Corrections to
Introduction to Complex Manifolds

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(11/19/24) **Page 15, last displayed equation:** In the formula for $a_{k_1 \dots k_n}$, insert a factor of $1/(2\pi i)^n$ on the right-hand side.

(6/15/24) **Page 25, fourth full paragraph, lines 4 and 8:** Change $\sigma \circ \pi$ to $\pi \circ \sigma$ (twice).

(6/15/24) **Page 25, fourth full paragraph, line 7:** Delete the repeated phrase "***rough* (*local or global*) *section of E*** is a map $\sigma : U \to E$."

(6/15/24) **Page 26, line 8 from the bottom:** Change $[Y_2, X_1]$ to $[Y_1, X_2]$.

(6/19/24) **Page 46, line 2:** Change $(\partial \overline{F}^k \partial \bar{z}^k)$ to $(\partial \overline{F}^k / \partial \bar{z}^j)$. [Insert missing slash and fix incorrect index.]

(11/27/24) **Page 50, Example 2.21, second line:** Change $\subseteq$ to $\in$.

(3/18/25) **Page 57, proof of Lemma 2.35, first paragraph:** Change $\sum_{j,k}$ to $\sum_{i,j}$ (twice).

(2/17/25) **Page 59, line 3:** Change "projective hypersurface" to "projective variety."

(2/17/25) **Page 59, Example 2.38, second line:** Change "holomorphic function" to "holomorphic polynomial."

(6/17/24) **Page 60, line 2 from the bottom:** Change $[x_0, \lambda_j(x_0), 1]$ to $[1, x_0, \lambda_j(x_0)]$.

(3/19/25) **Page 62, proof of Theorem 2.43, first paragraph, next-to-last line:** After $\mathbb{C}^{n+1} \setminus \{0\}$, insert "(see Lemma 3.38 below)."

(3/18/25) **Page 66, second paragraph, line 3:** Change "points p_j in K" to "points p_j in $\widehat{K}$."

(1/15/25) **Page 77, proof of Lemma 3.14, second paragraph:** Change "such a subsequence" to "such a sequence."

(7/11/24) **Page 78, proof of Theorem 3.13, last line of the first paragraph:** Change $\{\sigma_k\}_{k=1}^{\infty}$ to $\{\sigma_i\}_{i=1}^{\infty}$, and $\|\sigma_k\|$ to $\|\sigma_i\|$.

(7/11/24) **Page 79, first line:** Change $\mathbb{S}^{2n-1}$ to $\mathbb{S}^{2k-1}$.

(7/11/24) **Page 79, second paragraph:** Change σ_k to σ_i (four times).

(6/18/24) **Page 80, Example 3.21, lines 3 & 4:** "Holomorphic" is misspelled.

(2/3/25) **Page 81, first displayed equation:** Change the matrix to the following:

$$\tau_j^i = \begin{pmatrix} \alpha_j^i & \beta_j^i \\ 0 & \gamma_j^i \end{pmatrix}.$$

(6/18/24) **Page 91, two lines above the displayed equation:** Delete repeated "on."

(6/18/24) **Page 94, second-to-last line:** Change "Theorem 3.41" to "Theorem 3.39."

(7/10/24) **Page 97, last line of the proof:** Change "with $\sigma = s_j$" to "with $\sigma = s_j - f_j(p)s_0$ (so that $\widehat{\sigma} = f_j - f_j(p)$)."

(6/18/24) **Page 110, second full paragraph, first line:** Change z_1 and b_1 to z^1 and b^1, respectively, so the formula reads $\overline{D}_R(z^1) \supseteq D_\varepsilon(b^1)$.

(3/19/25) **Page 111, last displayed equation:** Swap $f \leftrightarrow g$, $f_1 \leftrightarrow g_1$, and $f_2 \leftrightarrow g_2$.

(7/16/24) **Page 112, line 4:** Change $\{0, \dots, q\}$ to $\{0, \dots, n\}$.

(1/13/25) **Page 112, second full paragraph, first line:** Change $\mathscr{C}_k^{0,q}(M)$ to $\mathscr{C}_k^{0,q}(U)$.

(7/16/24) **Page 113, last line of the proof:** Change $\bar\partial\big(\sum_I' \alpha_I \wedge \sigma_I\big)$ to $\bar\partial\big((-1)^p \sum_I' \alpha_I \wedge \sigma_I\big)$.

