Hands-On Explorations of Plane Transformations

James King
University of Washington
Department of Mathematics
king@uw.edu
http://www.math.washington.edu/~king

The "Plan"

- In this session, we will explore exploring.
- We have a big math toolkit of transformations to think about.
- We have some physical objects that can serve as a hands-on toolkit.
- We have geometry relationships to think about.
- So we will try out at many combinations as we can to get an idea of how they work out as a real-world experience.
- I expect that we will get some new ideas from each other as we try out different tools for various purposes.

Our Transformational Case of Characters

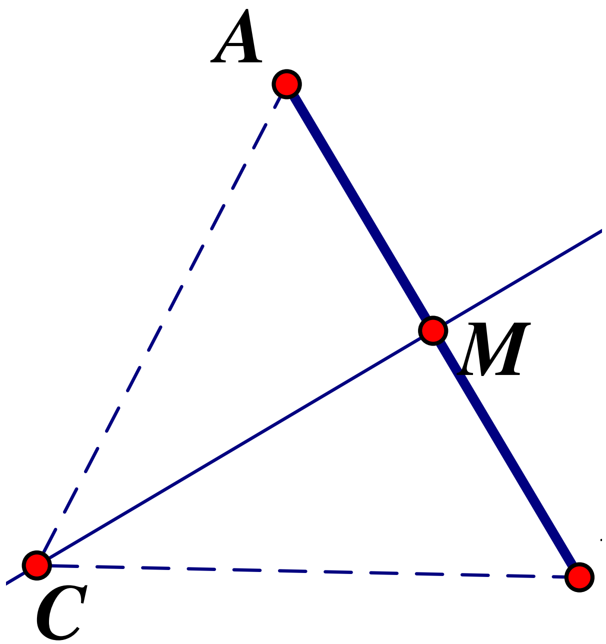
- Line Reflection
- Point Reflection (a rotation)
- Translation
- Rotation
- Dilation
- Compositions of any of the above

Our Physical Toolkit

- Patty paper
- Semi-reflective plastic mirrors
- Graph paper
- Ruled paper
- Card Stock
- Dot paper
- Scissors, rulers, protractors

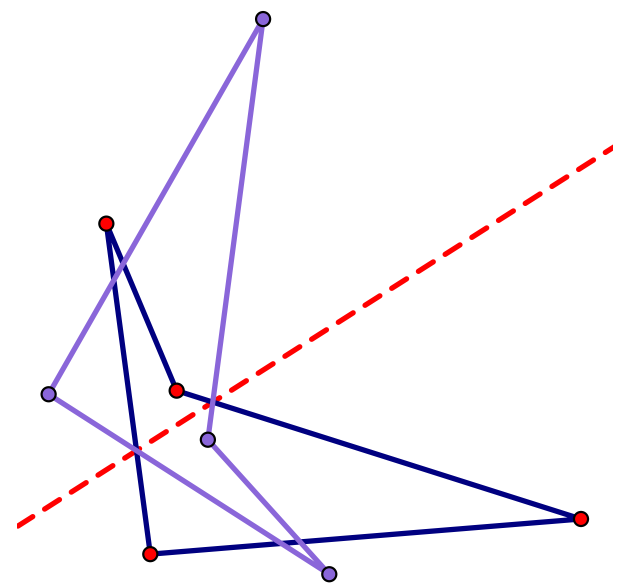
Reflecting a point

- As a first task, we will try out tools for line reflection of a point A to a point B. Then reflecting a shape.
- Suggest that you try the **semi-reflective mirrors** and the **patty paper** for folding and tracing. Also, graph paper is an option. Also, regular paper and cut-outs
- Note that pencils and overhead pens work on patty paper but not ballpoints. Also not that overhead dots are easier to see with the mirrors.
- Can we (or your students) conclude from your tool that the mirror line is the perpendicular bisector of AB?



Which tools best let you draw this reflection?

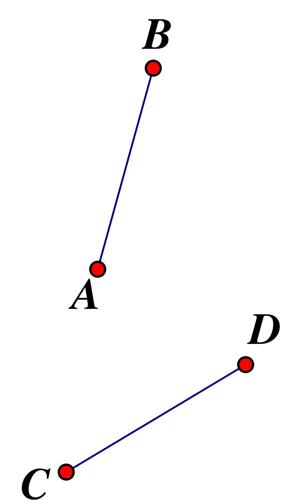
- When reflecting shapes, consider how to reflect some polygon when it is not all on one side of the mirror line.
- Otherwise students may be the wrong idea that reflection only works if the whole figure is on one side of the mirror line.



