
Notation Index

- * (Hodge star operator), 263
- ∞ (point at infinity), 56
- \sharp (sharp operator), 229
- $\langle \cdot, \cdot \rangle$ (Hermitian inner product), 76
- $\langle \cdot, \cdot \rangle$ (Kronecker pairing), 166
- (\cdot, \cdot) (global Hodge inner product), 257, 258
- $\|\cdot\|$ (global Hodge norm), 258
- $[\cdot, \cdot]$ (commutator bracket), 281
- $[\cdot]$ (singular homology class), 165
- $[\cdot]$ (cohomology class of a Čech cocycle), 149
- $[[\cdot]]$ (equivalence class in direct limit), 153
- \lrcorner (interior multiplication), 43
- \cong (isomorphic bundles), 25
- \cong (isomorphic sheaves), 126

- A^* (Hermitian adjoint of a matrix), 68
- A^* (adjoint of a linear map), 258
- A^* (cochain complex), 107
- $\text{Alb}(M)$ (Albanese variety), 300

- b (flat operator), 229
- \mathbb{B}^{2n} (unit ball), 4
- $b^k(M)$ (Betti number), 108
- $B(L)$ (base locus), 95
- $B^p(\mathcal{U}; \mathcal{S})$ (group of Čech coboundaries), 149
- $B_r(p)$ (open ball of radius r), 4
- $B(Z, W)$ (holomorphic bisectonal curvature), 253

- \mathcal{C} (sheaf of continuous functions), 123
- \mathcal{C}^* (sheaf of nonvanishing continuous functions), 123
- $c_1(\nabla)$ (Chern form of a connection), 202

- $c_1^{\mathbb{R}}(E)$ (first real Chern class), 203
- C_g (conjugation by g), 100
- $c(L)$ (sheaf-theoretic Chern class), 181
- $C^p(\mathcal{U}; \mathcal{S})$ (group of Čech cochains), 147
- $\text{ch}(K)$ (convex hull), 67
- $\text{Cl}(M)$ (divisor class group), 94
- $\text{Cl}^0(M)$ (divisor classes of degree 0), 296
- $\mathbb{C}\mathbb{P}^0$ (0-dimensional projective space), 5
- $\mathbb{C}\mathbb{P}^n$ (complex projective space), 5
- $(\mathbb{C}\mathbb{P}^n)^*$ (dual projective space), 54
- $\overline{\mathbb{C}\mathbb{P}^n}$ ($\mathbb{C}\mathbb{P}^n$ with opposite orientation), 101

- ∂ (conjugate Dolbeault operator), 105
- ∂ (singular boundary operator), 165
- $\bar{\partial}$ (Dolbeault operator), 105
- $\partial f / \partial z^i$ (complex partial derivative), 13
- $\partial f / \partial z^i$ (complex vector field), 27
- $\partial f / \partial \bar{z}^i$ (complex vector field), 27
- $\partial f / \partial x^j$ (derivative of complex-valued function), 17
- $\bar{\partial}_E$ (Cauchy–Riemann operator on bundle-valued forms), 116
- ∇ (connection), 193
- $\nabla\sigma$ (total covariant derivative of a section), 194
- δ (Čech coboundary operator), 148
- δ (singular coboundary operator), 166
- Δ (Laplace–Beltrami operator), 265
- Δ_d (Hodge Laplacian), 269
- $\Delta_{\bar{\partial}}$ (Dolbeault Laplacian), 274
- Δ_{∂} (conjugate Dolbeault Laplacian), 274
- Δ_k (standard simplex), 164
- D (exterior covariant derivative), 200