(6/18/24) **Page 115, third line:** Change "write β" to "write an arbitrary vector-valued form β."

(7/19/24) **Page 115, five lines below equation (4.17):** Change $E^{q+q'}(M; E)$ to $E^{q+q''}(M; E)$.

(6/18/24) **Page 115, eight lines below equation (4.17):** Change $\omega \in \mathscr{E}^q(\mathrm{End}(E))$ to $\omega \in \mathscr{E}^q(M; \mathrm{End}(E))$.

(7/19/24) **Page 116, third line from the bottom:** Change $\widetilde{\sigma}^k \tilde{s}_k$ to $\widetilde{\sigma}^k \otimes \tilde{s}_k$.

(6/18/24) **Page 123, three lines above Example 5.2:** Change $S_k(U)$ to $\mathcal{S}_k(U)$.

(3/16/25) **Page 131, proof of Lemma 5.8:** In the fourth line, replace U by B. In lines 5 and 6, replace $V \cap B$ by V.

(3/16/25) **Page 145, Problem 5-2:** Change "abelian groups" to "abelian groups, rings, vector spaces, or modules over a sheaf $\mathscr{R}$."

(6/17/24) **Page 145, Problem 5-7:** Change $\mathscr{H}\!\mathrm{om}$ to $\mathscr{H}\!\mathit{om}$ (three times). [This printing error, which also occurs on page 191, should be fixed in later printings and later versions of the ebook.]

(6/18/24) **Page 145, Problem 5-8(a):** Change "Exercise 5.6" to "Problem 5-1."

(4/7/25) **Page 148, proof of Lemma 6.1:** In the displayed equation, change all instances of $p + 1$ to $p + 2$ (five times). Here's the correct formula:

$$\begin{aligned}(\delta\delta c)_{\alpha_0\dots\alpha_{p+2}} &= \sum_{\substack{j\\0\le j\le p+2}} (-1)^j (\delta c)_{\alpha_0\dots\widehat{\alpha_j}\dots\alpha_{p+2}}\Big|_{U_0\cap\dots\cap U_{p+2}} \\ &= \sum_{\substack{j,k\\0\le k<j\le p+2}} (-1)^{j+k} c_{\alpha_0\dots\widehat{\alpha_k}\dots\widehat{\alpha_j}\dots\alpha_{p+2}}\Big|_{U_0\cap\dots\cap U_{p+2}} \\ &\quad + \sum_{\substack{j,k\\0\le j<k\le p+2}} (-1)^{j+k-1} c_{\alpha_0\dots\widehat{\alpha_j}\dots\widehat{\alpha_k}\dots\alpha_{p+2}}\Big|_{U_0\cap\dots\cap U_{p+2}},\end{aligned}$$

(4/7/25) **Page 156, three lines above equation (6.11):** Change $\rho^{\#}$ to ρ^*.

(7/15/24) **Page 156, two lines above equation (6.11):** Change "$\mathscr{A}$ and $\mathscr{B}$" to "$\mathscr{B}$ and $\mathscr{C}$."

(12/11/24) **Page 157, line 5:** Change $C^{p-1}(\mathscr{W}; \mathscr{C})$ to $C^{p-1}(\mathscr{W}; \mathscr{B})$.

(12/11/24) **Page 157, third paragraph, first line:** Replace $\beta_* \circ \mathscr{I}$ by $\mathscr{I} \circ \beta_*$.

(4/7/25) **Page 158, first displayed equation:** Add a restriction to $V_{x_0} \cap \dots \cap V_{x_p}$ on the right-hand side:

$$\tilde{b}_{x_0\dots x_p} = b^{(x_0)}_{\rho(x_0)\dots\rho(x_p)}\Big|_{V_{x_0}\cap\dots\cap V_{x_p}}.$$

(4/7/25) **Page 158, line below the first displayed equation:** Change b to $\tilde{b}$ (twice).

(4/7/25) **Page 162, Example 6.12, second paragraph:** Insert "and every integer $p \ge 1$" at the end of the first sentence of the paragraph.