Congruent Line Segments

- Here's food for thought: In CCSS, given the definition of congruence, we cannot assume without proof that two line segments of the same length are congruent. We must show that for any two such segments we can move one to the other by a sequence of rigid motions!

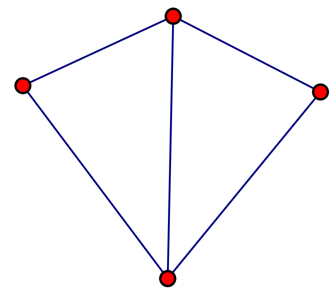
- So, as an exercise, draw two line segments of the same length and perform a sequence of reflections that will take one to the other. Do you think this can be justified in general?

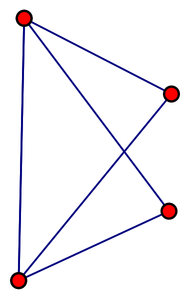


Proving SAS etc.

- To prove SAS, you can build on what we have done to move one side of the first triangle onto the second. Then either you are done already, or you get a figure like one of these.

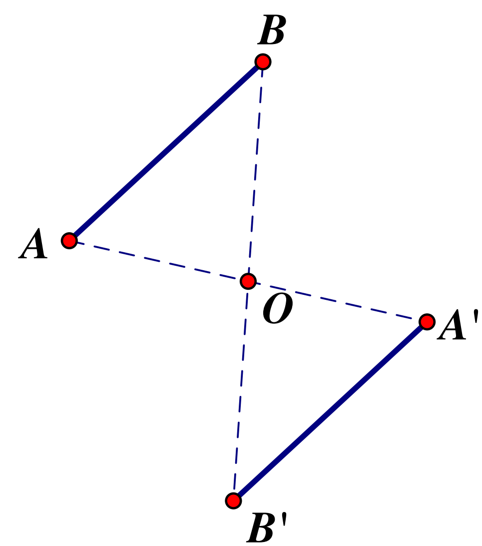
- Can you justify the final step?





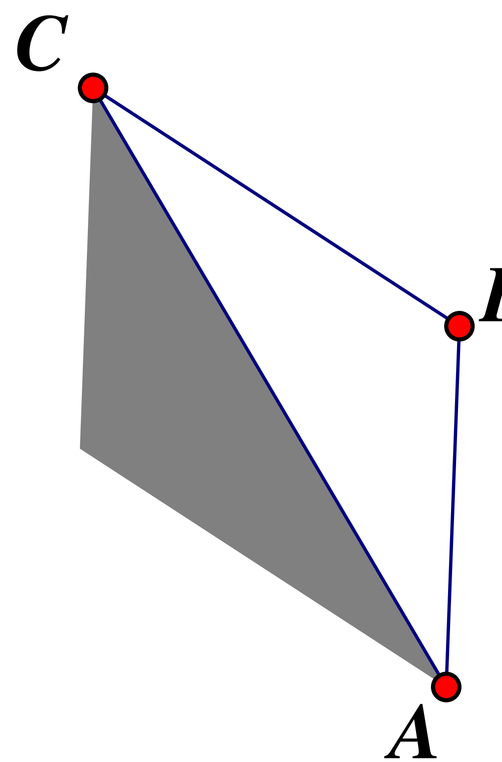
Point Reflection (half-turn)

- A *point reflection*, or a *half-turn* is an important special case of rotation: rotation by 180 degrees.
- It is not difficult to apply a half-turn to a point A with a straightedge and a piece of card (or a ruler) –or with patty paper or other tools.
- Reflect two points A and B with center O. Can you justify the claim that line A'B' is parallel to line AB? Does this mean also that line AB' is parallel to line A'B?
- Houston, we have a parallelogram!



