has a coordinate representation of the form $\tilde{F}(x, y) = (x, R(x, y))$. Then use the rank condition to show that $R|_M$ is independent of $y$.

Page 100, first sentence: At the end of the sentence, change “smooth embeddings” to “smooth embeddings of smooth manifolds.”

Page 100, proof of Proposition 5.4, next-to-last line: Change “It a homeomorphism” to “It is a homeomorphism.”

Page 104, line below the proof of Theorem 5.11: Change “See Theorem 5.31” to “See Problem 5-24.” [Problem 5-24 is a new problem, described later in this list. Theorem 5.31 is not appropriate in this situation because it applies only to manifolds without boundary.]

Page 105, line 4 from the bottom: Change $F$ to $\Phi$.

Page 112, Fig. 5.10: Interchange the labels $M$ and $N$ on the figure, to be consistent with the notation in Theorem 5.29.
Page 131, line 6: Change the definition of $\tilde{\psi}$ to $\tilde{\psi} = \pi \circ \psi|_{V_0}$. After the end of that sentence, insert the following: “To see that $\tilde{\psi}$ is a smooth coordinate map, let $i: V \hookrightarrow M$ be the inclusion map. Note first that for each $q \in V_0$, $x^{k+1}, \ldots, x^n$ are all constant on the image of $i$, so the image of $d_i q$ is contained in the span of $\partial / \partial x^1, \ldots, \partial / \partial x^k$. Since $d_i q$ is injective and its image has trivial intersection with $\text{Ker} d \tilde{\psi}_q$, it follows that $d \tilde{\psi}_q \circ d_i q$ is injective, so for dimensional reasons it is an isomorphism. Thus $\tilde{\psi} \circ i$ is a local diffeomorphism by the inverse function theorem. Since it is bijective from $V_0$ to its image, it is a diffeomorphism and hence a smooth coordinate map for $V$.”

Page 119, third line: Starting in the middle of that line, replace the rest of the proof with the following: “In the $H$ case, extend $F$ to a smooth map on an open subset of $\mathbb{R}^m$, and replace $U$ by that open subset; if we can show that the set of critical values of the extended map has measure zero, then the same is true of the set of critical values of $F$.”

Page 129, displayed equation near the bottom of the page: Change “ith partial derivatives” to “ith-order partial derivatives.”

Page 130, just below equation (6.2): Right after the displayed equation, insert “(where the component functions $F^2, \ldots, F^n$ might be different from the ones in the original coordinate chart).”

Page 131, two lines below the first displayed equation: Change $A'(R/K)^{k+1}$ to $A'(R\sqrt{m}/K)^{k+1}$.

Page 131, three lines below the first displayed equation: Insert “at most” before “$K^m$ balls.”

Page 131, second displayed equation: Change the left-hand side to $K^m(A')^n(R\sqrt{m}/K)^{n(k+1)}$, and in the next line, change the definition of $A''$ to $A'' = (A')^n(R\sqrt{m})^{n(k+1)}$.

Page 132, proof of Lemma 6.13, second paragraph: This argument does not apply when $\partial M \neq \emptyset$, because in that case $M \times M$ is not a smooth manifold with boundary. Instead, we can consider the restrictions of $\kappa$ to $(M \times \text{Int} M) \setminus \Delta_M$ and to $(M \times \partial M) \setminus \Delta_M$ (both of which are smooth manifolds with boundary), and note that there is a point $[v] \in \mathbb{R}P^{N-1}$ that is not in the image of $\tau$ or either of these restrictions of $\kappa$. [Thanks to David Iglesias Ponte for suggesting this correction.]
In case $M$ is an arbitrary compact subset of a larger manifold $zM$ with or without boundary, we can adapt this argument to obtain an embedding of a neighborhood of $M$ into $\mathbb{R}^{nm+m}$. After covering $M$ with finitely many regular coordinate balls or half-balls for $zM$, the argument above produces an injective immersion $F : \bigcup_i B_i \to \mathbb{R}^{nm+m}$, which is an embedding because its domain is compact; the restriction of this map to the union of the sets $B_i$ is the desired embedding. [This is needed in the ensuing argument for the noncompact case, because the sets $E_i$ might not be regular domains when $\partial M \neq \emptyset$.]

Page 134, displayed equations two-thirds of the way down the page: In the definition of $E_i$, there’s an “$i – 1$” that should be “$i – i$.” It should read $E_i = f^{-1}([b_{i-1}, a_{i+1}])$.

Page 134, just below the displayed equations two-thirds of the way down the page: Delete the sentence “By Proposition 5.47, each $E_i$ is a compact regular domain.” Two lines below that, replace “smooth embedding of $E_i$” with “smooth embedding of a neighborhood of $E_i$.”