- \mathbb{D} (unit disk), 4
 DF (differential of a smooth map), 30
 $D'F$ (holomorphic Jacobian), 31, 37
 $DF(p)$ (differential of a smooth map), 30
 $D_r^n(p)$ (polydisk of radius r), 4
 $D_r(p)$ (disk of radius r), 4
 d^* (formal adjoint of d), 262
 D_t (covariant derivative along a curve), 197
 $\dim_{\mathbb{C}}$ (complex dimension), 3
 $\dim_{\mathbb{R}}$ (real dimension), 3
 div (divergence), 265
 $\text{Div}(M)$ (group of divisors), 93
 $\text{Div}^0(M)$ (divisors of degree 0), 296, 311
 dV_g (Riemannian volume form), 237
 ε (exponential sheaf morphism), 127
 ε^J (wedge of basis 1-forms), 262
 \mathcal{E} (sheaf of smooth complex-valued functions), 123
 \mathcal{E}^* (sheaf of nonvanishing smooth functions), 123
 $\mathcal{E}(E)$ (sheaf of smooth sections of E), 123
 e_j (standard basis for \mathbb{C}^n), 74
 \mathcal{E}^k (sheaf of smooth complex-valued k -forms), 123
 $\mathcal{E}^k(E)$ (sheaf of bundle-valued k -forms), 123
 $\mathcal{E}^k(M)$ (sections of $\Lambda_{\mathbb{C}}^k M$), 104
 $\mathcal{E}^{p,q}$ (sheaf of smooth (p, q) -forms), 124
 $\mathcal{E}^{p,q}(E)$ (sheaf of smooth E -valued (p, q) -forms), 124
 $\mathcal{E}^{p,q}(M)$ (sections of $\Lambda^{p,q} M$), 104
 $\mathcal{E}^{p,q}(M; E)$ (bundle-valued (p, q) -forms), 116
 $\mathcal{E}^q(M; E)$ (bundle-valued q -forms), 114
 $\mathcal{E}_{\mathbb{R}}$ (sheaf of smooth real-valued functions), 124
 $E \times_M E$ (fiber product), 129
 $\text{End}(E)$ (endomorphism bundle), 81
 φ_f (section of H^d), 88
 (f) (divisor of a meromorphic function), 93
 \underline{F} (morphism of constant sheaves), 127
 $F_{i,k}$ (face map), 165
 F_p (stalk homomorphism), 128
 $[f]_p$ (germ of a function), 127
 $F_{\#}$ (coefficient homomorphism on singular cochains), 166
 F_* (coefficient homomorphism in singular cohomology), 166
 F_* (induced sheaf cohomology homomorphism), 153
 F^* (induced homomorphism on Dolbeault cohomology), 108
 $\Gamma(E)$ (space of smooth sections of E), 25
 $\Gamma_c(E)$ (space of compactly supported sections), 259
 \underline{G} (constant sheaf with coefficients in G), 123
 G/Γ (coset space), 10
 $G_k(V)$ (Grassmannian), 6
 G_p (skyscraper sheaf), 123
 g_{CH} (complex hyperbolic metric), 243
 $g_{\mathbb{E}}$ (standard metric on \mathbb{C}^n), 225
 g_{FS} (Fubini–Study metric), 235
 $\text{GL}(n, \mathbb{C})$ (group of invertible complex matrices), 10
 $\text{GL}(V)$ (group of linear automorphisms of V), 10
 grad (gradient), 265
 H (hyperplane bundle), 88, 90
 H^d (power of the hyperplane bundle), 88
 $H_{\text{dR}}^k(M)$ (de Rham cohomology), 108
 $H_{\text{dR}}^k(M; \mathbb{C})$ (complex de Rham cohomology), 143
 $H_{\text{dR}}^k(M; \mathbb{R})$ (real de Rham cohomology), 143
 $H_{\infty}^k(M)$ (homology of smooth chains), 175
 $H_k(M)$ (singular homology), 165
 $H_{\text{Sing}, \infty}^k(M; G)$ (cohomology of smooth cochains), 175
 $H_{\text{Sing}}^k(M; G)$ (singular cohomology), 166
 $H^{p,q}(M)$ (Dolbeault cohomology), 108
 $\mathcal{H}^{p,q}(M)$ (harmonic (p, q) -forms), 275
 $h^{p,q}(M)$ (Hodge number), 108
 $H^{p,q}(M; E)$ (Dolbeault cohomology with coefficients in E), 117
 $\mathcal{H}^{p,q}(M; E)$ ($\bar{\partial}_E$ -harmonic bundle-valued forms), 277
 $H^q(A^*)$ (cohomology of a cochain complex), 107
 $\mathcal{H}^q(M)$ (harmonic q -forms), 270
 $H(Z)$ (holomorphic sectional curvature), 241
 $\text{Hol}(p)$ (holonomy group), 254
 $\text{Hom}(E, E')$ (homomorphism bundle), 81
 \mathcal{I}_S (ideal sheaf of S), 124
 \mathcal{I}_S^2 (sheaf of holomorphic functions vanishing to second order on S), 124
 $\mathcal{I}_S(E)$ (sheaf of holomorphic sections vanishing on S), 124
 $\mathcal{I}_S^2(E)$ (sheaf of holomorphic sections vanishing to second order on S), 124

- J (almost complex structure), 39
 J (complex structure on a vector space), 34
 \bar{j} (barred index), 227
 J_M (almost complex structure), 36
 $\text{Jac}(M)$ (Jacobian variety), 305
 κ (Kronecker homomorphism), 167
 K (canonical bundle), 118
 K^* (anticanonical bundle), 118
 \widehat{K} (holomorphic hull), 65
 K_M (canonical bundle), 118
 K_M^* (anticanonical bundle), 118
 $\text{Ker } F$ (kernel of a sheaf morphism), 138
 $\Lambda_{\mathbb{C}}^k M$ (bundle of complex k -forms), 103
 $\Lambda^{p,q} M$ (bundle of (p, q) -forms), 104
 $L_{\mathbb{C}}$ (complexification of a linear map), 23
 L_{ω} (Lefschetz operator), 281
 $L_{\{p\}}$ (point bundle), 93
 $\lim_{\rightarrow} G_{\alpha}$ (direct limit), 128
 $[M]$ (fundamental homology class), 165
 M/Γ (quotient space by group action), 10
 ω_{CH} (complex hyperbolic Kähler form), 243
 $\omega_{\mathbb{E}}$ (standard Kähler form on \mathbb{C}^n), 225
 ω_{FS} (Fubini–Study Kähler form), 235
 Ω^p (sheaf of holomorphic p -forms), 124
 $\Omega^p(E)$ (sheaf of holomorphic E -valued p -forms), 124
 $\Omega^p(M)$ (space of holomorphic p -forms), 118
 $\Omega^p(M; E)$ (space of holomorphic E -valued p -forms), 118
 \mathcal{O} (sheaf of holomorphic functions), 124
 \mathcal{O}^* (sheaf of nonvanishing holomorphic functions), 124
 $\mathcal{O}(d)$ (algebraic geometry notation for H^d), 88, 137
 $\mathcal{O}(E)$ (sheaf of nonvanishing sections of E), 124
 $\mathcal{O}(M)$ (holomorphic functions on M), 7
 $\mathcal{O}(M; E)$ (space of global holomorphic sections), 72
 $\text{O}(n, \mathbb{C})$ (complex orthogonal matrices), 51
 $\mathcal{O}(U; E)$ (space of local holomorphic sections), 72
 \mathcal{P} (sheaf of pluriharmonic functions), 189
 P^{γ} (parallel transport operator), 254
 P^* (formal adjoint of P), 259
 $\mathbb{P}(V)$ (projectivization of V), 6
 $\text{Pic}(M)$ (Picard group), 83
 $\text{Pic}^0(M)$ (Picard variety), 182
 ρ (Ricci form), 247
 r_V^U (restriction map in a presheaf), 121
 Rc (Ricci curvature), 245
 Rm (Riemann curvature tensor), 239
 (σ) (divisor of a section of a line bundle), 94, 211
 \sum'_I (sum over increasing multi-indices), 105
 S (scalar curvature), 245
 $\widehat{\mathcal{S}}$ (sheaf of rough sections), 186
 $[s_0(p), \dots, s_m(p)]$ (associated map), 95
 $\mathcal{S}(K)$ (sections of \mathcal{S} over a closed subset, 190
 $\mathcal{S} \otimes_{\mathcal{R}} \mathcal{T}$ (tensor product sheaf), 134
 \mathcal{S}_p (stalk of a sheaf), 128
 $[s]_p$ (germ of a section), 128
 $\mathcal{S}(U)$ (sections of \mathcal{S} over an open subset, 121
 $\mathcal{S}|_V$ (restriction of a sheaf to an open subset), 123
 $s|_V$ (restriction of a section), 122
 $\text{sec}(v, w)$ (sectional curvature), 252
 sgn (sign of a permutation), 257
 $\text{Sing}_{\ell}^{\infty}(M)$ (smooth singular chain group), 175
 $\text{Sing}_{\ell}(M)$ (singular chain group), 165
 $\text{Sing}^k(M; G)$ (singular cochain group), 165
 $\text{SL}(2, \mathbb{Z})$ (integer matrices with determinant 1), 42
 $\text{SL}(n, \mathbb{C})$ (complex matrices with determinant 1), 51
 $\text{SO}(n, \mathbb{C})$ (subgroup of $\text{O}(n, \mathbb{C})$ with determinant 1), 51
 $\text{St } v$ (star of a vertex), 191
 Θ (curvature of a connection), 198
 Θ_j^k (curvature 2-forms), 199
 θ_j^k (connection 1-forms), 194
 $T_{\mathbb{C}} M$ (complexified tangent bundle), 26
 $T_{\mathbb{C}}^* M$ (complexified cotangent bundle), 26
 $T_j M$ (TM with complex structure), 38
 $T^{\prime} M$ (holomorphic tangent bundle), 37
 $T^{\prime\prime} M$ (antiholomorphic tangent bundle), 37
 $T_p^{\prime} M$ (holomorphic tangent space), 37
 $T_p^{\prime\prime} M$ (antiholomorphic tangent space), 37
 $U(n)$ (unitary group), 68
 V^{\prime} (i -eigenspace of complex structure), 34
 $V^{\prime\prime}$ ($(-i)$ -eigenspace of complex structure), 34

