Corrections to
Introduction to Smooth Manifolds (Second Edition)
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(8/16) Page 6, just below the last displayed equation: Change \( \varphi([x]) \) to \( \varphi_{n+1}[x] \). and in the next line, change \( x^i \) to \( x^{n+1} \). After “(Fig. 1.4),” insert “with similar interpretations for the other charts.”

(8/16) Page 7, Fig. 1.4: Both occurrences of \( x^i \) should be \( x^{n+1} \).

(12/18) Page 9, proof of Theorem 1.15: In the second line of the proof, replace “For each \( j \)” with “For each \( j \geq 0 \).” Then in the fourth-to-last line, replace “positive integers” by “nonnegative integers.”

(1/18) Page 20, Example 1.31: There are multiple errors in this example. Replace everything after the first two sentences by the following: For each \( i = 1, \ldots, n+1 \), let \( (U_i^\pm \cap S^n, \varphi_i^\pm) \) denote the graph coordinate charts we constructed in Example 1.4. For any distinct indices \( i \) and \( j \) and any choices of \( \pm \) signs, the transition maps \( \varphi_i^\pm \circ (\varphi_j^\pm)^{-1} \) and \( \varphi_i^\pm \circ (\varphi_j^\mp)^{-1} \) are easily computed. For example, in the case \( i < j \), we get the following formula for all \( u \) in the domain of \( \varphi_i^+ \circ (\varphi_j^+)^{-1} \):

\[
\varphi_i^+ \circ (\varphi_j^+)^{-1} (u^1, \ldots, u^n) = (u^1, \ldots, u^j, \sqrt{1-|u|}^{1/n}, u^{j+1}, \ldots, u^n)
\]

(with \( u^j \) omitted and the square root replacing \( u^j \)), and similar formulas hold in the other cases. When \( i = j \), the domains of \( \varphi_i^+ \) and \( \varphi_i^- \) are disjoint, so there is nothing to check. Thus, the collection of charts \( \{(U_i^\pm \cap S^n, \varphi_i^\pm)\} \) is a smooth atlas, and so defines a smooth structure on \( S^n \). We call this its standard smooth structure.

(6/13) Page 23, two lines below the first displayed equation: Change “any subspace \( S \subseteq V \)” to “any \( k \)-dimensional subspace \( S \subseteq V \)”.

(9/19) Page 24, first full paragraph, fourth line: Change “any subspace \( S \)” to “any \( k \)-dimensional subspace \( S \)”.

(12/18) Page 26, first line: Change \( U \cap \varphi^{-1}(\text{Int } \mathbb{H}^n) \) to \( \varphi^{-1}(\text{Int } \mathbb{H}^n) \).

(12/18) Page 27, last paragraph, sixth line: Change \( \bar{U} \cap \mathbb{H}^n \) to \( \bar{U} \cap U \).

(2/22) Page 29, proof of Theorem 1.46, second paragraph, line 4: Change \( \varphi(U \cap V) \) to \( \psi(U \cap V) \).

(10/8) Page 30, Problem 1-6: Interpret the formula for \( F_s \) to mean \( F_s(0) = 0 \) when \( s \leq 1 \).

(1/17) Page 31, Fig. 1.13: Change \{\( x^n = 0 \)\} to \{\( x^{n+1} = 0 \)\}.

(3/12) Page 31, Problem 1-11, next-to-last line: Change \( S^n \) to \( S^n \setminus \{N\} \).

(4/25) Page 45, second paragraph: Replace the last sentence of that paragraph with the following: “If \( N \) has empty boundary, we say that a map \( F: A \to N \) is smooth on \( A \) if it has a smooth extension in a neighborhood of each point: that is, if for every \( p \in A \) there exist an open subset \( W \subseteq M \) containing \( p \) and a smooth map \( \tilde{F}: W \to N \) whose restriction to \( W \cap A \) agrees with \( F \). When \( \partial N \neq \emptyset \), we say \( F: A \to N \) is smooth on \( A \) if for every \( p \in A \) there exist an open subset \( W \subseteq M \) containing \( p \) and a smooth chart \( (V, \psi) \) for \( N \) whose domain contains \( F(p) \), such that \( F(W \cap A) \subseteq V \) and \( \psi \circ F|_{W \cap A} \) is smooth as a map into \( \mathbb{R}^n \) in the sense defined above (i.e., it has a smooth extension in a neighborhood of each point).”
(7/23/14) Page 45, last displayed equation: The first $\pm$ sign should be $\subseteq$.

(9/15/19) Page 46, line 9: Change “on an open subset” to “on a nonempty open subset.”