Page 137, first paragraph under the subheading “Tubular Neighborhoods,” fifth line: Change $R^n$ to $\mathbb{R}^n$.

Page 138, proof of Theorem 6.23, end of the first paragraph: Change “standard coordinate frame” to “standard coordinate basis.”

Page 145, statement of Corollary 6.33: After “immersed submanifold,” insert “with $\dim S = \dim M$.”

Page 145, paragraph above Prop. 6.34: In the definition of smooth family of maps, replace “$F : M \times S \to N$” by “$F : N \times S \to M$.”

Page 146, equation (6.9): Should read $dF(T_{(p,s)}W) \subseteq T_qX$. [Change the equal sign to subset.]

Page 146, line below the last displayed equation: Change “$= T_qX$” to “$\subseteq T_qX$.”

Page 148, Problem 6-13: Delete part (c). [This statement is simply wrong. It is true with the added hypothesis that $F'$ is an embedding, but then it’s essentially just a restatement of part (b).]

Page 150, last line: Change “Theorem 20.16” to “Theorem 20.22.”

Page 160, first line: Change $R_{hh^{-1}}$ to $R_{h^{-1}h}$.

Page 164, just above the subheading: Replace the last line of the proof of Prop. 7.23 by “The action is smooth because each $\varphi$ can be written locally as a composition of a smooth local section followed by $\pi$.”

Page 169, first line: Change $\tilde{G}$ to $G$.

Page 169, statement of Theorem 7.35: Replace the phrase “closed Lie subgroups such that $N$ is normal” by “Lie subgroups such that $N$ is normal and closed.” [In fact, using the result of Theorem 19.25 later in the book, the hypothesis that $N$ is closed can also be omitted.]

Page 171, third line from the end of the proof: Change $E_i$ to $E_j$, so the formula reads $\rho_j^i(g) = \pi^t(g \cdot E_j)$.

Page 173, Problem 7-21: Replace the first sentence by “Prove that the groups in Problem 7-20 are isomorphic to direct products of the indicated groups in cases (a) and (c) if and only if $n$ is odd, and in cases (b) and (d) if and only if $n = 1$."

Page 178, Example 8.10(d): Change “Example 8.4” to “Example 8.5.”

Page 179, statement of Lemma 8.13: Change “local frame for $T \mathbb{R}^n$” to “local frame for $\mathbb{R}^n$.”

Page 184, Example 8.20, next-to-last line: Change $p = (u, v)$ to $q = (u, v)$. 
Page 196, proof of Proposition 8.45, next-to-last line: Should read “\( F_* \circ (F^{-1})_* = (F \circ F^{-1})_* = \text{Id}_{\text{Lie}(H)} \) and \( (F^{-1})_* \circ F_* = \text{Id}_{\text{Lie}(G)} \)”.

Page 208, first line: Change to “This is just the existence and smoothness statements of Theorem D.1 . . . .”

Page 213, first sentence of the last paragraph: The definition of \( t_0 \) should be \( t_0 = \sup \{ t \in \mathbb{R} : (t, p_0) \in W \} \).

Page 214, Fig. 9.6: The shaded area should be labeled \( W \), not \( D \).

Page 217, Fig. 9.7: Both occurrences of \( \phi \) should be \( \Phi \).

Page 219, second displayed equation: Change “\( V/ (0, p) = 0^+ \)” to “\( \Phi/(0, p) = 0^+ \)”.

Page 222, just below the section heading: Insert the following sentence: “On a manifold with boundary, the definitions of flow domain, flow, and infinitesimal generator of a flow are exactly the same as on a manifold without boundary.”

Page 223, line 2: Change \( \delta : M \to \mathbb{R}^+ \) to \( \delta : \partial M \to \mathbb{R}^+ \).

Page 223, proof of Theorem 9.26: There’s a gap in this proof, because it is not necessarily the case that \( M(a) \) is a regular domain in \( \text{Int} M \). To correct the problem, we have to choose our collar neighborhood more carefully. Replace the first sentence of the proof by the following paragraph:

