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Rational, Log Canonical, Du Bois Singularities II: Kodaira Vanishing and Small Deformations⋆

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Abstract. Kollár's conjecture, that log canonical singularities are Du Bois, is proved in the case of Cohen–Macaulay 3-folds. This in turn is used to derive Kodaira vanishing for this class of varieties. Finally it is proved that small deformations of Du Bois singularities are again Du Bois.

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The present article is a direct continuation of [Kovács99]. Please see the introduction of that article for details on Du Bois singularities, Steenbrink's conjecture, and Kollár's conjecture. There it was proved that Steenbrink's conjecture holds, namely rational singularities are Du Bois and some partial results were obtained regarding Kollár's conjecture, namely that under reasonable circumstances, Du Bois singularities should coincide with log canonical singularities. For the exact statements, please refer to [Kovács99].

The first result here is a strengthening of the one obtained there. In fact most of the ingredients are contained in that article, only the realization that this stronger statement follows was missing.

THEOREM 0.1. *Let X be a complex variety with log canonical Cohen–Macaulay singularities. Assume that either* $\dim X \leqslant 3$ *or* $2 \dim \mathrm{Sing} X + 1 < \dim X$. *Then X has Du Bois singularities.*

Combined with [Kovács99, 3.6] this implies the following.

COROLLARY 0.2. *Let X be a normal Gorenstein variety. Assume that either* $\dim X \leqslant 3$ *or* $2 \dim \mathrm{Sing} X + 1 < \dim X$. *Then X has Du Bois singularities if and only if it has log canonical singularities.*

This is a higher-dimensional generalization of [Steenbrink83, 3.8].

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As a simple application, it is shown that Kodaira vanishing holds in the following form for log canonical varieties:

THEOREM 0.3. *Let X be a proper log canonical Cohen–Macaulay variety such that either* $\dim X \leqslant 3$ *or* $2 \dim \mathrm{Sing} X + 1 < \dim X$. *Let* $\mathcal{L}$ *be an ample line bundle. Then* $H^i(X, \mathcal{L}^{-1}) = 0,\ i < \dim X$.

Note that the restriction on the dimension of the singular locus should be removable (I was unable to do that so far), but the Cohen–Macaulayness assumption is necessary, since Kodaira vanishing implies the dual form of Serre's vanishing which, in turn, implies Cohen–Macaulayness via Serre duality.

The second part of the article is concerned with small deformations. A basic question that comes up for every class of singularities is whether the class is invariant under small deformations. It is well known that rational singularities are invariant [Elkik78]. Steenbrink asked whether the same is true for Du Bois singularities [Steenbrink83].

The next result is an affirmative answer to this question. It was first proved for isolated Gorenstein singularities by [Ishii86].

THEOREM 0.4. *Small deformations of Du Bois singularities are again Du Bois.*

DEFINITIONS and NOTATION. The following, more general definition will replace the one used in [Kovács99].

A pair, (X, Δ), consisting of a normal variety X and a divisor Δ is said to have *log canonical* (resp. *log terminal, canonical*) *singularities* if $K_X + \Delta$ is $\mathbb{Q}$-Cartier and for any resolution of singularities $f : Y \to X$, with the collection of exceptional prime divisors $\{E_i\}$, there exist $a_i \in \mathbb{Q}$, $a_i \geqslant -1$ (resp. $a_i > -1$, $a_i \geqslant 0$) such that $K_Y + \tilde{\Delta} \equiv f^*(K_X + \Delta) + \sum a_i E_i$ where $\tilde{\Delta}$ is the proper tranform of Δ (cf. [KM98]).

X itself will be called *log canonical* (resp. *log terminal, canonical*) if $(X, \emptyset)$ has log canonical (resp. log terminal, canonical) singularities. This is the form these notions were used in [Kovács99]. Note that if K_X is $\mathbb{Q}$-Cartier, in particular if X is Gorenstein, and (X, Δ) is log canonical (resp. log terminal, canonical) for some Δ, then X itself is log canonical (resp. log terminal, canonical).