\bar{V}^* (space of conjugate-linear functionals),
311

$V_{\mathbb{C}}$ (complexification of a vector space), 23

$V_{\mathbb{R}}$ (underlying real vector space), 34

$[w]$ (point in projective space), 5

z^j (holomorphic coordinates), 3

\bar{z}^j (conjugate of z^j), 3

\bar{z}^j (conjugate of z^j), 227

\mathcal{Z}^k (sheaf of closed complex-valued
 k -forms), 123

$\mathcal{Z}^{p,q}$ (sheaf of $\bar{\partial}$ -closed (p, q) -forms), 124

$\mathcal{Z}^{p,q}(E)$ (sheaf of $\bar{\partial}$ -closed E -valued
 (p, q) -forms), 124

$Z^p(\mathcal{U}; \mathcal{E})$ (group of Čech cocycles), 149

Subject Index

- Abel, Niels Henrik, 301
- Abel–Jacobi theorem, 301
- Abel’s theorem, 311
- abelian variety, 337
- acyclic resolution, 159
- acyclic sheaf, 159
- adjoint
 - formal, 260
 - of a differential operator, 260
 - of a finite-dimensional linear map, 258
 - of a matrix, 68, 259
- adjoint representation
 - of a Lie algebra, 100
 - of a Lie group, 100
- adjunction formula, 120
- affine algebraic variety, 51
 - nonsingular, 51
 - smooth, 51
- affine coordinates, 6
- affine embedding, 55
 - standard, 55
- Akizuki, Yasuo, 319, 321, 322
- Akizuki–Nakano identity, 319
- Albanese map, 311
- Albanese variety, 298, 300, 301
 - universal property, 312
- algebraic variety
 - affine, 51
 - nonsingular, 51, 52
 - projective, 52
 - smooth, 51, 52
- almost complex manifold, 39
- almost complex structure, 39
 - compatible with a symplectic form, 287
- Alt convention for wedge products, 229
- ample line bundle, 98, 315
- analytic continuation of a local isometry, 244
- analytic function, 12
- analytic sheaf, 144
 - coherent, 144
- analytic variety, 53
- anticanonical bundle, 118
- antiderivation, 103, 113, 264
- antiholomorphic tangent bundle, 37
- antiholomorphic tangent space, 37
- associated line bundle, 91, 94
- associated map, 96
- atlas, 2
 - holomorphic, 2
 - smooth, 2
- Aubin, Thierry, 249
- automorphism, 7
 - of $\mathbb{C}\mathbb{P}^1$, 69
 - of $\mathbb{C}\mathbb{P}^n$, 310
- Bézout’s theorem, 61
- ball, 4
 - unit, 4
- base locus, 95
- base point, 95
- Betti number, 108, 143
- Bianchi identity
 - algebraic, 239
 - differential, 219
 - for a connection, 219
- bidegree, 104