“Theorem 9.25 shows that \( \partial M \) has a collar neighborhood \( C_0 \) in \( M \), which is the image of a smooth embedding \( E_0 : [0,1) \times \partial M \to M \) satisfying \( E_0(0,x) = x \) for all \( x \in \partial M \). Let \( f : M \to \mathbb{R}^+ \) be a smooth positive exhaustion function. Note that \( W = \{ (t,x) : f(E_0(t,x)) > f(x) - 1 \} \) is an open subset of \([0,1) \times \partial M\) containing \([0) \times \partial M\). Using a partition of unity as in the proof of Theorem 9.20, we may construct a smooth positive function \( \delta : \partial M \to \mathbb{R} \) such that \( (t,x) \in W \) whenever \( 0 \leq t < \delta(x) \). Define \( E : [0,1) \times \partial M \to M \) by \( E(t,x) = E_0(t \delta(x),x) \). Then \( E \) is a diffeomorphism onto a collar neighborhood \( C \) of \( \partial M \), and by construction \( f(E(t,x)) > f(x) - 1 \) for all \( (t,x) \in [0,1) \times \partial M \). We will show that for each \( a \in (0,1) \), the set \( E([0,a] \times \partial M) \) is closed in \( M \). Suppose \( p \) is a boundary point of \( E([0,a] \times \partial M) \) in \( M \); then there is a sequence \( \{ (t_i, x_i) \} \) in \([0,1] \times \partial M \) such that \( E(t_i, x_i) \to p \) in \( M \). Then \( f(E(t_i, x_i)) \) remains bounded, and thus \( f(x_i) < f(E(t_i, x_i)) + 1 \) also remains bounded. Since \( \partial M \) is closed in \( M \), \( f|\partial M \) is also an exhaustion function, and therefore the sequence \( \{ x_i \} \) lies in some compact subset of \( \partial M \). Passing to a subsequence, we may assume \( (t_i, x_i) \to (t_0, x_0) \), and therefore \( p = E(t_0, x_0) \in E([0,a] \times \partial M) \).

Then at the end of the first paragraph of the proof, add the following sentences:

“To see that \( M(a) \) is a regular domain, note first that it is closed in \( M \) because it is the complement of the open set \( C(a) \). Let \( p \in M(a) \) be arbitrary. If \( p \notin E([0,1] \times \partial M) \), then \( p \) has a neighborhood in \( \text{Int} M \) contained in \( M(a) \) by the argument above. If \( p \in E([0,1] \times \partial M) \), then \( p = E(a,x) \) for some \( x \in \partial M \), and \( C \) is a neighborhood of \( p \) in which \( M(a) \cap C \) is the diffeomorphic image of \( [a,1) \times \partial M \).”

Page 223, proof of Theorem 9.26, last line of the first paragraph: Change \( 0 \leq t < a \) to \( 0 \leq s < a \).

Page 225, Example 9.31: At the end of the example, insert the sentence “If \( n \geq 2 \), then \( M_1 \# M_2 \) is connected.”

Page 226, Example 9.32, fifth line: Replace the sentence beginning “It is a smooth manifold without boundary . . . .” by “It is a topological manifold without boundary, and can be given a smooth structure such that each of the natural maps \( M \to D(M) \) (induced by inclusion into the left and right summands of the disjoint union) is a smooth embedding.”
Page 230, line 1 and first displayed equation: Change \( \theta_1(x) \) to \( \theta_1(u) \) (twice).

Page 230, second paragraph: “from Case” should be “from Case 1.”

Page 230, fourth paragraph, last line: Change \([X, Y] \) to \([V, W] \).

Page 234, proof of Theorem 9.46, second paragraph: Replace the two parenthesized sentences by the following: “To see this, just choose \( \varepsilon_1 > 0 \) and \( U_1 \subseteq U \) such that \( \theta_1 \) maps \((-\varepsilon_1, \varepsilon_1) \times U_1 \) into \( U \), and then inductively choose \( \varepsilon_i \) and \( U_i \) such that \( \theta_i \) maps \((-\varepsilon_i, \varepsilon_i) \times U_i \) into \( U_{i-1} \). Taking \( \varepsilon = \min \{\varepsilon_i\} \) and \( Y = U_k \) does the trick.”

Page 241, Example 10.8, statement of Proposition 10.25, first line: Change “basis for” to “basis for homomorphisms.”

Page 241, Example 10.8, lines 6–8: Delete the second sentence of the hint. [Because \( N \) is inward-pointing along \( \partial M \), no integral curve that starts on \( \partial M \) can hit the boundary again, because the vector field would have to be tangent to \( \partial M \) or outward-pointing at the first such point.]

Page 248, first displayed equation: Should read

\[
V(t, p) = \left. \frac{\partial}{\partial s} \right|_{s=t} H_s(H_t^{-1}(p)).
\]

Page 248, Problem 9-22(c): Replace the problem statement by

(c) \( \frac{\partial u}{\partial x} + u \frac{\partial u}{\partial y} = -y \), \( u(0, y) = 0 \).

[Without this sign change, the third claim in Problem 9-23 is not true.]