(6/20/18) Page 47, proof of Theorem 2.29, second paragraph: Replace the first sentence of the paragraph by “Let $h: \mathbb{R}^n \to \mathbb{R}$ be a smooth bump function that is positive in $B_1(0)$ and zero elsewhere.”

(2/13/22) Page 49, Problem 2-10(c): Change “is an isomorphism” to “is a bijection.”

(1/20/22) Page 54, just after the first sentence: Insert “(The integral is a smooth function of $x$ by iterative application of Theorem C.14.)”

(11/17/12) Page 56, first displayed equation: Change $d.v/_{p}$ to $d_{p}.v/$.  

(1/21/21) Page 56, just below the last displayed equation: Replace “the last two equalities follow” by “the last equality follows.”

(5/4/13) Page 96, Problem 4-3: This problem probably needs a better hint. First, to get a good result, you’ll have to add the assumption that $\ker dF_p \not\subset T_p \partial M$. After choosing smooth coordinates, you can assume $M \subseteq \mathbb{H}^m$ and $N \subseteq \mathbb{R}^n$, and extend $F$ to a smooth function $\tilde{F}$ on an open subset of $\mathbb{R}^m$. If $\text{rank } F = r$, show that there is a coordinate projection $\pi: \mathbb{R}^n \to \mathbb{R}^r$ such that $\pi \circ \tilde{F}$ is a submersion, and apply the rank theorem to $\pi \circ \tilde{F}$ to find new coordinates in which $\tilde{F}$ has a coordinate representation of the form $\tilde{F}(x,y) = (x,R(x,y))$. Then use the rank condition to show that $R|_M$ is independent of $y$. 

(12/22/21) Page 100, first sentence: At the end of the sentence, change “smooth embeddings” to “smooth embeddings of smooth manifolds.”

(9/8/15) Page 100, proof of Proposition 5.4, next-to-last line: Change “It a homeomorphism” to “It is a homeomorphism.”

(7/8/19) Page 104, line below the proof of Theorem 5.11: Change “See Theorem 5.31” to “See Problem 5-24.” [Problem 5-24 is a new problem, described later in this list. Theorem 5.31 is not appropriate in this situation because it applies only to manifolds without boundary.]

(6/9/19) Page 105, line 4 from the bottom: Change $F$ to $\Phi$.  

(11/9/16) Page 112, Fig. 5.10: Interchange the labels $M$ and $N$ on the figure, to be consistent with the notation in Theorem 5.29.
Page 113, line 6: Change the definition of \( \tilde{\psi} \) to \( \tilde{\psi} = \pi \circ \psi |_{V_0} \). After the end of that sentence, insert the following: “To see that \( \tilde{\psi} \) is a smooth coordinate map, let \( i: V \hookrightarrow M \) be the inclusion map. Note first that for each \( q \in V_0 \), \( x^{k+1}, \ldots, x^n \) are all constant on the image of \( i \), so the image of \( d\tilde{\psi}_q \) is contained in the span of \( \partial/\partial x^1, \ldots, \partial/\partial x^k \). Since \( d\tilde{\psi}_q \) is injective and its image has trivial intersection with \( \text{Ker} d\tilde{\psi}_q \), it follows that \( d\tilde{\psi}_q \circ d\tilde{\phi}_q \) is injective, so for dimensional reasons it is an isomorphism. Thus \( \tilde{\psi} \circ i \) is a local diffeomorphism by the inverse function theorem. Since it is bijective from \( V \) to its image, it is a diffeomorphism and hence a smooth coordinate map for \( V \).”

Page 118, Fig. 5.13: Change the definition of \( z \) to “smooth map whose image is contained in \( \partial M \) to prove that \( \partial M \) has a unique smooth structure making it an embedded submanifold of \( M \).”

Page 120, proof of Proposition 5.46: At the beginning of the proof, insert this sentence: “Let \( F: D \hookrightarrow M \) denote the inclusion map.”

Page 121, line 5: Change \( x^n \) to \( x^m \).

Page 123, Problem 5-6: Add the assumption that \( m > 0 \).

Page 124: At the end of the page, add a new problem:

5-24. Suppose \( M \) is a smooth manifold with boundary, \( N \) is a smooth manifold, and \( F: N \rightarrow M \) is a smooth map whose image is contained in \( \partial M \). Show that \( F \) is smooth as a map into \( \partial M \), and use this to prove that \( \partial M \) has a unique smooth structure making it an embedded submanifold of \( M \).