- biholomorphic, 8
- biholomorphism, 7
 - local, 7
- bilinear relations, 340
- bisectional curvature, holomorphic, 253
- Bishop, Errett, 66
- blowdown map, 85
 - model, 85
- blowup, 85
 - at finitely many points, 87
 - is projective, 335
- Bochner, Salomon, 309, 310
- Bochner vanishing theorem, 309
- Borel, Armand, 41
- boundary, singular, 165
- bundle construction theorem, 73
- bundle homomorphism, 25
 - over M , 25
- bundle isomorphism, 25
 - holomorphic, 25
 - smooth, 25
- bundle-valued form, 114, 116
- C^k manifold, 1
- Calabi, Eugenio, 247
- Calabi–Yau manifold, 249, 339
- Calabi–Yau theorem, 247
- canonical bundle, 118
 - degree of, 253
 - of projective space, 118
- canonical curve, 339
- canonical holomorphic structure
 - on \mathbb{C}^n , 4
 - on an open subset of \mathbb{C}^n , 4
- canonical map, 339
- Cartan, Henri, 278
- Cartan’s theorems A & B, 278
- category, equivalence of, 133
- Cauchy–Riemann equations, 12, 14
 - inhomogeneous, 109
- Cauchy–Riemann operator, 106
- Čech, Eduard, 147
- Čech coboundary, 149
- Čech cochain, 147
- Čech cocycle, 149
- Čech cohomology, 147, 152
 - on an open cover, 148
- chain
 - singular, 165
 - smooth, 175
- chain complex, 107
- chain group, singular, 165
- chain map, 107
- chain rule
 - for holomorphic functions, 31
 - for smooth functions, 31
- chart, 2
- chart lemma, 3
- Chern, Shiing-Shen, 202
- Chern class
 - of a line bundle, 181
 - real, 203
 - sheaf-theoretic, 181
 - sheaf-theoretic and first real, 204
- Chern connection, 208
- Chern form, 202
- Chow, Wei-Liang, 53
- Chow’s theorem, 53, 291
 - for hypersurfaces, 291
- classification of smooth line bundles, 181
- coboundary
 - Čech, 149
 - singular, 166
- coboundary operator
 - Čech, 148
 - singular, 166
- cochain
 - Čech, 147
 - singular, 165
- cochain complex, 107
- cochain group, 147
- cochain homotopy formula, 151
- cochain map, 107
- cocompact subgroup, 12
- cocycle
 - Čech, 149
 - singular, 166
- cocycle condition, 73
- codimension, 48
- coefficient homomorphism, 166
- coherent analytic sheaf, 144
- cohomologous, 149
- cohomology
 - Čech, 147, 148
 - of a cochain complex, 107
 - of constant sheaves, 168
 - sheaf, 147, 152
 - singular, 166, 168
 - smooth singular, 175
- cohomology class, 149
 - integral, 178, 334
 - of type (p, q) , 294
- compatible almost complex structure, 287

- compatible charts
 - holomorphically, 2
 - smoothly, 2
- compatible with a metric, 195
- compatible with the holomorphic structure, 207
- complex-analytic function, 12
- complex coordinate frame, 29
- complex coordinate vector field, 29
- complex covector field, 26
- complex curve, 3, 41
- complex differential form, 26, 103
- complex dimension, 3
- complex Grassmannian, 6
- complex hyperbolic metric, 243, 252
- complex Lie group, 10, 51
- complex Lie subgroup, 51
- complex manifold, 1, 2
- complex Monge–Ampère equation, 248
- complex partial derivative, 13
- complex projective space, 5
- complex structure
 - on a vector bundle, 35
 - on a vector space, 34
 - traditional name for holomorphic structure, 3
- complex submanifold, 48
 - embedded, 48
 - immersed, 48
- complex surface, 3, 41
- complex torus, 11, 235
- complex vector bundle, 24
 - holomorphic, 71
 - smooth, 24
- complex vector field, 26
- complexification
 - of a linear map, 23
 - of a vector bundle, 25
 - of a vector space, 23
 - of the cotangent bundle, 26
 - of the tangent bundle, 26
- complexified cotangent bundle, 26
- complexified tangent bundle, 26
- component functions of a section, 72
- composition
 - of holomorphic functions, 16
 - of holomorphic maps, 8
 - of sheaf morphisms, 126
- conformal map, 44
- conformal metric, 251
- conjugate-linear, 24
- conjugate symmetry, 76
- conjugation
 - in a complexified vector bundle, 26, 42
 - in a complexified vector space, 24
 - in a Lie group, 100
 - of a quaternion, 43
 - of an octonion, 43
- connected sum, 100
- connecting homomorphism, 154
 - in sheaf cohomology, 155
- connection, 193
 - Chern, 208
 - compatible with a metric, 195
 - compatible with the holomorphic structure, 207
 - dual, 218
 - metric, 195
 - tensor product, 219
- connection forms, 194
- conormal bundle, 119
- constant holomorphic sectional curvature, 242, 243, 252
- constant presheaf, 145
- constant sheaf, 124
- contractible, 166
 - locally, 168
 - semilocally, 172
- contragredient, 80
- convex, holomorphically, 65
- convex hull, 65, 69
- coordinate chart, 2
 - holomorphic, 3
- coordinate frame, complex, 29
- coordinate representation, 7
- coordinate vector field, complex, 29
- coordinates
 - holomorphic, 3
 - homogeneous, 5
- covariant derivative, 193
 - along a curve, 197
 - exterior, 200
 - total, 194
- covering map, 8
 - holomorphic, 9
 - smooth, 9
- cubic, projective, 57
- curvature
 - holomorphic bisectional, 253
 - holomorphic sectional, 241
 - of a connection, 198
 - of a Kähler metric, 240