Page 254, paragraph beginning “With respect to,” third line: Replace \( V_p \times \mathbb{R}^k \) with \( U_a \times \mathbb{R}^k \).

Page 255, Example 10.8, line 5: Replace the phrase “a bijective map \( \Phi|_U : (\pi|_S)^{-1} (U \cap S) \to (U \cap S) \times \mathbb{R}^k \)” with “a bijective map from \((\pi|_S)^{-1} (U \cap S)\) to \((U \cap S) \times \mathbb{R}^k \).” [The notation \( \Phi|_U \) is inappropriate here.]

Page 255, Example 10.8, lines 6–8: Replace the sentence beginning with “If \( E \) is a smooth vector bundle” by the following: “If \( E \) is a smooth vector bundle and \( S \subseteq M \) is an embedded submanifold, it follows easily from the chart lemma that \( E|_S \) is a smooth vector bundle. If \( S \) is merely immersed, we give \( E|_S \) a topology and smooth structure making it into a smooth rank-\( k \) vector bundle over \( S \) as follows: For each \( p \in S \), choose a neighborhood \( U \) of \( p \) in \( M \) over which there is a local trivialization \( \Phi \) of \( E \), and a neighborhood \( V \) of \( p \) in \( S \) that is embedded in \( M \) and contained in \( U \). Then the restriction of \( \Phi \) to \( \pi^{-1}(V) \) is a bijection from \( \pi^{-1}(V) \) to \( V \times \mathbb{R}^k \), and we can apply the chart lemma to these bijections to yield the desired structure.”

Page 255, Example 10.8, last line: Change “over \( M \)” to “over \( S \).”

Page 260, two lines above Proposition 10.22: Change \( \tau^n(p) \) to \( \tau^k(p) \).

Page 261, statement of Proposition 10.25, first line: Change \( \pi' : E \to M' \) to \( \pi' : E' \to M' \).

Page 263, first full paragraph: In the first two lines of the paragraph, change \( \sigma_1, \sigma_2 \) to \( \tau_1, \tau_2 \) (twice).

Page 264, paragraph above the subheading, first sentence: “homomorphism” should be “homomorphisms.”

Page 265, proof of Lemma 10.32, fifth line: Change “basis for \( D_p \) at each point \( p \in U \)” to “basis for \( D_q \) at each point \( q \in U \).”

Page 267, proof of Lemma 10.35, lines 3 & 4: Change “single slice in some coordinate ball or half-ball” to “single slice or half-slice in some coordinate ball.”
Page 271, Problem 10-18: Change “a properly embedded” to “an embedded.”

Page 271, Problem 10-19(d): Add the following: [Hint: For the “only if” direction, to show that F is compact, use a finite number of local trivializations to construct a closed set over which E is trivial.]

Page 276, proof of Proposition 11.9, first line: Change “Theorem 10.4” to “Proposition 10.4.”

Page 278, Example 11.13, third line: Change “every coordinate frame” to “every coordinate coframe.”

Page 296, line 6 from the bottom: Change “closed forms” to “closed covector fields” (twice).

Page 301, Problem 11-10(c): Change S^2 to S^2.

Page 301, Problem 11-13: Add the assumption that n > 0.

Page 303, just below the commutative diagram: Insert this sentence: “A natural transformation is called a natural isomorphism if each map X is an isomorphism in D.”

Page 303, Problem 11-18(b) and (c): Change “natural transformation” to “natural isomorphism” in both parts.

Page 317, paragraph beginning “Any one”: At the end of the paragraph, add this sentence: “If A and B are tensor fields, then A˝B denotes the tensor field defined by .A˝B/ p D A p˝B p .”

Page 317, displayed equation just below the middle of the page: Change A^i_1:::i_k j_1:::i_l to A^i_1:::i_k j_1:::j_l on the third line of the display, and again on the line below the display. [The last lower index should be j_l, not i_l.]

Page 320, statement of Proposition 12.25: Change the domain and codomain of G: It should read G: P ! M.


Page 333, first line: Change U ≤ M to V ≤ M.

Page 345, Problem 13-10: In the last line of the problem statement, change L_x (g) > L_x (g') to L_x (g) ≥ L_x (g'), and delete the phrase “unless g is a reparametrization of g'.” [Because the definition of reparametrization that I’m using requires a diffeomorphism of the parameter domain, the original problem statement was not true.]

Page 356, proof of Lemma 14.10: At the beginning of the proof, insert “Let (E_1, . . . , E_n) be the basis for V dual to (e^i ).”

Page 356, Case 4, second line: Should read “brings us back to Case 3.”