Page 129, proof of Sard’s theorem, second paragraph: Just before the last sentence of the paragraph, insert the following: “In the \( \mathbb{H}^n \) case, extend \( F \) to a smooth map on an open subset of \( \mathbb{R}^m \), and replace \( U \) by that open subset; if we can show that the set of critical values of the extended map has measure zero, then the same is true of the set of critical values of \( F \).”

Page 129, displayed equation near the bottom of the page: Change “\( i \)th partial derivatives” to “\( i \)th-order partial derivatives.”

Page 130, just below equation (6.2): Right after the displayed equation, insert “(where the component functions \( F^2, \ldots, F^n \) might be different from the ones in the original coordinate chart).”

Page 131, two lines below the first displayed equation: Change \( A'(R/K)^k+1 \) to \( A'(R\sqrt{m}/K)^k+1 \).

Page 131, three lines below the first displayed equation: Insert “at most” before “\( K^m \) balls.”

Page 131, second displayed equation: Change the left-hand side to \( K^m(A')^n(R\sqrt{m}/K)^{n(k+1)} \), and in the next line, change the definition of \( A'' \) to \( A'' = (A')^n(R\sqrt{m})^{n(k+1)} \).

Page 132, proof of Lemma 6.13, second paragraph: This argument does not apply when \( \partial M \neq \emptyset \), because in that case \( M \times M \) is not a smooth manifold with boundary. Instead, we can consider the restrictions of \( \kappa \) to \( (M \times \text{Int} M) \sim \Delta_M \) and to \( (M \times \partial M) \sim \Delta_M \) (both of which are smooth manifolds with boundary), and note that there is a point \( [v] \in \mathbb{R}^{n-1} \) that is not in the image of \( \tau \) or either of these restrictions of \( \kappa \). [Thanks to David Iglesias Ponte for suggesting this correction.]
In case $M$ is an arbitrary compact subset of a larger manifold $\mathcal{M}$ with or without boundary, we can adapt this argument to obtain an embedding of a neighborhood of $M$ into $\mathbb{R}^{nm+\dim M}$. After covering $M$ with finitely many regular coordinate balls or half-balls for $\mathcal{M}$, the argument above produces an injective immersion $F: \bigcup B_i \to \mathbb{R}^{nm+\dim M}$, which is an embedding because its domain is compact; the restriction of this map to the union of the sets $B_i$ is the desired embedding. [This is needed in the ensuing argument for the noncompact case, because the sets $E_i$ might not be regular domains when $\partial M \neq \emptyset$.]
(7/25/16) Page 226, Example 9.32, fifth line: Replace the sentence beginning “It is a smooth manifold without boundary ...” by “It is a topological manifold without boundary, and can be given a smooth structure such that each of the natural maps $M \to D(M)$ (induced by inclusion into the left and right summands of the disjoint union) is a smooth embedding.”

(3/2/21) Page 230, line 1 and first displayed equation: Change $\theta_j(x)$ to $\theta_j(u)$ (twice).

(4/23/13) Page 230, second paragraph: “from Case” should be “from Case 1.”

(2/26/18) Page 230, fourth paragraph, last line: Change $\langle X, Y \rangle$ to $\langle V, W \rangle$.

(9/8/18) Page 234, proof of Theorem 9.46, second paragraph: Replace the two parenthesized sentences by the following: “(To see this, just choose $\varepsilon_1 > 0$ and $U_1 \subseteq U$ such that $\theta_1$ maps $(-\varepsilon_1, \varepsilon_1) \times U_1$ into $U$, and then inductively choose $\varepsilon_i$ and $U_i$ such that $\theta_i$ maps $(-\varepsilon_i, \varepsilon_i) \times U_i$ into $U_{i-1}$. Taking $\varepsilon = \min \{\varepsilon_i\}$ and $Y = U_k$ does the trick.)”

(5/29/16) Page 241, Example 9.52: At the end of the example, add the sentence “Note that $u$ is smooth on the open set $R^2 \setminus \{0\}$, which is a neighborhood of $S$.”

(6/4/14) Page 246, Problem 9-11: Delete the second sentence of the hint. [Because $N$ is inward-pointing along $\partial M$, no integral curve that starts on $\partial M$ can hit the boundary again, because the vector field would have to be tangent to $\partial M$ or outward-pointing at the first such point.]

(11/17/21) Page 248, first displayed equation: Should read
\[ V(t, p) = \frac{\partial}{\partial s} \big|_{s=0} H_s(H_t^{-1}(p)). \]

(11/12/16) Page 248, Problem 9-22(c): Replace the problem statement by

(c) $\frac{\partial u}{\partial x} + u \frac{\partial u}{\partial y} = -y, \quad u(0, y) = 0.$

[Without this sign change, the third claim in Problem 9-23 is not true.]