- curvature endomorphism field, 239
- curvature forms, 199
- curvature tensor, 239
- curve, complex, 3, 41
- cycle, singular, 165
- $\bar{\partial}_E$ -harmonic, 277
- $\bar{\partial}$ -Poincaré lemma, 109
- ∂ -harmonic form, 276
- $\bar{\partial}$ -harmonic form, 275
- $\partial\bar{\partial}$ -lemma
 - global, 248, 292
 - local, 113
- d -harmonic form, 276
- de Rham cohomology, 108, 143
 - with complex coefficients, 143
- de Rham theorem, 176
 - sheaf-theoretic version, 173
- de Rham–Weil theorem, 159
- decomposable form, 43
- defining function, local, 49
- degree
 - of a codimension-1 variety, 57
 - of a divisor, 211
 - of a line bundle, 182
 - of a smooth map, 338
- density, Riemannian, 257
- determinant bundle, 80
- determinant convention for wedge products, 229
- difference tensor, 202
- differential Bianchi identity, 219
- differential form
 - bundle-valued, 114, 116
 - complex, 103
 - endomorphism-valued, 114
 - of type (p, q) , 104
 - real, 103
- differential of a smooth map, 29
- differential operator, 259
 - elliptic, 268
 - order zero, 268
- dimension, 2
 - complex, 3
 - real, 3
- direct image sheaf, 146
- direct limit, 128
- direct sum of sheaves, 123
- direct system, 128
- directed set, 128
- discrete Lie group, 9
- disk, 4
- distribution, 40
- divergence, 265
- divisor, 93
 - effective, 93
 - linearly equivalent, 94
 - of a meromorphic function, 93
 - of a meromorphic section, 94
 - principal, 94
- divisor class group, 94
- Dolbeault, Pierre, 105, 173
- Dolbeault cohomology, 108
 - functoriality, 108
 - with bundle coefficients, 117, 276
- Dolbeault complex, 108
- Dolbeault Laplacian, 274
 - bundle-valued, 277
- Dolbeault operator, 105
- Dolbeault theorem, 173
- double skyscraper sheaf, 143, 316
- dual bundle, 79
- dual connection, 218
- dual metric, 210, 218, 219
- dual projective space, 54
- dual torus, 311
- E -valued form, 114, 116
- effective divisor, 93, 297
- Einstein summation convention, 2, 5, 27, 227
- elementary symmetric polynomial, 60
- elliptic complex, 312
- elliptic curve, 308
- elliptic differential operator, 268
- elliptic integral, 308
- embedded complex submanifold, 48
- embedding, 46
 - holomorphic, 46
- embedding theorem
 - Kodaira, 330, 334
 - Stein, 66
- empty set, sections in a presheaf, 122
- $\text{End}(E)$ -valued form, 114
- endomorphism bundle, 81
- endomorphism-valued form, 114
- equivalence of categories, 133
- étalé space, 129
 - of a presheaf, 129
 - of abelian groups, 129
 - of rings, 129
 - of vector spaces, 129
- étalé space morphism, 133
- Euclidean inner product, 4
- Euler characteristic of a sheaf, 158, 295

- Euler's identity, 63
- evaluation map, 316
- evenly covered, 8
- exact sequence
 - of abelian groups, 137
 - of cochain complexes, 154
 - of rings, 137
 - of sheaves, 138
 - of vector spaces, 137
 - short, 139
- exceptional hypersurface, 85
- exponential sheaf sequence, 142
 - smooth, 142
- extendible section, 197
- exterior covariant differentiation, 200
- face map, 165
- Fano manifold, 254
- Fermat curve, 59
- Fermat hypersurface, 59, 250
- Fermat's last theorem, 59
- fiber metric, Hermitian, 76
- fiber product, 75, 129, 146
- fine sheaf, 163
 - is acyclic, 163
- finiteness theorem, 278
- first real Chern class, 203
 - and sheaf-theoretic Chern class, 204
- flabby sheaf, 184
- flasque resolution, 184, 190
- flasque sheaf, 184
 - is acyclic, 188
- flat connection, 199
- flat operator (b), 229
- form
 - bundle-valued, 114, 116
 - complex, 103
 - endomorphism-valued, 114
 - of bidegree (p, q) , 104
 - of type (p, q) , 104
- formal adjoint, 260
- formally self-adjoint, 270
- fourfold, 3
- frame, local, 25
- Fredholm theorem, 269
 - for elliptic operators, 269
 - for operators with injective symbol, 312
- free action, 9
- free sheaf, 135
 - locally, 135
- Fubini, Guido, 235
- Fubini–Study metric, 235, 334
 - is homogeneous, 236
- fundamental 2-form, 224
- fundamental class of a manifold, 165
- GAGA principle, 53
- Gaussian curvature, 251
- Gaussian integer, 12
- Gaussian integers, 41
- genus-degree formula, 254
- genus of a compact Riemann surface, 183
- geometric normal bundle, 81
- germ
 - of a holomorphic function, 127
 - of a section, 128
- global $\partial\bar{\partial}$ -lemma, 248, 292
- global differential, 30
- global Hodge inner product, 257
- global section, 25
- gluing property of a sheaf, 122
- Godement, Roger, 184
- Godement resolution, 184, 190
- gradient, 265
- graph of a function, 46, 50
- Grassmannian, 6, 68
- Grothendieck, Alexander, 184
- group action, 9
 - free, 9
 - holomorphic, 9
 - proper, 9
- hard Lefschetz theorem, 288
- harmonic form, 255, 270, 275
- Hartogs, Friedrich, 13, 21
- Hartogs's extension theorem, 22
- Hermitian adjoint of a matrix, 68, 259
- Hermitian fiber metric, 76
- Hermitian inner product, 76
- Hermitian manifold, 224
- Hermitian metric, 224
- Hermitian vector bundle, 76
- Hilbert space, 258
- Hilbert space adjoint, 260
- Hodge, William V. D., 105, 256, 276, 281
- Hodge decomposition theorem, 286
- Hodge diamond, 287
- Hodge–Dolbeault theorem, 276
 - for bundle-valued forms, 277
- Hodge duality, 286
- Hodge inner product
 - global, 257
 - pointwise, 256
- Hodge Laplacian, 269