Let X be a complex scheme. Then $\mathrm{Sing}_r X$ (resp. $\mathrm{Sing}_{lt} X$) will denote the smallest closed subset of X, such that $X \setminus \mathrm{Sing}_r X$ has rational (resp. $(X \setminus \mathrm{Sing}_{lt} X, D)$ has log terminal for some D). Note that $\mathrm{Sing}_r X \subseteq \mathrm{Sing}_{lt} X$ by [Elkik81].

For all other definitions and notation please refer to [Kovács99].

1. Du Bois Singularities

The original construction of Du Bois' complex, $\underline{\Omega}^{\cdot}_X$, is based on simplicial resolutions. The reader interested in the details is referred to the original article. Note also that a simplified construction was later obtained by [GNPP88] via the general theory of cubic resolutions. An easily accessible introduction can be found in [Steenbrink85].

Hyperresolution will refer to either simplicial or cubic. Formally the construction of $\underline{\Omega}^{\cdot}_X$ is the same regardless the resolution used and no specific aspects of either resolution will be used. The basic results regarding $\underline{\Omega}^{\cdot}_X$ that are essential in the sequel are summarized in [Kovács99, 1.1]. In the rest of the article (1.1.1–6) will refer to [Kovács99, 1.1.1–6]. In addition to those one more property will be used:

THEOREM 1.1 [DuBois81].

(1.1.7) *Let* $\epsilon_.: X_. \to X$ *be a hyperresolution. Then* $\underline{\Omega}^0_X \simeq_{qis} R\epsilon_{.*}\mathcal{O}_{X_.}$.

Remark. 1.1.0. Let $f: Y \to X$ be a resolution of singularities of X. Then by (1.1.1) and (1.1.2) the natural morphism $\mathcal{O}_X \to Rf_*\mathcal{O}_Y$ factors through $\underline{\Omega}^0_X$. Furthermore, if X is proper, then by (1.1.4) $H^i(X, \mathbb{C}) \to \mathbb{H}^i(X, \underline{\Omega}^0_X)$ is surjective for all i.

DEFINITION 1.2 [Steenbrink83]. X is said to have *Du Bois singularities* if $\mathcal{O}_X \to \underline{\Omega}^0_X$ is a quasi-isomorphism. (i.e., $h^0(\underline{\Omega}^0_X) \simeq \mathcal{O}_X$ and $h^i(\underline{\Omega}^0_X) = 0$ for all $i \neq 0$.) In particular, if X is proper and has Du Bois singularities, then $H^i(X, \mathbb{C}) \to H^i(X, \mathcal{O}_X)$ is surjective for all i.

1.3. Let X be a complex scheme that one would like to prove to have Du Bois singularities. Let $F^{\cdot}$ be a complex such that $\mathcal{O}_X \to \underline{\Omega}^0_X \to F^{\cdot} \xrightarrow{+1}$ forms a distinguished triangle and let $\mathrm{Sing}_{DB} X = \cup \mathrm{Supp} h^i(F^{\cdot})$ the union of the supports of the cohomology sheaves of $F^{\cdot}$. Then $\mathrm{Sing}_{DB} X$ is the *non-Du Bois locus* of X. By taking general hyperplane sections, as in [Kollár95, 12.8], one may assume that $\dim \mathrm{Sing}_{DB} X \leqslant 0$. Therefore as long as the assumptions on X are invariant under taking hyperplane sections, one can restrict to the case when the possibly non-Du Bois locus is at most a set of finite points.

LEMMA 1.4. *Let X be a complex scheme with a finite set of points, P, such that* $X \setminus P$ *has only Du Bois singularities. Then* $H^i_P(X, \mathcal{O}_X) \to \mathbb{H}^i_P(X, \underline{\Omega}^0_X)$ *is surjective for all i.*

Proof. Since the statement is local, one may assume that X is affine. Let $F^{\cdot}$ be the complex defined in (1.3). By assumption P contains $\mathrm{Sing}_{DB} X$.