Page 368, second paragraph: At the end of the first sentence of the paragraph, insert “(see pp. 341–343).”

Page 368, paragraph below equation (14.25): Change TM to T R^3 (twice).

Page 371, three lines above (14.31): Change that sentence to “The only terms in this sum that can possibly be nonzero are those for which J has no repeated indices and m is equal to one of the indices in J, say m = j_p.”

Page 374, Problem 14-2: Add “[Hint: One way to approach this is to prove first that a k-covector ω is decomposable if and only if the map from R^n to Λ^{k-1}(R^n*) given by v ! v ∧ . ω has (n − k)-dimensional kernel.]”

Page 377, line 4: Change “is a simply” to “is simply.”

Page 382, proof of Proposition 15.6, second paragraph: In the first sentence of the paragraph, after “smooth chart,” insert “with connected domain.”

Page 386, just above Proposition 15.24: After “determines an orientation on ∂M,” insert “if M is oriented.”

Page 388, last paragraph: Change “Proposition 13.6” to “Corollary 13.8.”
(7/20/17) Page 389, Exercise 15.30: Change “a local isometry” to “an orientation-preserving local isometry.”

(5/9/20) Page 397, Problem 15-1: At the end of the last sentence, add “when \( n > 1 \).”

(5/14/20) Page 397, Problem 15-3: Change \( \mathbb{H}^n \) to \( \mathbb{H}^{n+1} \) (twice).

(5/28/22) Page 397, Problem 15-4: Change the first sentence to “Let \( \phi \) be the flow of a smooth vector field on an oriented smooth manifold.” [The stated result is true also for manifolds with boundary and for nonmaximal flows, but to prove it, one must first do a little work to generalize some of the results of Theorem 9.12 to more general flows.]

(4/26/14) Page 402, lines 2–3: There should not be a paragraph break before “and.”

(3/14/16) Page 403, just after the last displayed equation: Add “(In the \( H^n \) case, apply Theorem C.26 to the interiors of \( D \) and \( E \) considered as subsets of \( \mathbb{R}^n \)).”

(5/28/18) Page 409, line 2: Change ‘\( i \)’ to ‘\( \partial \).’

(6/2/16) Page 416, line 3 from the bottom: Change “\( \cdot/\mathcal{D}_p \)” to “\( \cdot/\mathcal{D}_{p/\mathcal{D}} \).”

(9/25/19) Page 418, statement of Proposition 16.21: Delete “compact,” and change “\( n \)-manifold” to “\( (n+1) \)-manifold.”

(6/24/18) Page 419, proof of Theorem 16.25, first paragraph: Replace the second and third sentences of the paragraph by the following: “By means of smooth charts and a partition of unity, we may reduce the theorem to the cases in which \( M = \mathbb{R}^n \), \( M = \mathbb{H}^n \), or \( M = \mathbb{R}^n \). The \( \mathbb{R}^n \) and \( \mathbb{H}^n \) cases are treated just as before.”

(9/3/23) Page 423, just above equation (16.11): Change “\( \beta : \mathcal{X}(M) \to \Omega^{n-1}(M) \)” to “\( \beta : T^*M \to \Lambda^{n-1}T^*M \).”

(7/22/15) Page 424, second displayed equation: Change \( \iota^*_S \beta(X) \) to \( \iota^*_M \beta(X) \).

(2/18/13) Page 426, three lines below the section heading: “cam” should be “can.”

(2/11/15) Page 430, Proposition 16.38(c): This statement is wrong. Change it to “If \( F \) is smooth, then \( F^*\mu \) is a continuous density on \( M \); and if \( F \) is a local diffeomorphism, \( F^*\mu \) is smooth.”

(5/31/22) Page 435, Problem 16-4: Change “manifold with boundary” to “manifold with nonempty boundary.”

(7/27/16) Page 439, Problem 16-23: The formula for \( g \) should be

\[
g = \frac{dx^2 + dy^2}{(1 - x^2 - y^2)^2}.
\]

(2/19/13) Page 444, two lines below equation (17.4): Change \( T_{(q,s)}M \to T_{(q,s)}(M \times \mathbb{R}) \).

(6/6/18) Page 447, Corollary 17.15: Change “every closed form is exact” to “every closed \( p \)-form is exact for \( p \geq 1 \).”

(5/15/15) Page 450, proof of Theorem 17.21, line 5: Change \( H^1_{\text{dr}}(\mathbb{S}^n) \to H^1_{\text{dr}}(\mathbb{S}^1) \).

