Doing Math With Teachers

In Washington State

Northwest Mathematics Interaction

- NWMI is a local outreach group of the Park City Mathematics Institute, based in the University of Washington Mathematics Department, and led by a team of secondary teachers and higher ed mathematicians. Since 1995 NWMI has offered math workshops and summer programs for teachers (especially around geometry) sometimes working with math-science partnership projects.

Several Flavors of Doing Math

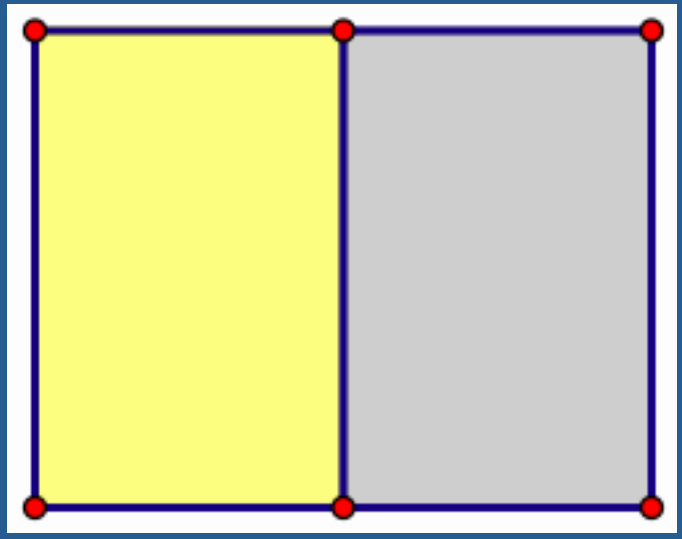
- Doing math related to the math that teachers teach ("I can do this with my kids").
- Doing math that is intrinsically interesting and challenging.
- Doing math that showcases big ideas.
- Doing math that approaches the realm of research.

Proportion and Similarity

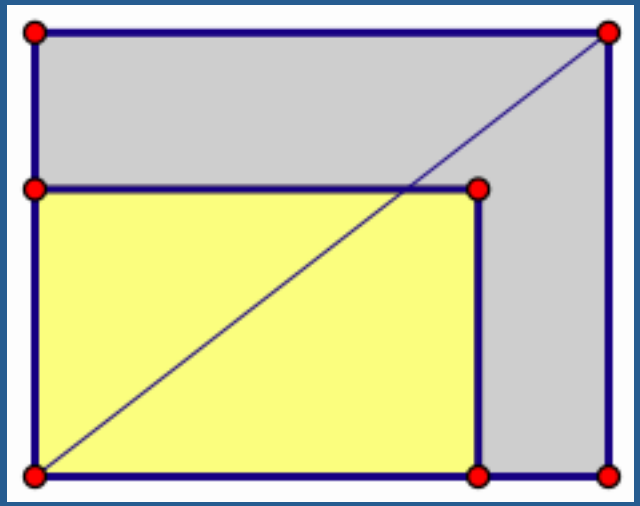
- Proportional reasoning is one of the big ideas of middle school math.
- An approach to similarity via dilations is central to the Common Core State Standards but is not how many teachers think of similarity.

Half-rectangles and Similarity

- Fold a rectangular sheet of paper in half to form a new rectangle. Is the new rectangle similar to the old one?

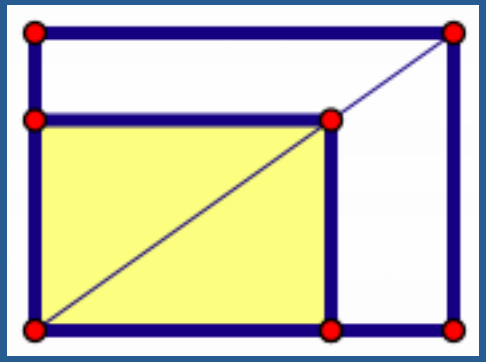


Not Similar for 8.5 x 11



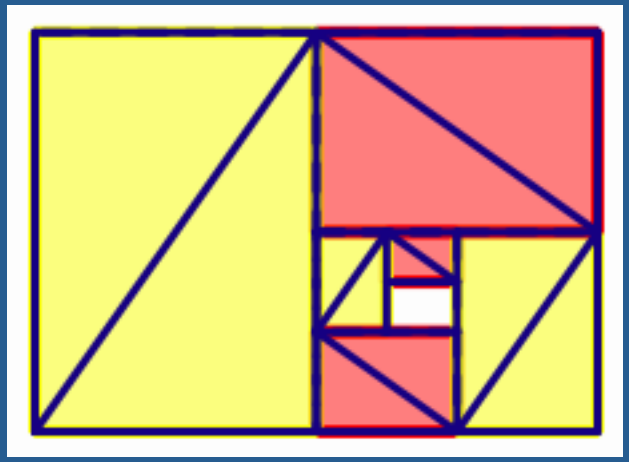
- This raises question about meaning of similarity for figures that are not triangles.
- One approach by checking ratios of corresponding sides.
- A second approach uses the geometry of dilations.