Next let $\bar{X}$ be a projective closure of X, and let $Q = \bar{X} \setminus X$ and $Z = P \cup Q$. Then $\bar{X} \setminus Z \simeq X \setminus P$ has only Du Bois singularities, i.e., $\mathcal{O}_{\bar{X}\setminus Z} \simeq_{qis} \underline{\Omega}^0_{\bar{X}\setminus Z}$.
Now by (1.1.4) the composition,

$$H^i(\bar{X}, \mathbb{C}) \to H^i(\bar{X}, \mathcal{O}_{\bar{X}}) \to \mathbb{H}^i(\bar{X}, \underline{\Omega}^0_{\bar{X}})$$

is surjective for all i. Then in the commutative diagram,

$$\begin{array}{ccccccc} H^{i-1}(\bar{X} \setminus Z, \mathcal{O}_{\bar{X}\setminus Z}) & \to & H^i_Z(\bar{X}, \mathcal{O}_{\bar{X}}) & \to & H^i(\bar{X}, \mathcal{O}_{\bar{X}}) & \to & H^i(\bar{X} \setminus Z, \mathcal{O}_{\bar{X}\setminus Z}) \\ \downarrow \alpha & & \downarrow \beta & & \downarrow \gamma & & \downarrow \delta \\ \mathbb{H}^{i-1}(\bar{X} \setminus Z, \underline{\Omega}^0_{\bar{X}\setminus Z}) & \to & \mathbb{H}^i_Z(\bar{X}, \underline{\Omega}^0_{\bar{X}}) & \to & \mathbb{H}^i(\bar{X}, \underline{\Omega}^0_{\bar{X}}) & \to & \mathbb{H}^i(\bar{X} \setminus Z . \underline{\Omega}^0_{\bar{X}\setminus Z}) \end{array}$$

the rows are exact, α and δ are isomorphisms, and γ is surjective. Hence β is surjective by the 5-lemma.

Observe that $P \cap Q = \emptyset$, since $\dim P \leqslant 0$, so

$$H^i_Z(\bar{X}, \mathcal{O}_{\bar{X}}) \simeq H^i_P(\bar{X}, \mathcal{O}_{\bar{X}}) \oplus H^i_Q(\bar{X}, \mathcal{O}_{\bar{X}}),$$
$$\mathbb{H}^i_Z(\bar{X}.\underline{\Omega}^0_{\bar{X}}) \simeq \mathbb{H}^i_P(\bar{X}, \underline{\Omega}^0_{\bar{X}}) \oplus \mathbb{H}^i_Q(\bar{X}, \underline{\Omega}^0_{\bar{X}}),$$

and by excision

$$H^i_P(\bar{X}, \mathcal{O}_{\bar{X}}) \simeq H^i_P(X, \mathcal{O}_X) \quad \text{and} \quad \mathbb{H}^i_P(\bar{X}, \underline{\Omega}^0_{\bar{X}}) \simeq \mathbb{H}^i_P(X, \underline{\Omega}^0_X).$$

Therefore $H^i_P(X, \mathcal{O}_X) \to \mathbb{H}^i_P(X, \underline{\Omega}^0_X)$ is surjective. □

COROLLARY 1.5 [Kovács99, 2.2]. *Let X be a complex scheme with a finite set of points, P, such that $X \setminus P$ has only Du Bois singularities. Further assume that $H^i_P(X, \mathcal{O}_X) \to \mathbb{H}^i_P(X, \underline{\Omega}^0_X)$ is injective for all $i = 0, \ldots, \dim X$. Then X has Du Bois singularities.*

Proof. By (1.4) $H^i_P(X, \mathcal{O}_X) \to \mathbb{H}^i_P(X, \underline{\Omega}^0_X)$ is also surjective, hence an isomorphism, so $\mathbb{H}^i_P(X, F^\cdot) = 0$. The cohomology sheaves of $F^\cdot$ are supported on P, so $\mathbb{H}^i(X \setminus P, F^\cdot) = 0$ for all i. Hence, $\mathbb{H}^i(X, F^\cdot) = \mathbb{H}^i_P(X, F^\cdot) = 0$. Using again that $\dim P \leqslant 0$, one finds that $\mathbb{H}^i(X, F^\cdot) = H^0(X, h^i(F^\cdot))$, so in fact $h^i(F^\cdot) = 0$ for all i, thus $\mathcal{O}_X \simeq \underline{\Omega}^0_X$. □

DEFINITION 1.6. X will be said to have *semi-Du Bois singularities* if $h^i(\underline{\Omega}^0_X) = 0$ for $i \neq 0$.