- Hodge metric, 334
 Hodge numbers, 108, 276
 of $\mathbb{C}\mathbb{P}^n$, 289
 of a Riemann surface, 289
 Hodge star operator, 262, 263
 on a Hermitian manifold, 274
 on a Riemann surface, 274
 on a Riemannian manifold, 262
 Hodge theorem, 255
 for elliptic complexes, 313
 for Kähler manifolds, 286
 for Riemannian manifolds, 272
 Hodge theory, 256
 holomorphic action, 9
 holomorphic atlas, 2
 holomorphic bisectional curvature, 253
 holomorphic bundle isomorphism, 25
 holomorphic coordinate chart, 3
 holomorphic coordinates, 3
 holomorphic covering map, 9
 holomorphic embedding, 46
 holomorphic form, 118
 is closed and harmonic, 292
 is parallel, 310
 holomorphic function, 7
 of one variable, 12
 of several variables, 13
 vector-valued, 13
 holomorphic hull, 65
 holomorphic immersion, 46
 holomorphic implicit function theorem, 46
 holomorphic inverse function theorem, 45
 holomorphic Jacobian, 31, 37
 holomorphic manifold, 2
 holomorphic map
 between manifolds, 7
 between open subsets of \mathbb{C}^n , 2
 holomorphic normal bundle, 81
 holomorphic quotient manifold theorem, 10
 holomorphic rank theorem, 46
 holomorphic section, 9, 72
 holomorphic sectional curvature, 241
 constant, 242, 243, 252
 holomorphic slice, 48
 holomorphic structure, 2
 determined by an atlas, 3
 on \mathbb{C}^n , 4
 on a manifold, 2
 on an open subset of \mathbb{C}^n , 4
 holomorphic subbundle, 76
 holomorphic submersion, 46
 holomorphic tangent bundle, 37
 holomorphic tangent space, 37
 holomorphic vector bundle, 25, 71
 holomorphic vector field, 99
 holomorphically compatible charts, 2
 holomorphically convex, 65
 holonomy group, 254, 339
 homogeneous coordinates, 5
 homogeneous function, 83
 homogeneous polynomial, 52
 homogenization of a polynomial, 59
 homologous, 165
 homology
 of a chain complex, 107
 singular, 165
 smooth singular, 175
 homology class, 165
 homomorphism bundle, 81
 homomorphism of vector bundles, 25
 Hopf manifold, 11, 238
 horizontal tangent space, 251
 hull
 convex, 65, 69
 holomorphic, 65
 hyperbolic metric, complex, 243, 252
 hyperelliptic curve, 338
 hyperplane, projective, 54
 hyperplane at infinity, 55
 hyperplane bundle, 90
 is positive, 320
 hypersurface, 48
 in projective space, 291
 ideal sheaf, 125, 133, 137
 identity theorem, 18
 for manifolds, 19
 image of a sheaf morphism, 138
 immersed complex submanifold, 48
 immersion, 46
 holomorphic, 46
 implicit function theorem, 46
 inclusion morphism, 127
 induced cohomology homomorphism, 107
 injective resolution, 184
 injective sheaf, 184
 is acyclic, 188
 is flasque, 185
 injective sheaf morphism, 139
 injective symbol, 312
 inner product, Hermitian, 76
 integrable almost complex structure, 40
 integral cohomology class, 178, 294, 334

- integration over a smooth chain, 176
 interior multiplication, 43, 113, 264
 interior product, *see* interior multiplication
 inverse function theorem, 45
 inverse image sheaf, 146
 invertible sheaf, 137
 involutivity, 40
 irreducible, 51
 isolated singularities, 22
 isomorphic bundles, 25
 isomorphic presheaves, 126
 isomorphic sheaves, 126
 isomorphism
 of presheaves, 126
 of sheaves, 126
 of vector bundles, 25
 Iwasawa manifold, 12, 41, 292
 standard, 12

 Jacobi, Carl Gustav Jacob, 301
 Jacobi inversion theorem, 311
 Jacobian, holomorphic, 31, 37
 Jacobian variety, 305, 339

 K3 surface, 250
 k -form, complex, 103
 Kähler, Erich, 250
 Kähler class, 227
 Kähler–Einstein metric, 249
 Kähler form, 226
 is harmonic, 309
 Kähler identities, 281
 for bundle-valued forms, 317
 Kähler manifold, 226
 Kähler metric, 226
 on $\mathbb{C}P^n$, 235
 on \mathbb{C}^n , 234
 on a Riemann surface, 235
 on complex hyperbolic space, 243, 252
 on complex tori, 235
 Kähler potential, 234
 Kähler symmetries of the curvature, 240
 kernel of a sheaf morphism, 138
 Kodaira, Kunihiko, 250, 315, 322
 Kodaira embedding theorem, 330, 334
 geometric version, 334
 line bundle version, 330
 Kodaira–Nakano–Akizuki vanishing
 theorem, 321
 Kodaira–Thurston manifold, 288
 Kodaira vanishing theorem, 317, 322
 Kronecker homomorphism, 167

 Kronecker pairing, 166
 and de Rham cohomology, 178
 Kummer, Ernst, 250

 L^2 inner product, 258
 Laplace–Beltrami operator, 265, 308
 Laplacian
 Dolbeault, 274
 Hodge, 269
 lattice, 11
 Lefschetz, Solomon, 281, 288
 Lefschetz decomposition theorem, 309
 Lefschetz operator, 281
 Lefschetz theorem on $(1, 1)$ -classes, 294
 Lefschetz theorem, hard, 288
 Leray, Jean, 121
 Levi-Civita connection, 198, 207
 Lie group
 complex, 10, 51
 discrete, 9
 Lie subgroup, complex, 51
 lift of a vector field, 61
 line, projective, 54
 line bundle, 25
 associated with a divisor, 94
 associated with a hypersurface, 91
 classification, 181, 290
 degree, 182
 negative, 216, 337
 on projective space, 290
 positive, 216, 320
 linear subspace, projective, 54
 linearly equivalent divisors, 94
 Liouville’s theorem, 19
 local biholomorphism, 7
 local $\partial\bar{\partial}$ -lemma, 113
 local defining function, 49
 system of, 91
 local frame, 25
 local isometry, 243
 local operator, 259
 local section
 of a bundle, 25
 of a continuous map, 123
 of a covering map, 9
 of a map, 47
 local trivialization, 24
 locality property of a sheaf, 122
 locally contractible, 168
 locally decomposable, 43
 locally finitely generated sheaf, 144
 locally free sheaf, 135