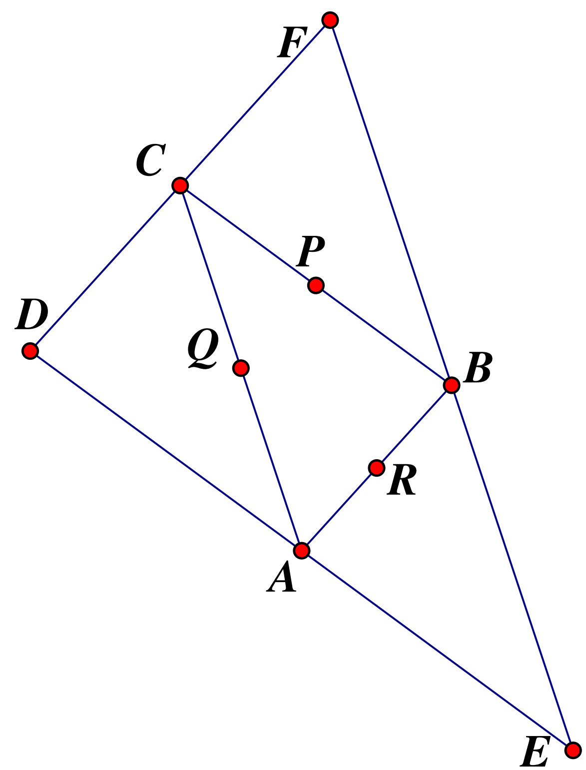
Parallelogram from 3 points

- Given a triangle ABC, find the center of a half-turn that will construct a point D so that ABCD is a parallelogram.
- Or else that ABDC is a parallelogram, or ...
- How does this figure compare with what you would get from a line reflection?



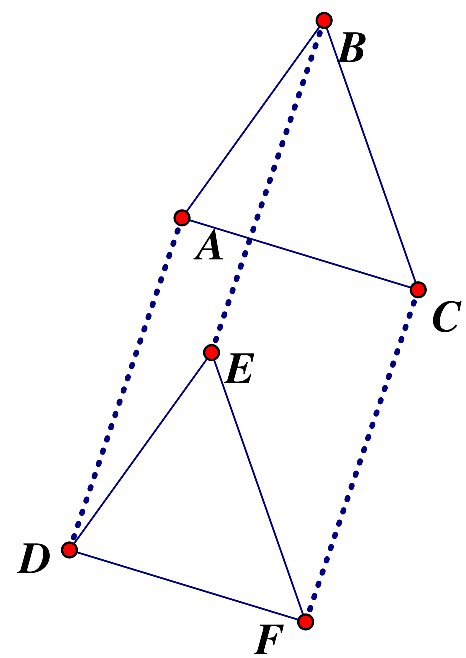
Hmm!

- If you create 3 parallelograms from ABC all in the same figure, this is what you get! A rich figure.
- Can you find any examples of a composition of two half-turns here?



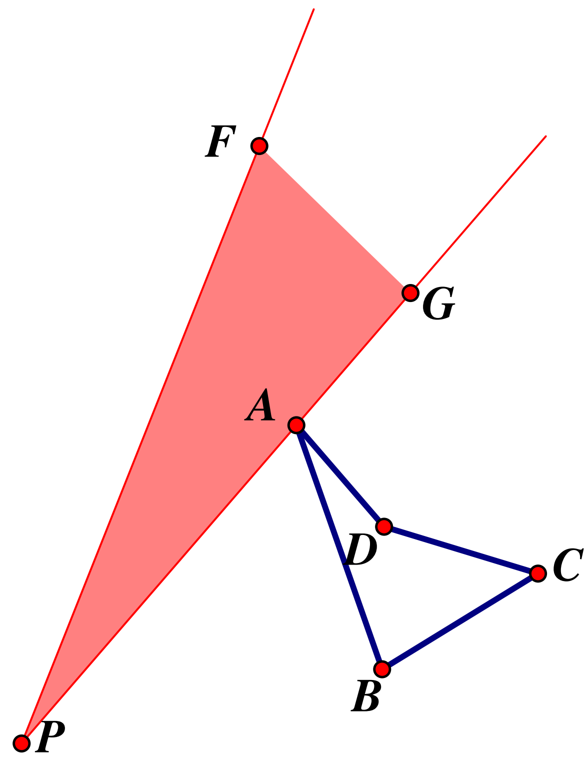
Translation: Harder than it looks

- Translations are commonly viewed as the easy transformations to model hands-on: just slide!
- But how can you be sure that your freehand slide does not have some rotation in it? We need a careful slide.
- Suggestion. We know how to use half-turns to construct parallelograms. Can you use this a a practical way to translate one polygon to another?
- Other ideas? Graph paper?