PROPOSITION 1.7 [DuBois81, 4.9]. *Every curve has semi-Du Bois singularities.*

THEOREM 1.8. *Let X be a complex variety with log canonical Cohen–Macaulay singularities. Assume that either* $\dim \operatorname{Sing} X + \dim \operatorname{Sing}_r X + 1 < \dim X$ *or* $\operatorname{Sing} X$ *has semi-Du Bois singularities. Then X has Du Bois singularities.*

Proof. First assume that X has Gorenstein singularities. By (1.1.5)

$$\dim \operatorname{Supp} h^i(\underline{\Omega}^0_{\operatorname{Sing} X}) \leqslant \dim \operatorname{Sing} X - i$$

so the assumptions imply that

$$\dim \operatorname{Supp} h^i(\underline{\Omega}^0_{\operatorname{Sing} X}) < \dim X - \dim \operatorname{Sing}_r X - 1 - i \quad \text{for all } i > 0.$$

Then the statement follows by [Kovács99, 3.4].

The statement is local, so in the general case one can take the index 1 cover, X', of X which has log canonical Cohen–Macaulay singularities of index 1 (cf. [Reid87, 3.6), hence it is Gorenstein. Then X' has Du Bois singularities by the first part, so X has Du Bois singularities by [Kovács99, 2.5]. □

COROLLARY. *Let X be a complex 3-fold with log canonical Cohen–Macaulay singularities. Then X has Du Bois singularities.*

2. Kodaira Vanishing for Log Canonical Singularities

The following theorem represents a culmination of the work of several authors: Tankeev, Ramanujam, Miyaoka, Kawamata, Viehweg, Kollár, Esnault-Viehweg. It should be regarded as the ultimate generalization of the Kodaira vanishing theorem.

THEOREM 2.1 [Kollár95, 9.12]. *Let X be a proper variety and L a line bundle on X. Let $L^n \simeq \mathcal{O}_X(D)$, where $D = \sum d_i D_i$ is an effective divisor and let s be a global section whose zero divisor is D. Assume that $0 < d_i < n$ for every i. Let Z be the normalization of taking the n^{th} root of s. Assume further that*

$$H^j(Z, \mathbb{C}_Z) \to H^j(Z, \mathcal{O}_Z) \qquad (\star)$$

is surjective. Then for any $b_i \geqslant 0$ the natural map

$$H^j\Big(X, L^{-1}\Big(-\sum b_i D_i\Big)\Big) \to H^j(X, L^{-1})$$

is surjective.

Combined with (1.9), this will easily imply that Kodaira vanishing holds for log canonical Cohen–Macaulay threefolds.

THEOREM 2.2. *Let X be a proper log canonical Cohen–Macaulay variety such that either $\dim X \leqslant 3$ or $\dim \operatorname{Sing} X + \dim \operatorname{Sing}_{lt} X + 1 < \dim X$. Let $\mathcal{L}$ be an ample line bundle. Then $H^i(X, \mathcal{L}^{-1}) = 0, \quad i < \dim X$.*

Proof. Let $m \gg 0$ and $s \in H^0(X, \mathcal{L}^m)$ generic. Then $D = (s = 0)$ is reduced and irreducible and intersects the singular locus transversally, so (X, D) is log canonical. Let $\mathcal{A} = \oplus_{i=0}^{m-1} \mathcal{L}^{-i}$ with the $\mathcal{O}_X$-algebra structure induced by s. Let $\pi\colon Z = \operatorname{Spec}_X \mathcal{A} \to X$ be the mth root cover of X. X is Cohen–Macaulay, hence so is $\mathcal{A} \simeq \pi_* \mathcal{O}_Z$. Thus Z is Cohen–Macaulay and then normal by Serre's criterion.