For what rectangles is the half-rectangle similar?



- Equate ratios of sides, $a/b = b/(a/2)$ implies $(b/a)^2 = 2$.
- Think of similar rectangle as dilations of original. If area = ½ original, then ratio of dilation must be square root of ½.
- This shape of paper is actually used world-wide as A4 paper (and A3, A2, etc)

Related Questions



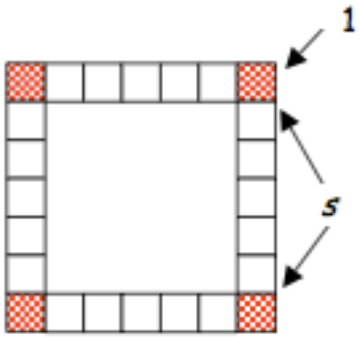
- Form a spiral of similar half-rectangles. What is the limit point? One can use dilations but there is also entry by forming a table of coordinates of corresponding points, leading to geometric series.
- This line of thought can be extended further by introducing the golden rectangle and its spiral.

Variations on a well-known Professional Development Problem

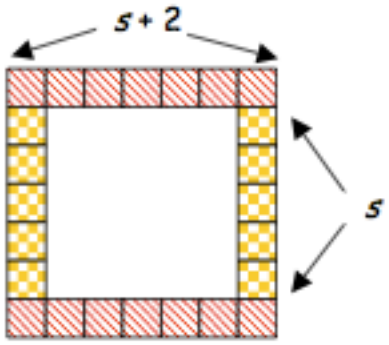
- The Pool Border problem: What is the area of a border of uniform width added to a rectangular pool (garden? Rug?).

Solutions relate Algebra & Geometry

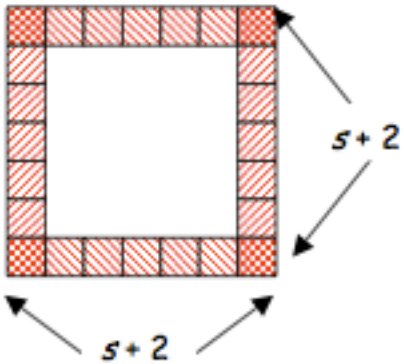
1. Sides + corners:
   $n = 4s + 4$



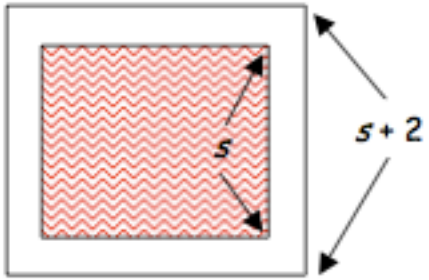
2. Top/bottom and 2 sides:
   $n = 2(s + 2) + 2s$