(3/19/25) **Page 163, Example 6.13(c):** In the first two lines, change "Every skyscraper sheaf" to "Every skyscraper sheaf on a Hausdorff space," and "on a topological space" to "on a Hausdorff space."

(3/19/25) **Page 165, four lines above Propositino 6.15:** Change "cohomology class" to "homology class."

(4/7/25) **Page 169, line 2:** Change "0-chain" to "0-cochain."

(3/19/25) **Page 169, third line from the bottom:** Add missing left parenthesis in $H^k_{\mathrm{Sing}+}(M;G)$.

(4/7/25) **Page 170, last paragraph:** Eliminate property (iv) (it is redundant), and delete the rest of the last sentence on the page beginning with "and (iv) because …."

(4/7/25) **Page 171, line above equation (6.24):** Change (iv) to (i).

(11/27/24) **Page 178, statement of Corollary 6.26, last line:** Change "cohomologous" to "homologous."

(7/15/24) **Page 179, third line from the bottom:** Change "cycles" to "cocycles."

(7/15/24) **Page 181, line 10:** Change $\delta^*([L])$ to $\delta_*([L])$.

(3/19/25) **Page 183, line 3:** Change $H^2(M;\mathscr{O}^*)$ to $H^1(M;\mathscr{O}^*)$, and delete the spurious colon after the arrow.

(6/17/24) **Page 191, Problem 6-9(b):** Change $\mathscr{H}$om to $\mathscr{H}om$.

(6/17/24) **Page 191, Problem 6-10, first paragraph:** Change $\mathrm{St}\, v_0 \cap \cdots \cap \mathrm{St}\, v_p$ to $\mathrm{St}\, v_0 \cup \cdots \cup \mathrm{St}\, v_p$.

(6/17/24) **Page 193, second paragraph, last line:** Change "Chapter 3" to "Chapter 6."

(7/19/24) **Page 198, first full paragraph, second line:** Change $[0,b]$ to $[t_0,b]$.

(6/17/24) **Page 198, Proposition 7.8(b):** Change ∇E to $\nabla\sigma$.

(7/19/24) **Page 199, line below (7.9):** Change "If we interpret θ^l_j and Θ^l_j as the local expressions for $\mathrm{End}(E)$-valued forms" to "If we interpret the matrix $\theta = \left(\theta^l_j\right)$ as the local expression for an $\mathrm{End}(E)$-valued form."

(7/19/24) **Page 199, nine lines from the bottom:** Change $\tilde\theta = \tau^{-1}d\tau$ to $\tilde\theta = \tau^{-1}\theta\tau$.

(7/19/24) **Page 200, line below the second displayed equation:** Change $\partial_{j+1}\sigma$ to $\nabla_{\partial_{j+1}}\sigma$.

(7/19/24) **Page 201, part (iii) of the statement of Proposition 7.11:** Change "$D^2\alpha = \Theta\wedge\alpha$" to "For $\gamma \in \mathscr{E}^q(M;E)$, $D^2\gamma = \Theta\wedge\gamma$."

(7/20/24) **Page 203, lines 2 & 3:** Change "a section of $\mathscr{E}^1(\mathrm{End}(E))$" to "an element of $\mathscr{E}^1(M;\mathrm{End}(E))$."

(7/19/24) **Page 203, second line after the proof of Proposition 7.13:** Change "fiber metric on ∇" to "fiber metric on E."

(6/17/24) **Page 204, paragraph above Theorem 7.14:** Change "Chapter 5" to "Chapter 6."

(6/17/24) **Page 206, second line of the Remark:** Change $\delta^*([L])$ to $\delta_*([L])$.

(11/27/24) **Page 211, proof of Theorem 7.22, third line of the proof and third line from the bottom of the page:** Change L to L_D (twice).

(11/27/24) **Page 212, equation (7.21) and the third line below it:** Change L to L_D (twice).

(6/17/24) **Page 215, last displayed equation:** Change $\int_0^\infty$ to $\int_1^\infty$.

(6/17/24) **Page 219, Problem 7-5, second line:** Change "Problem 7-5" to "Problem 7-4."

(11/27/24) **Page 224, paragraph before Lemma 8.5:** Change "Hermitian structure" to "Hermitian metric."