(11/16/20) Page 254, paragraph beginning “With respect to,” third line: Replace $V_p \times R^k$ with $U_\alpha \times R^k$.

(11/4/21) Page 255, Example 10.8, line 5: Replace the phrase “a bijective map $\Phi|_U: (\pi|_S)^{-1}(U \cap S) \to (U \cap S) \times R^k$” with “a bijective map from $(\pi|_S)^{-1}(U \cap S)$ to $(U \cap S) \times R^k$.” [The notation $\Phi|_U$ is inappropriate here.]

(6/17/19) Page 255, Example 10.8, lines 6-8: Replace the sentence beginning with “If $E$ is a smooth vector bundle” by the following: “If $E$ is a smooth vector bundle and $S \subseteq M$ is an embedded submanifold, it follows easily from the chart lemma that $E|_S$ is a smooth vector bundle. If $S$ is merely immersed, we give $E|_S$ a topology and smooth structure making it into a smooth rank-$k$ vector bundle over $S$ as follows: For each $p \in S$, choose a neighborhood $U$ of $p$ in $M$ over which there is a local trivialization $\Phi$ of $E$, and a neighborhood $V$ of $p$ in $S$ that is embedded in $M$ and contained in $U$. Then the restriction of $\Phi$ to $\pi^{-1}(V)$ is a bijection from $\pi^{-1}(V)$ to $V \times R^k$, and we can apply the chart lemma to these bijections to yield the desired structure.”

(3/30/21) Page 255, Example 10.8, last line: Change “over $M$” to “over $S$.”

(11/27/20) Page 260, two lines above Proposition 10.22: Change $r^\alpha(p)$ to $r^k(p)$.

(10/22/18) Page 261, statement of Proposition 10.25, first line: Change $\pi': E \to M'$ to $\pi': E' \to M'$.

(4/2/21) Page 263, first full paragraph: In the first two lines of the paragraph, change $\sigma_1, \sigma_2$ to $\tau_1, \tau_2$ (twice).
Page 264, paragraph above the subheading, first sentence: “homomorphism” should be “homomorphisms.”

Page 265, proof of Lemma 10.32, fifth line: Change “basis for $D_p$ at each point $p \in U$” to “basis for $D_q$ at each point $q \in U$.”

Page 267, proof of Lemma 10.35, lines 3 & 4: Change “single slice in some coordinate ball or half-ball” to “single slice or half-slice in some coordinate ball.”

Page 271, Problem 10-18: Change “a properly embedded” to “an embedded.”

Page 271, Problem 10-19(d): Add the following: [Hint: For the “only if” direction, to show that $F$ is compact, use a finite number of local trivializations to construct a closed set over which $E$ is trivial.]

Page 276, proof of Proposition 11.9, first line: Change “Theorem 10.4” to “Proposition 10.4.”

Page 278, Example 11.13, third line: Change “every coordinate frame” to “every coordinate coframe.”

Page 296, line 6 from the bottom: Change “closed forms” to “closed covector fields” (twice).

Page 301, Problem 11-10(c): Change $S^2$ to $\mathbb{S}^2$.

Page 301, Problem 11-13: Add the assumption that $n > 0$.

Page 303, just below the commutative diagram: Insert this sentence: “A natural transformation is called a natural isomorphism if each map $\lambda_X$ is an isomorphism in $D$.”

Page 303, Problem 11-18(b) and (c): Change “natural transformation” to “natural isomorphism” in both parts.

Page 317, paragraph beginning “Any one”: At the end of the paragraph, add this sentence: “If $A$ and $B$ are tensor fields, then $A \otimes B$ denotes the tensor field defined by $(A \otimes B)_p = A_p \otimes B_p$.”

Page 317, displayed equation just below the middle of the page: Change $A_{i_1\ldots i_k}$ to $A_{j_1\ldots j_l}$ on the third line of the display, and again on the line below the display. [The last lower index should be $j_l$, not $i_l$.]

Page 320, statement of Proposition 12.25: Change the domain and codomain of $G$: It should read $G: P \to M$.

Page 320, Proposition 12.25(e): Should read $(F \circ G)^*B = G^*(F^*B)$.

Page 333, first line: Change $U \subseteq M$ to $V \subseteq M$.

Page 345, Problem 13-10: In the last line of the problem statement, change $L_{\bar{\gamma}}(\gamma) > L_{\bar{\gamma}}(\gamma)$ to $L_{\bar{\gamma}}(\gamma) \geq L_{\bar{\gamma}}(\gamma)$, and delete the phrase “unless $\bar{\gamma}$ is a reparametrization of $\gamma$.” [Because the definition of reparametrization that I’m using requires a diffeomorphism of the parameter domain, the original problem statement was not true.]

