CORRECTIONS TO
Axiomatic Geometry
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(1/13/16) **Preface, page xv, end of the first full paragraph:** Change "Dominic Joyce" to "David Joyce."

(8/28/18) **Page 8, lines 4 and 5:** Replace the clause beginning "then pick it up" with the following: "then pick it up and mark off a segment of that length on a line somewhere else."

(8/28/18) **Page 8, paragraph 2, second sentence:** Replace the sentence with this: "After Proposition I.3 is proved, given any sufficiently long line segment, you can mark off a part of that segment having the same length as another segment somewhere else, just as we typically do with a physical compass."

(6/18/19) **Page 41, last line of the proof of Theorem 2.23:** Change ℓ to ℓ'.

(8/28/18) **Page 61, line above Corollary 3.10:** Change "next lemma" to "next corollary."

(8/28/18) **Page 61, proof of Corollary 3.10, second line:** After "by definition," add "(since the ruler postulate guarantees that there is at least one coordinate function for ℓ)."

(8/28/18) **Page 71, paragraph below Fig. 3.6:** After the definitions of ***endpoint*** and ***interior point***, insert the following sentence: "(Corollary 3.41 below will show that these concepts are consistently defined, independently of how the ray is presented. But for simplicity, we will go ahead and refer to "the" endpoint and "the" interior points of a ray, knowing that this terminology will be justified later.)"

(8/28/18) **Page 75, bottom of the page:** After the last line on the page, insert the following sentence: "(The next theorem and its corollary use only the definition of a ray together with Lemma 3.30 and Theorem 3.35, which did not depend on knowing that endpoints and interior points were consistently defined.)"

(8/28/18) **Page 89, proof of Theorem 4.8:** Replace the first two sentences of the proof by the following: "Given that $\overrightarrow{a} * \overrightarrow{b} * \overrightarrow{c}$, choose some half-rotation $\mathrm{HR}(\overrightarrow{r}, P)$ containing all three rays a corresponding coordinate function g such that either $g(\overrightarrow{a}) < g\big(\overrightarrow{b}\big) < g(\overrightarrow{c})$ or $g(\overrightarrow{a}) > g\big(\overrightarrow{b}\big) > g(\overrightarrow{c})$; after interchanging the names of $\overrightarrow{a}$ and $\overrightarrow{c}$ if necessary, we may as well assume that the first set of inequalities holds."

(3/1/17) **Page 93, proof of Theorem 4.16, second paragraph:** Replace the phrase "We noted above that $\overrightarrow{c}'$ and $\overrightarrow{a}$ lie on opposite sides of $\overleftrightarrow{b}$" with "By the X-lemma, $\overrightarrow{c}'$ and $\overrightarrow{a}$ lie on opposite sides of $\overleftrightarrow{b}$."

(4/13/15) **Page 98, Lemma 4.26, part (a):** Add the hypothesis that no two of the rays are collinear.

(5/8/15) **Page 142, proof of Theorem 7.1, third paragraph:** Change "AFD and $AF'D$ are a linear pair" to "ACD and $A'CD$ are a linear pair."

(5/8/15) **Page 224, proof of Theorem 12.14:** Replace the first sentence of the proof by "Let $ABCD$ be a rectangle whose shorter side has length a and whose longer side has length c, and define $b = c - a$ (see Fig. 12.14)."

(5/8/15) **Page 225, first full paragraph:** Replace the beginning of the first sentence by "Conversely, if $ABCD$ is a rectangle in which the side lengths are a and $c = \varphi a$, let $b = c - a$; then a straightforward computation shows ...."

(7/31/18) **Page 244, sketch of the proof of Theorem 13.19, Step 5:** There is a gap in the argument for Step 5: Because the cosine function is defined in terms of the relationship between distances and angles, which might behave differently in different models, it's not immediately clear that the cosine functions defined in $\mathcal{M}$ and $\mathcal{M}'$ are equal to each other. To show that they are, let cos and cos$'$ denote the cosine functions in the models $\mathcal{M}$ and $\mathcal{M}'$, respectively, and define sin and sin$'$ similarly. (The primes do *not* represent derivatives here.) Let us say that a number $\alpha \in [0, 180]$ is a ***matched number*** if $\cos\alpha = \cos'\alpha$ and $\sin\alpha = \sin'\alpha$. It follows directly from the definitions that 0, 90, and 180 are matched numbers, and consideration of isosceles right triangles shows that 45 is also matched. Using induction on n together with the angle-sum formulas (Thm. 13.15), one can show that if α is a matched number between 0 and 90, then so is $n\alpha$ for any positive integer n such that $n\alpha < 90$. The double-angle formulas (Cor. 13.16) then can be can be used to derive formulas expressing $\sin \frac{1}{2}\alpha$ and $\cos \frac{1}{2}\alpha$ in terms of $\sin \alpha$ and $\cos \alpha$ (and similarly for $\sin'$ and $\cos'$), and then another inductive argument based on these formulas shows that if $\alpha \in (0, 90)$ is matched, then so is $(1/2^m)\alpha$ for every positive integer m. The upshot is that every number between 0 and 90 of the form $(n/2^m)45$ for positive integers m and n is matched. (A rational number whose denominator is a power of 2 is called a ***dyadic rational***, so this shows that all dyadic rational multiples of 45 in $(0, 90)$ are matched numbers.) Because there is a dyadic rational between any two distinct real numbers, an argument by contradiction just like the last step of the proof of Theorem 11.7 shows that every real number between 0 and 90 is matched. Finally, the formulas $\sin \theta = \sin(180° - \theta)$ and $\cos \theta = -\cos(180° - \theta)$ (and their counterparts for $\sin'$ and $\cos'$) show that every number between 90 and 180 is matched, thus completing the proof.

(7/31/18) **Page 248, proof of Theorem 14.3:** After the second sentence of the proof, insert this: "A line through O can only contain two points on the circle by the result of Exercise 3G, so we may assume O is not collinear with A, B, C."

(7/25/16) **Page 304, Construction Problem 16.22:** In the first line of the solution, change the word **Proof** to **Solution**. In the next-to-last line, change ABC_nC_n to ABC_nD_n.

(1/13/16) **Page 452, reference [Euc98]:** Change "Dominic E. Joyce" to "David E. Joyce."