(8/14/17) Page 451, proof of Corollary 17.25, next-to-last line: Change \( \text{Id}_{H^p_{\text{dr}}(S)} \to \text{Id}_{H^{p-1}_{\text{dr}}(S)} \).

(11/24/17) Pages 455–456, Proof of Theorem 17.32: The proof given in the book is incorrect, because the \( V_i \)’s might not be connected, so Theorem 17.30 does not apply to them. Here’s a corrected proof.

**Lemma.** If \( M \) is a noncompact connected manifold, there is a countable, locally finite open cover \( \{V_i\}_{i=1}^\infty \) of \( M \) such that each \( V_i \) is connected and precompact, and for each \( j \), there exists \( k > j \) such that \( V_j \cap V_k \neq \emptyset \).
Proof. Let \( \{ W_j \}_{j=1}^{\infty} \) be a countably infinite, locally finite cover of \( M \) by precompact, connected open sets (such a cover exists by Prop. 1.19 and Thm. 1.15). By successively deleting unneeded sets and renumbering, we can ensure that no \( W_j \) is contained in the union of the other \( W_i \)’s.

Let \( Y_1 = \bigcup_{i=2}^{\infty} W_i \). Because \( M \) is connected, each component of \( Y_1 \) meets \( W_1 \), and by local finiteness of \( \{ W_j \} \), there are only finitely many such components. Such a component is precompact in \( M \) if and only if it is a union of finitely many \( W_i \)’s. Let \( V_1 \) be the union of \( W_i \) together with all of the precompact components of \( Y_1 \), and let \( X_1 \) be the union of all \( W_i \)’s not contained in \( V_1 \). Then \( V_1 \) is connected and precompact, and \( X_1 \) has no precompact components. Proceeding by induction, suppose we have defined connected, precompact open sets \( V_1, \ldots, V_m \) whose union contains \( W_1 \cup \cdots \cup W_m \), and such that the union \( X_m \) of all the \( W_i \)’s not contained in \( V_1 \cup \cdots \cup V_m \) has no precompact components. Let \( j_m \) be the smallest index such that \( W_{j_m} \) is not contained in \( V_1 \cup \cdots \cup V_m \), and let \( Y_{m+1} \) be the union of all \( W_i \)’s other than \( W_{j_m} \) not contained in \( V_1 \cup \cdots \cup V_m \). Any precompact component of \( Y_{m+1} \) must meet \( W_{j_m} \), because otherwise, it would be a precompact component of \( X_m \). Let \( V_{m+1} \) be the union of \( W_{j_m} \) with all of the precompact components of \( Y_{m+1} \). As before, \( V_{m+1} \) is precompact and connected, and the union \( X_{m+1} \) of the \( W_i \)’s not contained in \( V_1 \cup \cdots \cup V_{m+1} \) has no precompact components. Then by construction, for each \( j \), the set \( X_j = \bigcup_{i>j} V_i \) has no precompact components. If some \( V_j \) does not meet \( V_k \) for any \( k > j \), then \( V_j \) itself is a precompact component of \( X_{j-1} \), which is a contradiction. Thus for each \( j \), there is some \( k > j \) such that \( V_j \cap V_k \neq \emptyset \).

Proof of Theorem 17.32. Choose an orientation on \( M \). Let \( \{ V_j \}_{j=1}^{\infty} \) be an open cover of \( M \) satisfying the conclusions of the preceding lemma. For each \( j \), let \( K(j) \) denote the least integer \( k > j \) such that \( V_j \cap V_k \neq \emptyset \), and let \( \theta_j \) be an \( n \)-form compactly supported in \( V_j \cap V_{K(j)} \) whose integral is \( 1 \). Let \( \{ \psi_j \}_{j=1}^{\infty} \) be a smooth partition of unity subordinate to \( \{ V_j \}_{j=1}^{\infty} \).

Now suppose \( \omega \) is any \( n \)-form on \( M \), and let \( \omega_j = \psi_j \omega \) for each \( j \). Let \( c_1 = \int_{V_1} \omega_1 \), so that \( \omega_1 - c_1 \theta_1 \) is compactly supported in \( V_1 \) and has zero integral. It follows from Theorem 17.30 that there exists \( \eta_1 \in \Omega_{e}^{n-1}(V_1) \) such that \( d\eta_1 = \omega_1 - c_1 \theta_1 \). Suppose by induction that we have found \( \eta_1, \ldots, \eta_m \) and constants \( c_1, \ldots, c_m \) such that for each \( j = 1, \ldots, m, \eta_j \in \Omega_{e}^{n-1}(V_j) \) and

\[
\frac{d\eta_j}{(\ast)} = \left( \omega_j + \sum_{i : K(i)=j} c_i \theta_i \right) - c_j \theta_j.
\]