Page 355, proof of Lemma 14.10: At the beginning of the proof, insert “Let $(E_1, \ldots, E_n)$ be the basis for $V$ dual to $(e^i)$.”

Page 356, Case 4, second line: Should read “brings us back to Case 3.”

Page 368, second paragraph: At the end of the first sentence of the paragraph, insert “(see pp. 341–343).”

Page 368, paragraph below equation (14.25): Change $TM$ to $T\mathbb{R}^3$ (twice).
The only terms in this sum that can possibly be nonzero are those for which \( J \) has no repeated indices and \( m \) is equal to one of the indices in \( J \), say \( m = j_p \).

Add “[Hint: One way to approach this is to prove first that a \( k \)-covector \( \omega \) is decomposable if and only if the map from \( \mathbb{R}^n \) to \( \Lambda^{k-1}(\mathbb{R}^n^*) \) given by \( v \mapsto v \cdot \omega \) has \((n-k)\)-dimensional kernel.]”

Page 377, line 4: Change “is a simply” to “is simply . ”

Page 386, just above Proposition 15.24: After “determines an orientation on \( \partial M \),” insert “if \( M \) is oriented.”

Page 388, last paragraph: Change “Proposition 13.6” to “Corollary 13.8.”

Page 389, Exercise 15.30: Change “a local isometry” to “an orientation-preserving local isometry.”

Page 397, Problem 15-1: At the end of the last sentence, add “when \( n > 1 \).”

Page 397, Problem 15-3: Change \( \mathbb{B}^n \) to \( \mathbb{B}^n+1 \) (twice).

Page 397, Problem 15-4: Change the first sentence to “Let \( \theta \) be the flow of a smooth vector field on an oriented smooth manifold.” [The stated result is true also for manifolds with boundary and for nonmaximal flows, but to prove it, one must first do a little work to generalize some of the results of Theorem 9.12 to more general flows.]

Page 402, lines 2–3: There should not be a paragraph break before “and.”

Page 403, just after the last displayed equation: Add “(In the \( \mathbb{H}^n \) case, apply Theorem C.26 to the interiors of \( D \) and \( E \) considered as subsets of \( \mathbb{R}^n \).)”

Page 409, line 2: Change \( \varphi_i \) to \( \varphi \).

Page 415, paragraph above Example 16.19: Change “interior charts and charts with corners” to “interior charts, boundary charts, and charts with corners.”

Page 416, line 3 from the bottom: Change “\( \gamma(0) = p \)” to “\( \gamma(0) = \psi(p) \).”

Page 418, statement of Proposition 16.21: Delete “compact,” and change “\( n \)-manifold” to “\( (n+1) \)-manifold.”

Page 419, proof of Theorem 16.25, first paragraph: Replace the second and third sentences of the paragraph by the following: “By means of smooth charts and a partition of unity, we may reduce the theorem to the cases in which \( M = \mathbb{R}^n \), \( M = \mathbb{H}^n \), or \( M = \mathbb{R}^n_{++} \). The \( \mathbb{R}^n \) and \( \mathbb{H}^n \) cases are treated just as before.”

Page 423, just above equation (16.11): Change \( \beta : \mathfrak{X}(M) \to \Omega^{n-1}(M) \)” to “\( \beta : TM \to \Lambda^{n-1}T^*M \)”

Page 424, second displayed equation: Change \( t^*_S \beta(X) \) to \( t^*_{SM} \beta(X) \).

Page 426, three lines below the section heading: “cam” should be “can.”

Page 430, Proposition 16.38(c): This statement is wrong. Change it to “If \( F \) is smooth, then \( F^* \mu \) is a continuous density on \( M \); and if \( F \) is a local diffeomorphism, \( F^* \mu \) is smooth.”
Page 435, Problem 16-4: Change “manifold with boundary” to “manifold with nonempty boundary.”

Page 439, Problem 16-23: The formula for \( g \) should be

\[
g = \frac{dx^2 + dy^2}{(1 - x^2 - y^2)^2}.
\]

Page 444, two lines below equation (17.4): Change \( T_{(q,q)} M \) to \( T_{(q,q)}(M \times \mathbb{R}) \).

Page 444, two lines below equation (17.4): Change \( j \) we have defined connected, precompact open sets meet \( V \) by construction, for each \( j \) and the union \( X \) of all \( W_j \) is connected and precompact, and for each \( j \) and the union \( Y \) of all \( W_j \)

Page 450, proof of Theorem 17.21, line 5: Change \( H^1_{dR}(S^n) \) to \( H^1_{dR}(S^1) \).