- locally zero, 170
- long exact sequence, 154
 - in sheaf cohomology, 154
- loop, 254
- manifold
 - C^k , 1
 - complex, 1, 2
 - smooth, 1, 2
 - topological, 1
- maximal atlas
 - holomorphic, 2
 - smooth, 2
- maximum principle, 19
- meromorphic function, 93
- meromorphic section, 93
- metric connection, 195
- Möbius transformation, 56
- model blowdown map, 85
- Monge–Ampère equation, complex, 248
- Montel’s theorem, 20
- morphism
 - of presheaves, 126
 - of sheaves, 126
 - \mathcal{R} -module, 126
- multiplicity of a zero, 13
- musical isomorphism, 229
- Nakano, Shigeo, 319, 321, 322
- Narasimhan, Raghavan, 66
- natural coordinates for T^*M , 267
- negative $(1, 1)$ -form, 215
- negative line bundle, 216, 337
- Newlander, August, 40
- Newlander–Nirenberg theorem, 40
- Nijenhuis tensor, 42
- Nirenberg, Louis, 40
- nondegenerate 2-form, 227
- nondegenerate bilinear form, 273
- nonsingular variety
 - affine, 51
 - projective, 52
- nontrivial section of a vector bundle, 25
- norm with respect to a fiber metric, 76
- normal bundle
 - geometric, 81
 - holomorphic, 81
- normal covering map, 8
- normal subgroup, 8, 10
- octonions, 43
- open submanifold, 5, 50
- order
 - of a pole, 93
 - of a zero, 13
- orientation of a complex manifold, 32
- oriented connected sum, 100
- (p, q) -form, 104
- parallel local frame, 199
- parallel section, 198
 - along a curve, 197
- parallel transport, 197, 254
- partition of unity
 - sheaf, 163
 - smooth, 162, 163
 - topological, 163
- period lattice, 300
- period matrix, 300
- periods of a Riemann surface, 300
- Picard, Charles Émile, 83
- Picard group, 83, 181
 - and sheaf cohomology, 179
- Picard variety, 182, 298, 301
- plane, projective, 54
- Plücker embedding, 69
- pluriharmonic function, 119, 189
- Poincaré lemma, 109
 - for the Dolbeault operator, 109
- point bundle, 93
- pointwise Hodge inner product, 256
- pole of a meromorphic function, 93
- polydisk, 4
- polynomial, 17
 - homogeneous, 52
- positive $(1, 1)$ -form, 215, 216, 320
- positive definite, 76
- positive line bundle, 216, 320
- positive (p, p) -form, 220
- presheaf, 121
 - constant, 145
 - of abelian groups, 122
 - of rings, 122
 - of vector spaces, 122
 - that is not a sheaf, 125
 - with values in a category, 122
- presheaf isomorphism, 126
- presheaf morphism, 126
- primed summation sign, 105
- primitive cohomology class, 309
- principal divisor, 94, 311
- principal symbol of a differential operator, 267
- projective algebraic variety, 52

- nonsingular, 52
- smooth, 52
- projective completion, 59
- projective cubic, 57
- projective hyperplane, 54
- projective hypersurface, 291
 - diffeomorphic, 62
 - is connected, 291
- projective line, 54
- projective linear subspace, 54
- projective manifold, 52
- projective plane, 54
- projective quadric, 57
- projective quartic, 57
- projective quintic, 57
- projective space, complex, 5
- projective tangent space, 55
- projective transformation, 47
- projectively equivalent, 54
- projectivization of a vector space, 6
- proper action, 9
- pseudoconvex, 67
- pullback bundle, 75, 210
- pullback metric, 210
- pullback of sections, 75, 76

- quadric, projective, 57
- quartic, projective, 57
- quaternions, 43
- quintic, projective, 57
- quotient bundle, 80
- quotient manifold theorem, 10
- quotient sheaf, 133

- \mathcal{R} -module morphism, 126
- rank of a quadratic polynomial, 57
- rank theorem, 46
- rank-nullity law, 158
- rational function is holomorphic, 17
- real-analytic manifold, 1
- real Chern class, 203
- real differential form, 103
- real dimension, 3
- real vector, 24
- refinement of an open cover, 150
- refining map, 150
- regular level set, 50
- regular point, 51
- regular value, 50
- resolution of a sheaf, 159
 - acyclic, 159
 - flasque, 184, 190
 - Godement, 184, 190
 - injective, 184
- restricting domains or codomains, 50
- restriction map in a presheaf, 121
- restriction of a bundle, 80
- restriction of a sheaf to an open subset, 123
- resultant, 60
 - of several homogeneous polynomials, 62
- Ricci curvature, 245
- Ricci flat, 250
- Ricci form, 247
 - and first Chern class, 247
- Riemann, Bernhard, 297
- Riemann bilinear relations, 340
- Riemann curvature tensor, 239
- Riemann form, 336
- Riemann–Roch theorem, 295, 297
- Riemann sphere, 56
- Riemann surface, 41
- Riemannian covering, 244
- Riemannian density, 257
- Riemannian submersion, 251
- ring
 - has multiplicative identity, 125
 - presheaf of, 122
 - sheaf of, 125
- Roch, Gustav, 297
- rough section, 25, 129
 - of a continuous map, 123