4. Four sides – 4 corners:
   $n = 4(s + 2) - 4$

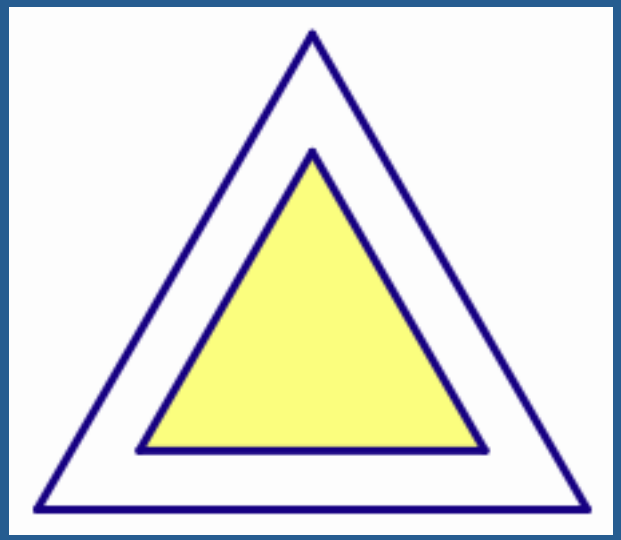


5. Whole area – area of pool:
   $n = (s + 2)^2 - s^2$
   s = length of the pool side



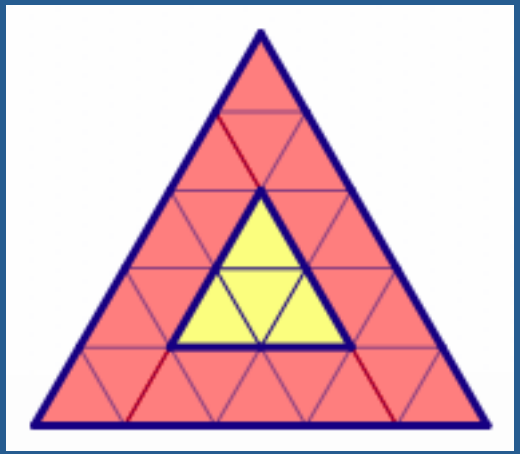
A Triangular or Cubical Border Problem

- Once the rectangular border problem has been discussed, one can pose the same problem for a new shape: a triangle ... or a cube.



Or …Triangular Tiles Generate Numerical Formulas

- In this figure, the yellow triangle is formed from tiles and has a side length $n = 2$. What is the number of tiles in the border for this case and for general n?



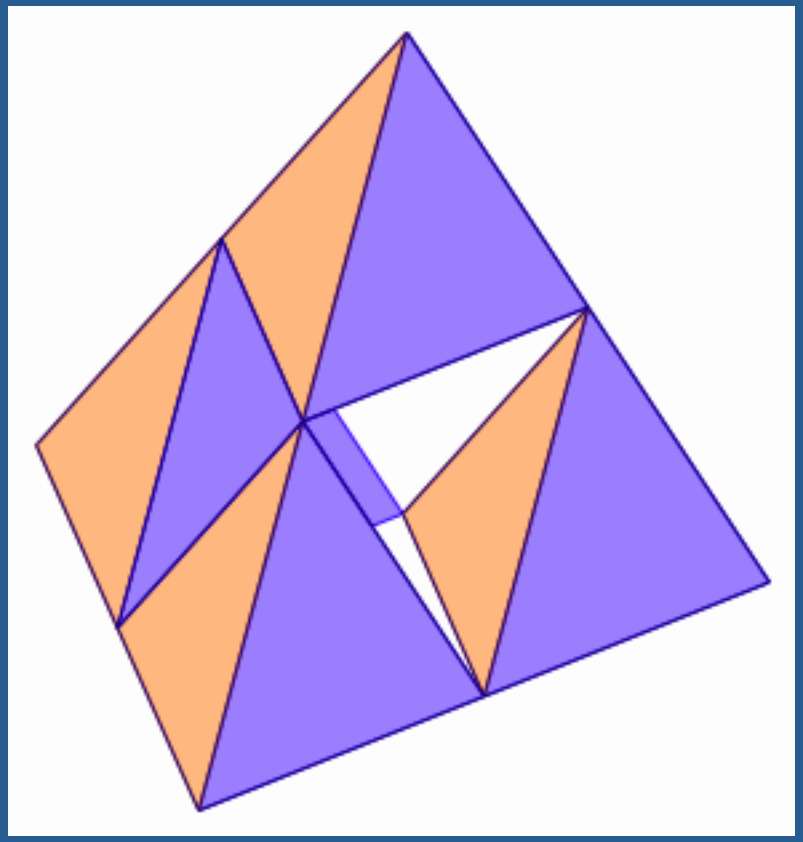
NWMI Likes 3D

3D Geometry

- The geometry of space is not much emphasized in school math and is not a strong point with most teachers.
- In our summer program we spend a lot of time with polyhedra and on topics such as volume.