(11/27/24) **Page 241, last two displayed equations:** The fractions $\frac{1}{8}$, $\frac{1}{8}$, and $\frac{1}{2}$ should be replaced with $\frac{1}{16}$, $\frac{1}{16}$, and $\frac{1}{4}$ (twice).

(11/27/24) **Page 253, Problem 8-14(a):** Replace the displayed equation by

$$B(Z, W) = \sec(\operatorname{Re} Z, \operatorname{Re} W) + \sec(\operatorname{Re} Z, \operatorname{Im} W).$$

(3/19/25) **Page 257, first paragraph, next-to-last line:** Change "it descends" to "then $q \circ G_1$ descends."

(11/27/24) **Page 268, sentence after the first displayed equation:** Replace "is just a smooth bundle homomorphism $P : \Gamma(E) \to \Gamma(F)$" by "can be identified with a smooth bundle homomorphism from E to F."

(1/28/25) **Page 268, last displayed equation:** Change $C\xi^2$ to $C\eta^2$.

(11/27/24) **Page 273, statement of Corollary 9.22, second line:** "manifold" is misspelled.

(1/28/25) **Page 273, lines 4 & 3 from the bottom:** Change $H^{N-k}(M; E)$ and $H^{N-k}(M; E^*)$ to $H^{N-k}(M; \mathbb{C})$ and $H^{N-k}(M; \mathbb{C})^*$, respectively.

(1/28/25) **Page 274, Lemma 9.23(a):** Change $\mathscr{E}^{p+q}(M)$ to $\mathscr{E}^{p,q}(M)$.

(1/28/25) **Page 279, proof of Theorem 9.37, first displayed equation:** The last three occurrences of η in that equation should be changed to ζ:

$$\int_M \eta \wedge \zeta = \int_M \bar{\partial}_E \gamma \wedge \zeta = \int_M \bar{\partial}(\gamma \wedge \zeta) = \int_M d(\gamma \wedge \zeta) = 0.$$

(11/27/24) **Page 283, last paragraph of the proof of Proposition 9.38:** Change Theorem 8.10(e) to Theorem 8.10(d).

(11/27/24) **Page 289, Corollary 9.47:** Change b_k to b^k and b_{k+2} to b^{k+2} (twice).

(11/27/24) **Page 296, proof of Theorem 9.61, line 6:** Change $\chi(L \otimes L^k_{\{p\}})$ to $\chi(\mathcal{O}(L \otimes L^k_{\{p\}}))$ and $\chi(L)$ to $\chi(\mathcal{O}(L))$.

(9/21/24) **Page 311, Problem 9-13:** There are several errors in the problem statement. First, replace the second sentence by the following: "Define the ***dual torus*** to be the group $T^* = \operatorname{Hom}(\Lambda, \mathbb{C}^*)$ (the set of homomorphisms from the lattice Λ to the group of nonzero complex numbers, which is a group under pointwise multiplication)." Then in part (a), replace $e^{2\pi i \operatorname{Im} f(v)}$ by $e^{2\pi i f(v)}$, and replace the displayed equation by

$$\Lambda^* = \{f \in \overline{V}^* : f(v) \in \mathbb{Z} \text{ for all } v \in \Lambda\}.$$

(9/21/24) **Page 311, Problem 9-14:** In the third line and in the displayed equation, change Λ to Γ (to be consistent with the notation in the proofs of Theorems 9.68 & 9.69).

(11/27/24) **Page 318, first line of the proof:** Change "local frame" to "local orthonormal frame."

(11/27/24) **Page 319, statement of Lemma 10.3:** Change $\mathscr{E}^{1,1}(M; E)$ to $\mathscr{E}^{1,1}(M; \operatorname{End}(E))$.

(1/28/25) **Page 321, proof of Proposition 10.5, last sentence:** Change F to f.

(1/28/25) **Page 331, expression (10.14):** Remove spurious last right parenthesis.

(1/28/25) **Page 331, last paragraph, first line:** Change π_* to π^*.

(11/27/24) **Page 333, third display:** Remove spurious last right parenthesis.

(1/28/25) **Page 335, just above the last displayed equation:** Change $\operatorname{Sing}^2_\infty(\widetilde{M})$ to $\operatorname{Sing}^\infty_2(\widetilde{M})$.

(1/28/25) **Page 338, Problem 10-5, last line:** Remove spurious "that."