Let

\[
c_{j+1} = \int_{V_{j+1}} \left( \omega_{j+1} + \sum_{i : K(i)=j+1} c_i \theta_i \right).
\]

Then by Theorem 17.30, there exists \( \eta_{j+1} \in \Omega_{e}^{n-1}(V_{j+1}) \) satisfying the analog of \((\ast)\) with \( j \) replaced by \( j+1 \). Set \( \eta = \sum_{j=1}^{\infty} \eta_j \), with each \( \eta_j \) extended to be zero on \( M \sim V_j \). By local finiteness, this is a smooth \((n-1)\)-form on \( M \). It satisfies

\[
d\eta = \omega + \sum_{j=1}^{\infty} \left( \sum_{i : K(i)=j} c_i \theta_i \right) - \sum_{j=1}^{\infty} c_j \theta_j.
\]

Each term \( c_i \theta_i \) appears exactly once in the first sum above, so the two sums cancel each other. \( \square \)

(7/27/16) Page 457, line below the second displayed equation: Change “Theorem 17.31” to “Theorem 17.30.”

(7/12/16) Page 463, line above equation (17.15): Insert missing space before “Similarly.”

(7/13/16) Page 464, end of proof of Corollary 17.42: Insert “Note that this construction produces a form \( \sigma \) whose support is contained in \( U \cap V \).” [This might be useful for solving Problem 18-6.]

(7/12/16) Page 471, last paragraph: Replace the sentence starting “The hardest part . . .” with “The hardest part is showing that the singular chain complex of \( M \) can be replaced by a chain complex built out of simplices whose images lie in either \( U \) or \( V \), without changing the homology.”
(9/12/17) Page 487, Problem 18-1, first line: Change “an oriented smooth manifold” to “a smooth manifold.”

(8/8/18) Page 489, Problem 18-7(b): Add to the hint: “In order to use Lemma 17.27, you’ll need to prove the following fact: Every bounded convex open subset of \( \mathbb{R}^n \) is diffeomorphic to \( \mathbb{R}^n \).” To prove this, let \( U \) be such a subset, and without loss of generality assume 0 \( \in U \). First show that there exists a smooth nonnegative function \( f \in C^\infty(U) \) such that \( f(0) = 0 \) and \( f(x) \geq 1/d(x) \) away from a small neighborhood of 0, where \( d(x) \) is the distance from \( x \) to \( \partial U \). Next, show that \( g(x) = 1 + \int_0^1 t^{-1} f(tx) \, dt \) is a smooth positive exhaustion function on \( U \) that is nondecreasing along each ray starting at 0. Finally, show that the map \( F: U \to \mathbb{R}^n \) given by \( F(x) = g(x)x \) is a bijective local diffeomorphism. Also, you may use the fact that the conclusion of the five lemma is still true even if the appropriate diagram commutes only up to sign.”

(1/15/13) Page 491, Example 19.1(c): Delete the word “unit.”


(12/17/15) Page 492, proof of Proposition 19.2, fourth line: Change “Given \( p \in M \)” to “Given \( p \in U \).”


(6/1/20) Page 512, Problem 20-11(b): Here’s a better hint, which doesn’t require proving part (a) first: “[Hint: Consider the graph of \( F \) as a subgroup of \( G \times H \).]”

(10/18/17) Page 542, middle of the paragraph before Example 21.3: Change “the action of \( \mathbb{R}^k \) on \( \mathbb{R}^n \)” to “the action of \( \mathbb{R}^k \) on \( \mathbb{R}^k \times \mathbb{R}^n \).”

(2/25/18) Page 548, last two lines: Allen Hatcher’s name is misspelled. (Sorry, Allen.)
Change the first phrase of that sentence to “Second, if \( p, p' \in E \) are in different orbits and \( \pi(p) \neq \pi(p') \). Then add the following sentences at the end of the proof: “If \( p \) and \( p' \) are in different orbits and \( \pi(p) = \pi(p') \), let \( W \) be an evenly covered neighborhood of \( \pi(p) \), and let \( V, V' \) be the components of \( \pi^{-1}(W) \) containing \( p \) and \( p' \), respectively. For any \( g \in \text{Aut}_\pi(E) \), a simple connectedness argument shows that \( g \cdot V \) is a component of \( \pi^{-1}(W) \); if it had nontrivial intersection with \( V \) it would have to be equal to \( V \), which would imply \( g \cdot p = p' \), a contradiction.”

Insert “a” before “2-covector.”

The coordinates should be \( x^1, \ldots, x^n, y^1, \ldots, y^n \). (The last coordinate is \( y^n \), not \( x^n \).)