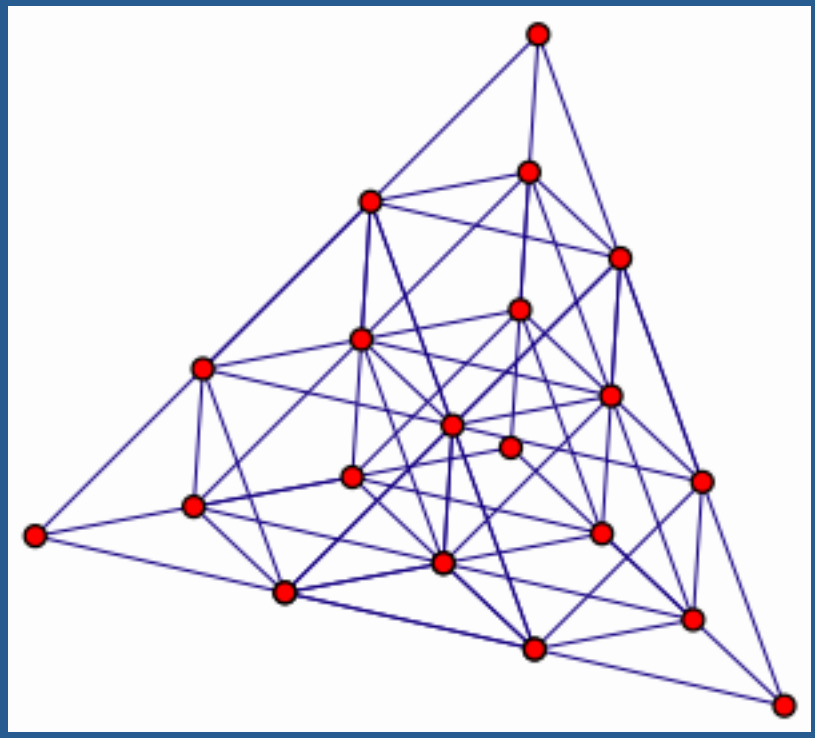
Straw Kites

- Teachers construct tetrahedral kites from straws and tissue.
- Then one can pose questions about shape and volume.
- What is the shape and volume of the “hole” in this kite?



More Tetrahedra in a Kite

- By stacking unit tetrahedra to create a tetrahedron with side n, one fills a volume with tetrahedra and octahedra.
- How many of each as a function of n? Lots of relations with Pascal's triangle and difference equations.

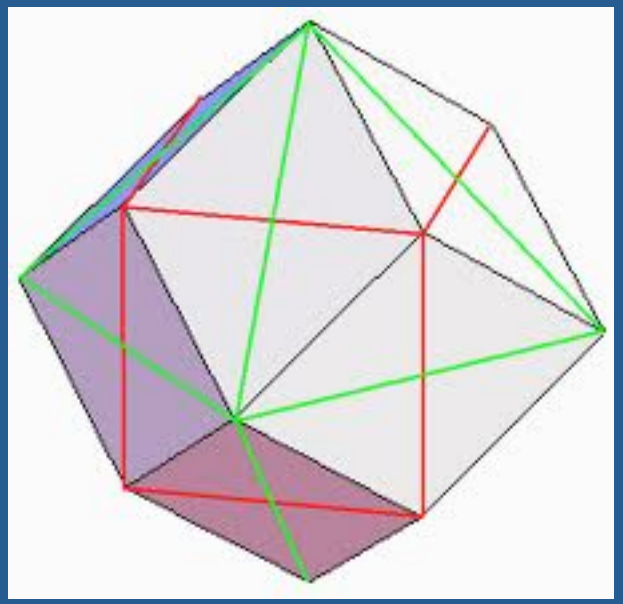


A Side Comment about Difference Equations

- Difference tables are now big in secondary school.
- Common understanding that constant differences come from linear functions, and constant second differences from quadratics.
- But teachers generally have not encountered difference equations in college math courses.
- Also, they do not recognize Pascal's Triangle as the Mother of all Difference Tables for polynomials.