- saturated subset, 63
- scalar curvature, 245
- section
 - holomorphic, 72
 - local, 25
 - of a bundle, 25
 - of a continuous map, 123
 - of a presheaf, 121
 - of a vector bundle along a curve, 197
 - over the empty set, 122
 - rough, 25, 129
- sectional curvature, holomorphic, 241, 242
- Segre embedding, 68
- self-adjoint, formally, 270
- semilocally contractible, 172
- separates directions, 64, 97
- separates points, 64, 97
- Serre, Jean-Pierre, 41, 53, 278
- Serre duality, 278
- sesquilinearity, 76
- sharp operator (\sharp), 229
- sheaf, 122

- flabby, 184
- flasque, 184
- free, 135
- injective, 184
- invertible, 137
- locally free, 135
- of discontinuous sections, 187
- of \mathcal{E} -modules, 125
- of modules over a sheaf of rings, 125
- of \mathcal{O} -modules, 125
- of \mathcal{R} -modules, 125
- of rings, 125
- of rough sections, 186
- of \mathbb{Z} -modules, 126
- sheaf cohomology, 152
 - Čech, 147
 - functoriality, 153
 - long exact sequence, 154
- sheaf isomorphism, 126
- sheaf morphism, 126
 - composition, 126
 - injective, 139
 - surjective, 139
- sheaf partition of unity, 163
- sheaf-theoretic Chern class, 181
 - and first real Chern class, 204
- sheafification, 131
 - universal property, 131
- short exact sheaf sequence, 139
- sign of a permutation, 257
- simple zero, 13
- simplex
 - singular, 164
 - smooth singular, 175
 - standard, 164
- simplicial cohomology, 191
- simplicial complex, 191
- singular boundary operator, 165
- singular chain, 165
 - smooth, 175
- singular chain group, 165
- singular coboundary, 166
- singular coboundary operator, 166
- singular cochain, 165
- singular cochain group, 165
- singular cocycle, 166
- singular cohomology, 166
- singular cycle, 165
- singular homology, 165
 - smooth, 175
- singular point of a variety, 51
- singular simplex, 164
 - smooth, 175
- singularity, isolated, 22
- skyscraper sheaf, 124
 - double, 143, 316
- slice, 48
- slice chart, 48
- smooth atlas, 2
- smooth bundle isomorphism, 25
- smooth chain, 175
- smooth complex vector bundle, 24
- smooth covering map, 9
- smooth exponential sheaf sequence, 142
- smooth manifold, 1, 2
- smooth map, 2
- smooth partition of unity, 162, 163
- smooth singular chain, 175
- smooth singular cohomology, 175
- smooth singular homology group, 175
- smooth singular simplex, 175
- smooth structure, 2
- smooth variety
 - affine, 51
 - projective, 52
- smoothly compatible charts, 2
- soft sheaf, 190
- stalk
 - of a presheaf, 128
 - of an étalé space, 129
 - of the sheaf of holomorphic functions, 127
- stalk homomorphism, 128
- standard affine embedding, 55
- standard basis of \mathbb{C}^n , 35
- standard holomorphic structure on a vector space, 5
- standard Iwasawa manifold, 12
- standard metric on \mathbb{C}^n , 225, 234
- standard simplex, 164
- standard symplectic form, 225
- star of a simplex, 191
- star operator, *see* Hodge star operator
- Stein embedding theorem, 66
- Stein manifold, 65
- Study, Eduard, 235
- subbundle, 76
- submanifold, 48
 - complex, 48
 - embedded, 48
 - immersed, 48
 - open, 5
- submersion, 46

- holomorphic, 46
- subpresheaf, 123
- subsheaf, 123
- summation convention, 2, 5, 27, 227
- superstring theory, 250
- support of a sheaf morphism, 163
- surface
 - complex, 3, 41
 - Riemann, 41
- surjective sheaf morphism, 139
- symbol of a differential operator
 - principal, 267
 - total, 266
- symmetric polynomial, 60
 - fundamental theorem on, 60
- symplectic form, 227, 287
 - standard, 225
- symplectic manifold, 287
- system of local defining functions, 91
- tangent bundle
 - antiholomorphic, 37
 - holomorphic, 37
- tautological bundle, 84, 100
- Taylor series, 18
- tensor product bundle, 80
- tensor product connection, 219
- tensor product metric, 210
- tensor product sheaf, 134
- threefold, 3
- Thurston, William, 287, 288
- topological manifold, 1
- topological partition of unity, 163
- torsion-free connection, 207
- torsion subgroup, 179
- torus, complex, 11, 235
- total covariant derivative, 194
- total derivative, 29
- total symbol of a differential operator, 266
- transition function, 2, 71
- transverse to a submanifold, 50
- triangulation, 165
 - smooth, 165
- trivial sheaf, 139
- trivial vector bundle, 25
- trivializing cover, 25
- type of a differential form, 104
- \mathcal{U} -small chain, 171
- \mathcal{U} -small simplex, 171
- uniformization theorem, 249
- unit ball, 4
- unit disk, 4
- unitary group, 68
- universal coefficient theorem, 167
- universal property
 - of sheafification, 131
 - of the Albanese variety, 312
- vanishing
 - simply, 90
 - to second order, 124, 137, 316
- variety
 - affine, 51
 - algebraic, 51, 52
 - analytic, 53
 - determined by a section, 90
 - nonsingular, 51, 52
 - projective, 52
 - smooth, 51, 52
- vector bundle
 - complex, 24
 - Hermitian, 76
 - holomorphic, 25, 71
 - smooth, 24
- vector field, holomorphic, 99
- vector space, holomorphic structure on, 5
- very ample line bundle, 97, 315
- wedge product
 - Alt convention, 229
 - determinant convention, 229
 - with a bundle-valued form, 114, 115
 - with an endomorphism-valued form, 115
- Weil, André, 250
- Whitney embedding theorem, 64
- Whitney sum, 80
- Yau, Shing-Tung, 247, 249
- Zariski topology, 53, 183
- zero section, 25
- zigzag lemma, 154