Let $B = (\pi^* D)_{\mathrm{red}}$ and r the index of K_X. Since $X$and Z are smooth in codimension 2, it follows, that $\pi^*(r(K_X + D)) = r(K_Z + B)$. Furthermore, $\pi^* D = mB$, so B and hence K_Z is $\mathbb{Q}$-Cartier. Then by [Kollár *et al.* 92, 20.3.3] Z is log canonical. Similarly $\dim \operatorname{Sing}_r Z \leqslant \dim \operatorname{Sing}_{lt} Z \leqslant \dim \operatorname{Sing}_{lt} X$, so either $\dim Z \leqslant 3$ or $\dim \operatorname{Sing} Z + \dim \operatorname{Sing}_r Z + 1 < \dim Z$. Therefore Z has Du Bois singularities by (1.9).

Now (2.1) implies that $H^i(X, L^{-N}) \to H^i(X, L^{-1})$ is surjective for all $N \geqslant 0$. Finally Serre's vanishing implies that $H^i(X, L^{-N}) = 0$ for $N \gg 0$ and $i < \dim X$. $\square$

3. Deformations

The following notation will be fixed through the rest of the article:

3.1. Let (S, s) be the germ of a smooth complex curve and let u be a local uniformizing parameter at $s \in S$. Further let $f\colon X \to S$ be an algebraic fibre space and $t = f^* u$. Note that the ideal sheaf of X_s is generated by t.

THEOREM 3.2. *Assume that X_s has Du Bois singularities. Then X has Du Bois singularities as well.*

Proof. Let $P = \mathrm{Sing}_{DB} X$, the non-Du Bois locus of X. By taking general hyperplane sections one can assume that P is a finite set of points contained in X_s (cf. (1.3)).

Let $\epsilon_\cdot\colon X_\cdot \to X$ be a hyperresolution of X. Then there exists a hyperresolution, $\mu_\cdot\colon Z_\cdot \to X_s$, of X_s such that it factors though $(\epsilon_\cdot)_s$.

$$\begin{array}{ccccc}
Z_\cdot & \xrightarrow{\nu} & (X_\cdot)_s & \to & X_\cdot \\
 & \mu_\cdot\searrow & \downarrow & & \downarrow \epsilon_\cdot \\
 & & X_s & \hookrightarrow & X \\
 & & f_s \downarrow & & \downarrow f \\
 & & \{s\} & \hookrightarrow & S
\end{array}$$

Now $\underline{\Omega}^0_X \simeq_{qis} R\epsilon_{\cdot *}\mathcal{O}_{X_\cdot}$ and $\underline{\Omega}^0_{X_s} \simeq_{qis} R\mu_{\cdot *}\mathcal{O}_{Z_\cdot}$ by (1.1.7). One also has two natural morphisms,

$$\rho\colon \mathcal{O}_{X_s} \to R\epsilon_{\cdot *}\mathcal{O}_{(X_\cdot)_s} \quad \text{and} \quad \tau\colon R\epsilon_{\cdot *}\mathcal{O}_{(X_\cdot)_s} \to R\mu_{\cdot *}\mathcal{O}_{Z_\cdot} \simeq_{qis} \underline{\Omega}^0_{X_s}.$$

Since X_s has Du Bois singularities, $\tau \circ \rho$ is a quasi-isomorphism.

Furthermore, one has the following commutative diagram where the rows form distinguished triangles in the derived category of $\mathcal{O}_X$-modules.

$$\begin{array}{ccccccc}
\mathcal{O}_X & \xrightarrow{t\cdot} & \mathcal{O}_X & \longrightarrow & \mathcal{O}_{X_s} & \xrightarrow{+1} \\
\downarrow & & \downarrow & & \downarrow \rho & \\
\underline{\Omega}^0_X & \xrightarrow{t\cdot} & \underline{\Omega}^0_X & \longrightarrow & R\epsilon_{\cdot *}\mathcal{O}_{(X_\cdot)_s} & \xrightarrow{+1} \\
 & & & & \downarrow \tau & \\
 & & & & \underline{\Omega}^0_X &
\end{array} \tag{3.2.1}$$

Since $\tau \circ \rho$ is a quasi-isomorphism,

$$\gamma\colon H^j_P(X_s, \mathcal{O}_{X_s}) \hookrightarrow \mathbb{H}^j_P(X_s, R\epsilon_{\cdot *}\mathcal{O}_{(X_\cdot)_s})$$

is injective for all j.

