
Contents

Preface	ix
Chapter 1. The Basics	1
Definitions	1
Holomorphic Maps	7
Covering Manifolds and Quotient Manifolds	8
Some Complex Analysis	12
The Complexified Tangent and Cotangent Bundles	23
Almost Complex Structures	34
Problems	41
Chapter 2. Complex Submanifolds	45
Variations on the Inverse Function Theorem	45
Complex Submanifolds	48
Complex Submanifolds of Projective Spaces	52
The Holomorphic Embedding Problem	64
Problems	68
Chapter 3. Holomorphic Vector Bundles	71
Holomorphic Bundle Tools	71
Holomorphic Line Bundles	81
Line Bundles over Projective Space	84
Applications of Holomorphic Line Bundles	90
Problems	99

Chapter 4. The Dolbeault Complex	103
Decomposing Differential Forms by Type	103
A Poincaré Lemma for the Dolbeault Operator	109
Bundle-Valued Forms	114
Problems	118
Chapter 5. Sheaves	121
Definitions	121
The Étale Space of a Presheaf	127
Exact Sequences of Sheaves	137
Problems	144
Chapter 6. Sheaf Cohomology	147
Definitions	147
The Long Exact Cohomology Sequence	154
Acyclic Resolutions	159
Sheaf Cohomology and Singular Cohomology	164
Applications of Sheaf Cohomology	173
Other Sheaf Cohomology Theories	183
Problems	189
Chapter 7. Connections	193
Connections on Complex Vector Bundles	193
Curvature	198
The First Real Chern Class	202
The Chern Connection	207
Problems	218
Chapter 8. Hermitian and Kähler Manifolds	221
Hermitian Metrics on the Tangent Bundle	221
Kähler Metrics	226
Examples of Kähler Metrics	234
Curvature of Kähler Metrics	239
Ricci and Scalar Curvatures	245
Problems	251

Chapter 9. Hodge Theory	255
The Hodge Inner Product	256
Elliptic Differential Operators	265
Hodge Theory on Riemannian Manifolds	269
Hodge Theory on Complex Manifolds	274
Hodge Theory on Kähler Manifolds	281
Applications of Hodge Theory	287
Problems	308
Chapter 10. The Kodaira Embedding Theorem	315
Preliminaries	315
The Kodaira Vanishing Theorem	317
Proof of the Embedding Theorem	330
Applications of the Embedding Theorem	335
Problems	337
Bibliography	343
Notation Index	347
Subject Index	351