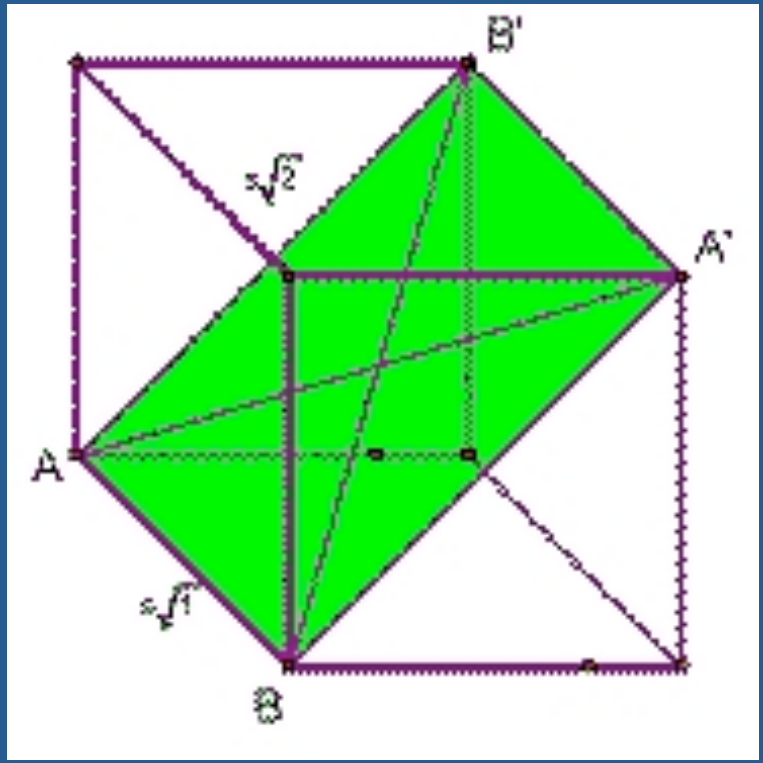
More Space-Filling

- Other 3D shapes and nets are the subject of exploration, including the favorite space-filling Rhombic Dodecahedron



Back to A4 paper

- The Rhombic Dodecahedron can be built by adding a pyramid to each face of a cube. And the triangular faces of the pyramid come from the “A4” paper shape, which is the diagonal rectangle of a cube.



The Mathematics of Klee and Grunbaum: July 2010

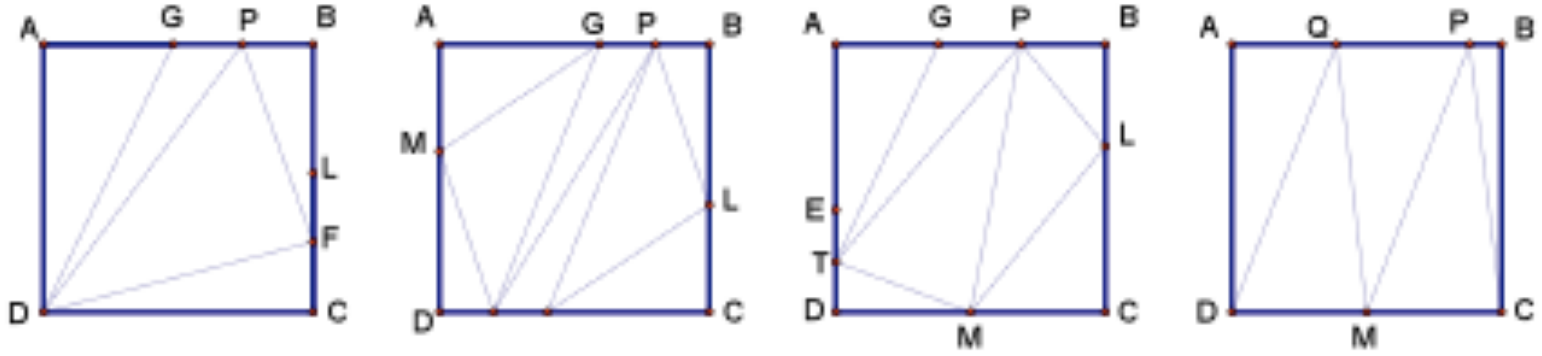
- A conference was held in July 2010 on the UW campus to mark the contributions to mathematics of Vic Klee and Branko Grunbaum (mostly geometry and discrete math).
- Brian Hopkins created a three-day minicourse for teachers around this conference, picking accessible talks and conferring with speakers.
- The teachers attended and then spent time debriefing.

Outcome from the conference: a Course doing Math with Teachers

- Philip Mallinson from Exeter took what he learned from mathematicians at the conference about research on folding and designed a two-day NWMI course for summer 2011.

To fold a polyhedron from a square

Depending on the crease pattern you use you can fold tetrahedra, pentahedra, hexahedra and octahedra from a square. Each of the crease patterns will produce a continuum of non-congruent polyhedra. Below are four of the possible crease patterns.



PCMI Math Course Problems

- A big source of really interesting math problems in our program are the problems from the morning math course at the Park City Mathematics Institute's Secondary School Teacher Program.
- However, since these problems originate with the PROMYS program described by Glenn Stevens on this panel, we will not discuss them here.

Final Word

- Teachers really like math that seems relevant to their jobs teaching students.
- But they also like doing math because it is enjoyable and they rarely have a chance to do math for its own sake.
- So there is worthwhile math to do with teachers that can address either of these likes – and sometimes it turns out that the aims coincide.

NWMI Website

- The NWMI website is given below, or go to link on James King's pages at the University of Washington Math Department.

www.math.washington.edu/nwmi