Delete the spurious word “theorem” at the end of the line.

Replace the sentence starting “On the other hand” by this: “On the other hand, the left-hand side is just the ordinary \( t \)-derivative of a time-dependent tensor on a fixed vector space, and expanding in terms of a basis shows that it satisfies a similar product rule.”

Change “\( V \wedge W \mathcal{J} M \)” to “\( V \wedge W \mathcal{J} TM \);” and change \( \mathcal{J} \) to \( \mathcal{J} \).

Change \( R^{2n+1} \sim \{0\} \) to \( R^{2n+2} \sim \{0\} \).

Page 583, third displayed equation: Should read
\[
T \mathcal{J} d\Theta = -2 \sum_{i=1}^{n+1} (x^i dx^i + y^i dy^i) = -d(|x|^2 + |y|^2).
\]

Page 583, two lines below the third displayed equation: The formula for \( d\Theta(N, T) \) should be \( d\Theta(N, T) = 2(|x|^2 + |y|^2) \).

Page 584, Exercise 22.29: Part (b) should read
\[
(b) \ T = \frac{\partial}{\partial z};
\]

Page 584, paragraph above Theorem 22.33: Change all occurrences of \( \theta \) in this paragraph to \( \psi \), to avoid confusion with the use of \( \theta \) for a contact form elsewhere in this section.

Page 585, statement of Theorem 22.34, last line: Change \( H \) to \( F \).

Page 587, equation (22.27): Change both occurrences of \( \sigma(s) \) to \( \sigma(x) \).

Page 591, Problem 22.5: Add the hypothesis \( n > 0 \).

Page 592, Problem 22.15: Add the hypothesis that \( M \) is connected.

Page 608, Proposition A.41(a): Insert the following phrase at the beginning of this statement: With the exception of the word “closed” in part (d).

Page 616, Proposition A.77(b), last line: Change \( \tilde{f}(0) \) to \( \tilde{f}_0(0) \).

Page 619, proof of Lemma B.2, fourth line: Replace “By Exercise B.1(b)” with “If \( w_1 \) is equal to one of the \( u_i \)’s, then the ordered \( (n + 1) \)-tuple \( (u_1, v_1, \ldots, v_n) \) is linearly dependent; if not, then by Exercise B.1(b), . . . .”

Page 632, Exercise B.29: Change “by a matrix” to “by a certain matrix” (twice).
(12/19/18) Page 637, Exercise B.42: Delete the words “is a homeomorphism that.” [Checking that it’s a homeomorphism requires the norm topology, which is not defined until later on that page.]

(9/6/16) Page 637, Exercise B.44: Change “basis map” to “basis isomorphism.”

(12/19/18) Page 653, proof of Proposition C.21, second paragraph, second line: Change $f$ to $f_D$.

(2/25/18) Page 658, two lines above (C.15): Change $B_{\delta}(0)$ to $\overline{B}_{\delta}(0)$.

(2/25/18) Page 660, display (C.20): Change $F^{-1}(x)$ to $F^{-1}(y)$.

(1/18/21) Page 664, statement of Theorem D.1(b): After the phrase “Any two differentiable solutions to (D.3)–(D.4),” insert “defined on intervals containing $t_0$.”

(12/2/15) Page 666, just below the fifth display: After the sentence ending “by our choice of $\delta$ and $\epsilon$,” insert “(If $t < t_0$, interchange $t$ and $t_0$ in the second line above.)”

(1/18/21) Page 664, statement of Theorem D.4: After the phrase “any two differentiable solutions to (D.3)–(D.4),” insert “defined on intervals containing $t_0$.”

(1/18/21) Page 668, paragraph below equation (D.10): In the fourth line of the paragraph, change $\overline{W}$ to $W$.

(1/18/21) Page 670, displayed inequality between (D.17) and (D.18): Change $n$ to $n^2$.

(1/18/21) Page 670, last line: Change $n$ to $n^2$ in the definition of $B$.

(1/18/21) Page 671, inequality (D.19): Change $n$ to $n^2$ (twice).

(12/15/20) Page 671, just below (D.19): Replace the sentence “Since the expression on the right can be made as small as desired by choosing $h$ and $\overline{h}$ sufficiently small, this shows . . .” by the following: “Thus the expression on the left can be made as small as desired by choosing $h$ and $\overline{h}$ sufficiently small. This shows . . .”

(6/11/19) Page 692: Under the entry for “Form,” delete the references to page 294 for “closed” and page 292 for “exact.”

(2/25/18) Page 693: The index entry for “Hatcher, Allen” is misspelled.