Pages 455–456, Proof of Theorem 17.32: The proof given in the book is incorrect, because the \( V_i \)'s might not be connected, so Theorem 17.30 does not apply to them. Here's a corrected proof.

Lemma. If \( M \) is a noncompact connected manifold, there is a countable, locally finite open cover \( \{V_j\}_{j=1}^\infty \) of \( M \) such that each \( V_j \) is connected and precompact, and for each \( j \), there exists \( k > j \) such that \( V_j \cap V_k \neq \emptyset \).

Proof. Let \( \{W_j\}_{j=1}^\infty \) be a countably infinite, locally finite cover of \( M \) by precompact, connected open sets (such a cover exists by Prop. 1.19 and Thm. 1.15). By successively deleting unneeded sets and renumbering, we can ensure that no \( W_j \) is contained in the union of the other \( W_i \)'s.

Let \( Y_1 = \bigcup_{i=2}^\infty W_i \). Because \( M \) is connected, each component of \( Y_1 \) meets \( W_1 \), and by local finiteness of \( \{W_j\} \), there are only finitely many such components. Such a component is precompact in \( M \) if and only if it is a union of finitely many \( W_i \)'s. Let \( V_1 \) be the union of \( W_1 \) together with all of the precompact components of \( Y_1 \), and let \( X_1 \) be the union of all \( V_i \)'s not contained in \( V_1 \). Then \( V_1 \) is connected and precompact, and \( X_1 \) has no precompact components. Proceeding by induction, suppose we have defined connected, precompact open sets \( V_1, \ldots, V_m \) whose union contains \( W_1 \cup \cdots \cup W_m \), and such that the union \( X_m \) of all the \( W_i \)'s not contained in \( V_1 \cup \cdots \cup V_m \) has no precompact components. Let \( j_m \) be the smallest index such that \( W_{j_m} \) is not contained in \( V_1 \cup \cdots \cup V_m \), and let \( Y_{m+1} \) be the union of all \( W_i \)'s other than \( W_{j_m} \), not contained in \( V_1 \cup \cdots \cup V_m \). Any precompact component of \( Y_{m+1} \) must meet \( W_{j_m} \), because otherwise, it would be a precompact component of \( X_m \). Let \( V_{m+1} \) be the union of all \( W_i \)'s not contained in \( V_1 \cup \cdots \cup V_m \) and \( Y_{m+1} \). As before, \( V_{m+1} \) is precompact and connected, and the union \( X_{m+1} \) of the \( W_i \)'s not contained in \( V_1 \cup \cdots \cup V_{m+1} \) has no precompact components. Then by construction, for each \( j \), the set \( X_j = \bigcup_{i>j} V_i \) has no precompact components. If some \( V_j \) does not meet \( V_k \) for any \( k > j \), then \( V_j \) itself is a precompact component of \( X_{j-1} \), which is a contradiction. Thus for each \( j \), there is some \( k > j \) such that \( V_j \cap V_k \neq \emptyset \).

Proof of Theorem 17.32. Choose an orientation on \( M \). Let \( \{V_j\}_{j=1}^\infty \) be an open cover of \( M \) satisfying the conclusions of the preceding lemma. For each \( j \), let \( K(j) \) denote the least integer \( k > j \) such that \( V_j \cap V_k \neq \emptyset \), and let \( \theta_j \) be an \( n \)-form compactly supported in \( V_j \cap V_{K(j)} \) whose integral is 1. Let \( \{\psi_j\}_{j=1}^\infty \) be a smooth partition of unity subordinate to \( \{V_j\}_{j=1}^\infty \).

Now suppose \( \omega \) is any \( n \)-form on \( M \), and let \( \omega_j = \psi_j \omega \) for each \( j \). Let \( c_1 = \int_{V_1} \omega_1 \), so that \( \omega_1 - c_1 \theta_1 \) is compactly supported in \( V_1 \) and has zero integral. It follows from Theorem 17.30 that there exists \( \eta_1 \in \Omega_c^{n-1}(V_1) \) such that \( d\eta_1 = \omega_1 - c_1 \theta_1 \). Suppose by induction that we have found \( \eta_1, \ldots, \eta_m \) and constants \( c_1, \ldots, c_m \) such that for each \( j = 1, \ldots, m \), \( \eta_j \in \Omega_c^{n-1}(V_j) \) and