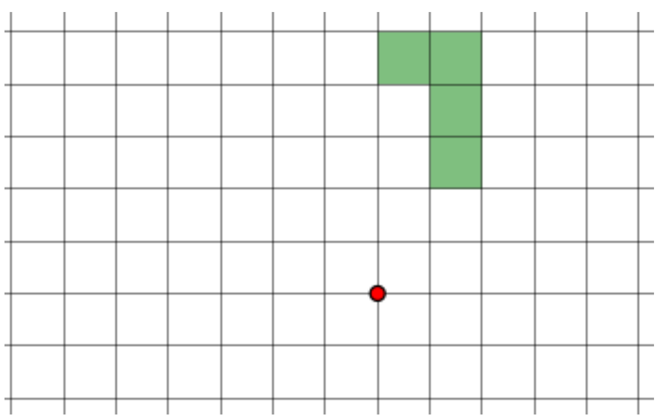
Rotation

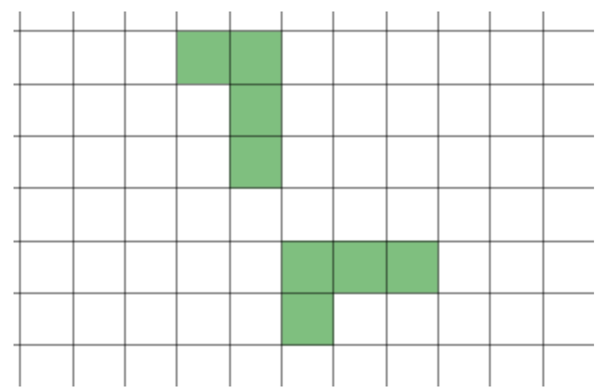
- Rotate ABCD with center P an angle GPF. What to do?
- One idea: Use a wedge of cardstock like the red shape and use it to rotate the rays PA, PB, PC, PD and then mark off the lengths PA, PB, PC, PD to get the rotated shape.
- Second idea: reflect ABCD in the line PA and then reflect again in the angle bisector of GPF. Does this work?



The Coordinate Plane

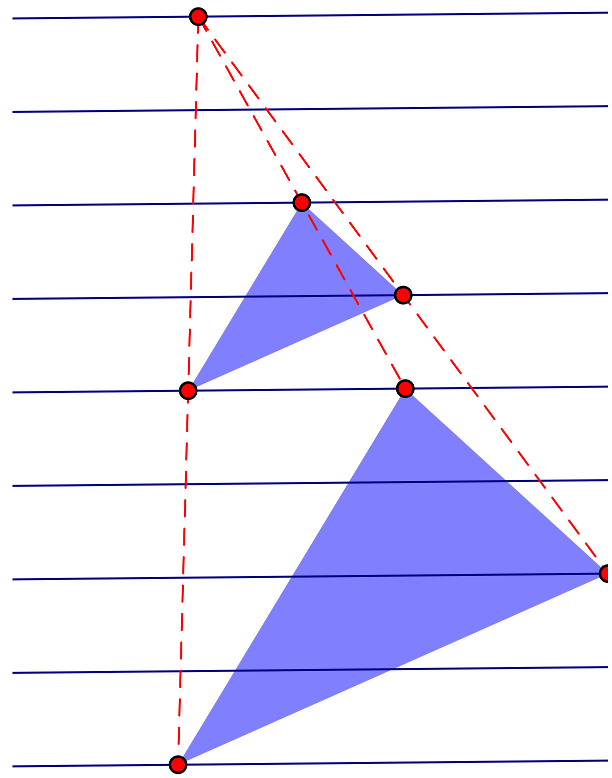
- Graph paper!
- Examples: In the left figure, rotate the shape by 90 degrees with the center point shown.
- In the right figure, find the center and angle of rotation that takes one shape to the other.





Dilation by notebook paper

- Euclid did not have notebook paper. You have lots.
- So you can dilate a shape like this. Just count the spaces between the line and take the ratio to see the ratio of dilation.



Dilation by dot paper

- Lots of similar triangles in this paper. Look for a dilation? Find the center.
- Or start with a corner triangle and dilate it to a bigger one.