Note that by (1.4) $\alpha_j : H^j_P(X, \mathcal{O}_X) \twoheadrightarrow \mathbb{H}^j_P(X, \underline{\Omega}^0_X)$ is surjective for all j.

Note also that since $P \subseteq X_s$, t annihilates P, hence for all $a \in H^i_P(X, \mathcal{O}_X)$ there exists an integer m such that $t^m \cdot a = 0$.

Now apply $\mathbb{H}_{\mathbb{P}}$ to (3.2.1).

$$\begin{array}{ccccccc}
H^{i-1}_P(X, \mathcal{O}_X) & \xrightarrow{\delta_1} & H^{i-1}_P(X_s, \mathcal{O}_{X_s}) & \xrightarrow{\beta_1} & H^i_P(X, \mathcal{O}_X) & \xrightarrow{t\cdot} & H^i_P(X, \mathcal{O}_X) \\
\downarrow \alpha_{i-1} & & \downarrow \gamma & & \downarrow \alpha_i & & \downarrow \\
\mathbb{H}^{i-1}_P(X, \underline{\Omega}^0) & \xrightarrow{\delta_2} & \mathbb{H}^{i-1}_P(X_s, R\epsilon_{\cdot *}\mathcal{O}_{(X_\cdot)_s}) & \xrightarrow{\beta_2} & \mathbb{H}^i_P(X, \underline{\Omega}^0_X) & \xrightarrow{t\cdot} & \mathbb{H}^i_P(X, \underline{\Omega}^0_X)
\end{array}$$

Let $a \in H^i_P(X, \mathcal{O}_X)$, $a \neq 0$, and choose m and such that $t^m \cdot a = 0$, but $t^{m-1} \cdot a \neq 0$. Assume that $\alpha_i(a) = 0$. Then $\alpha_i(t^{m-1} \cdot a) = t^{m-1} \cdot \alpha_i(a) = 0$. Now $t \cdot (t^{m-1} \cdot a) = 0$, hence there exists a $b \in H^{i-1}_P(X_s, \mathcal{O}_{X_s})$ such that $\beta_1(b) = t^{m-1} \cdot a$. Then $\beta_2(\gamma(b)) = 0$, so there exists a $c \in \mathbb{H}^{i-1}_P(X, \underline{\Omega}^0_X)$ such that $\delta_2(c) = \gamma(b)$. α_{i-1} is surjective, so there exists a $d \in H^{i-1}_P(X, \mathcal{O}_X)$ such that $\alpha_{i-1}(d) = c$. Thus $\gamma(b) = \delta_2(\alpha_{i-1}(d)) = \gamma(\delta_1(d))$, and then $b = \delta_1(d)$, since γ is injective. But then $t^{m-1} \cdot a = \beta_1(b) = 0$, contrary to our assumption.

$$
\begin{tikzcd}
d \arrow[r, mapsto, "\delta_1"] \arrow[d, mapsto, "\alpha_{i-1}"] & b \arrow[r, "\delta_1"] \arrow[d, mapsto, "\gamma"] & t^{m-1} \cdot a \arrow[r, "t\cdot"] \arrow[d, mapsto, "\alpha_i"] & 0 \\
c \arrow[r, mapsto, "\delta_2"] & \gamma(b) \arrow[r, mapsto, "\beta_2"] & 0 &
\end{tikzcd}
$$

Therefore α_i is injective for all i and then X is Du Bois by (1.5). □

COROLLARY 3.3. *Du Bois singularities are stable under small deformations.*

COROLLARY 3.4. *Assume that X is a normal Gorenstein variety and X_s has log canonical singularities and either* $\dim X_s \leqslant 3$ *or* $\dim \mathrm{Sing} X_s + \dim \mathrm{Sing}_r X_s + 1 < \dim X_s$. *Then X has log canonical singularities.*

Proof. X has Du Bois singularities by (3.2), since X_s has Du Bois singularities by (1.9). Then X has log canonical singularities by [Kovács99, 3.6]. □

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