\[
d\eta_j = \left( \omega_j + \sum_{i: K(i) = j} c_i \theta_i \right) - c_j \theta_j.
\]
Let
\[ c_{j+1} = \int_{V_{j+1}} \left( \omega_{j+1} + \sum_{i:K(i) = j+1} c_i \theta_i \right). \]
Then by Theorem 17.30, there exists \( \eta_{j+1} \in \Omega^{n-1}_c(V_{j+1}) \) satisfying the analog of (*) with \( j \) replaced by \( j + 1 \). Set \( \eta = \sum_{j=1}^{\infty} \eta_j \), with each \( \eta_j \) extended to be zero on \( M \setminus V_j \). By local finiteness, this is a smooth \((n-1)\)-form on \( M \). It satisfies
\[ d\eta = \omega + \sum_{j=1}^{\infty} \left( \sum_{i:K(i) = j} c_i \theta_i \right) - \sum_{j=1}^{\infty} c_j \theta_j. \]
Each term \( c_i \theta_i \) appears exactly once in the first sum above, so the two sums cancel each other. \( \square \)
Page 524, first paragraph, last line: Change “$U_i \subseteq U_0$ and $\tilde{U}_i \subseteq \tilde{U}_0$” to “$U_i \subseteq U_0$, $V_i \subseteq \Phi(\tilde{U}_0)$, and $\tilde{U}_i \cap b \subseteq U_0$.”

Page 528, line 9: Change two instances of $(g, p)$ in subscripts to $(g, q)$.

Page 528, just below the displayed equation in the middle of the page: The smoothness of the map $\sigma_q$ is not quite immediate from the definition. Replace the three sentences beginning “It follows” with this: “Because $S_p$ is a weakly embedded submanifold by Theorem 19.17, to show that $\sigma_q$ is a smooth local section of $S_p$, it suffices to show that it is smooth into $G \times M$ and takes its values in $S_p$. The first component function is smooth as a map into $G$ by smoothness of group multiplication. To show that the second component is smooth into $M$ as a function of $\tilde{X}$ (and therefore of $\exp X$), you need to use the argument sketched out just below equation (20.10): as in the proof of Prop. 20.8, apply the fundamental theorem on flows to the vector field $\mathcal{E}(p, X) = (\tilde{X}_p, 0)$ on $M \times q$. A straightforward computation shows that $\gamma(t) = (g \exp tX, \eta(\tilde{X})(t, q))$ is an integral curve of $\tilde{X}$ starting at $(g, q)$, from which it follows easily that $\sigma_q(g \exp X) = \gamma(1) \in S_p$.”

Page 537, Problem 20-6(a): Change $B \in \mathfrak{gl}(n, \mathbb{R})$ to $B \in \mathfrak{s}l(n, \mathbb{R})$.

Page 538, Problem 20-11(b): Here’s a better hint, which doesn’t require proving part (a) first: “[Hint: Consider the graph of $F$ as a subgroup of $G \times H$.]”

Page 542, middle of the paragraph before Example 21.3: Change “the action of $\mathbb{R}^k$ on $\mathbb{R}^n$” to “the action of $\mathbb{R}^k$ on $\mathbb{R}^k \times \mathbb{R}^n$.”

Page 543, 6th line from the bottom: Change “subsequence of $\mathcal{G}_K$” to “sequence in $\mathcal{G}_K$.”

Page 548, last two lines: Allen Hatcher’s name is misspelled. (Sorry, Allen.)

Page 549, proof of Proposition 21.12, last sentence: Change the first phrase of that sentence to “Second, if $p, p' \in E$ are in different orbits and $\pi(p) \neq \pi(p')$, ...” Then add the following sentences at the end of the proof: “If $p$ and $p'$ are in different orbits and $\pi(p) = \pi(p')$, let $W$ be an evenly covered neighborhood of $\pi(p)$, and let $V, V'$ be the components of $\pi^{-1}(W)$ containing $p$ and $p'$, respectively. For any $g \in \text{Aut}_\pi(E)$, a simple connectedness argument shows that $g \cdot V$ is a component of $\pi^{-1}(W)$; if it had nontrivial intersection with $V$ it would have to be equal to $V$, which would imply $g \cdot p = p'$, a contradiction.”

Page 567, two lines above Proposition 22.8: Insert “a” before “2-covector.”

Page 568, Example 22.9(a), first line: The coordinates should be $(x^1, \ldots, x^n, y^1, \ldots, y^n)$. (The last coordinate is $y^n$, not $x^n$.)

Page 571, line below equation (22.5): Delete the spurious word “theorem” at the end of the line.

Page 572, middle of the page: Replace the sentence starting “On the other hand” by this: “On the other hand, the left-hand side is just the ordinary $t$-derivative of a time-dependent tensor on a fixed vector space, and expanding in terms of a basis shows that it satisfies a similar product rule:”

Page 573, statement of Proposition 22.15, second line: Change “$V : J \times M$” to “$V : J \times M \to TM$”; and change $\psi$ to $\theta$.

Page 583, line 4: Change $\mathbb{R}^{2n+1} \smallsetminus \{0\}$ to $\mathbb{R}^{2n+2} \smallsetminus \{0\}$.

Page 583, third displayed equation: Should read

$$T \cdot d\Theta = -2 \sum_{i=1}^{n+1} (x^i \, dx^i + y^i \, dy^i) = -d(|x|^2 + |y|^2).$$
(7/26/16) Page 583, two lines below the third displayed equation: The formula for \( d\mathcal{N}(N, T) \) should be \( d\mathcal{N}(N, T) = 2(|x|^2 + |y|^2) \).

(11/28/12) Page 584, Exercise 22.29: Part (b) should read

\[
(b) \quad T = \frac{\partial}{\partial z};
\]

(8/14/14) Page 584, paragraph above Theorem 22.33: Change all occurrences of \( \theta \) in this paragraph to \( \psi \), to avoid confusion with the use of \( \theta \) for a contact form elsewhere in this section.

(11/24/17) Page 585, statement of Theorem 22.34, last line: Change \( H \) to \( F \).

(11/17/12) Page 587, equation (22.27): Change both occurrences of \( \sigma(s) \) to \( \sigma(x) \).

(6/7/22) Page 591, Problem 22-5: Add the hypothesis \( n > 0 \).

(11/18/17) Page 592, Problem 22-15: Add the hypothesis that \( M \) is connected.

(9/22/15) Page 608, Proposition A.41(a): Insert the following phrase at the beginning of this statement: With the exception of the word “closed” in part (d).

(7/22/13) Page 616, Proposition A.77(b), last line: Change \( f_{\mathfrak{f}}.0/ \) to \( \mathfrak{f} e_{\mathfrak{f}}.0/ \).

(12/19/18) Page 619, proof of Lemma B.2, fourth line: Replace “By Exercise B.1(b)” with “If \( w_1 \) is equal to one of the \( v_i \)'s, then the ordered \( (n + 1) \)-tuple \( (w_1, v_1, \ldots, v_n) \) is linearly dependent; if not, then by Exercise B.1(b), …”

(9/1/16) Page 632, Exercise B.29: Change “by a matrix” to “by a certain matrix” (twice).

(12/19/18) Page 637, Exercise B.42: Delete the words “is a homeomorphism that.” [Checking that it’s a homeomorphism requires the norm topology, which is not defined until later on that page.]

(9/6/16) Page 637, Exercise B.44: Change “basis map” to “basis isomorphism.”

(12/19/18) Page 653, proof of Proposition C.21, second paragraph, second line: Change \( f \) to \( f_{D} \).

(2/25/18) Page 658, two lines above (C.15): Change \( B_{\delta}(0) \) to \( \overline{B}_{\delta}(0) \).

(2/25/18) Page 660, display (C.20): Change \( F^{-1}(x) \) to \( F^{-1}(y) \).

(1/18/21) Page 664, statement of Theorem D.1(b): After the phrase “Any two differentiable solutions to (D.3)–(D.4),” insert “defined on intervals containing \( t_0 \).”

(12/2/15) Page 666, just below the fifth display: After the sentence ending “by our choice of \( \delta \) and \( \varepsilon \),” insert “(If \( t < t_0 \), interchange \( t \) and \( t_0 \) in the second line above.)”

(1/18/21) Page 664, statement of Theorem D.4: After the phrase “any two differentiable solutions to (D.3)–(D.4),” insert “defined on intervals containing \( t_0 \).”

(1/18/21) Page 668, paragraph below equation (D.10): In the fourth line of the paragraph, change \( \overline{W} \) to \( W \).

(1/18/21) Page 670, displayed inequality between (D.17) and (D.18): Change \( n \) to \( n^2 \).

(1/18/21) Page 670, last line: Change \( n \) to \( n^2 \) in the definition of \( B \).

(1/18/21) Page 671, inequality (D.19): Change \( n \) to \( n^2 \) (twice).
Page 671, just below (D.19): Replace the sentence “Since the expression on the right can be made as small as desired by choosing $h$ and $\tilde{h}$ sufficiently small, this shows ...” by the following: “Thus the expression on the left can be made as small as desired by choosing $h$ and $\tilde{h}$ sufficiently small. This shows ...”

Page 692: Under the entry for “Form,” delete the references to page 294 for “closed” and page 292 for “exact.”

Page 693: The index entry for “Hatcher, Allen” is misspelled.