Session 6: The Power of Polynomials

73. Expand (i.e., write without parentheses) each of the following.
    (a) $(a+b)^2$
    (b) $(a+b)^3$
    (c) $(a+b)^4$
    (d) $(a+b)^5$

74. What patterns do you observe in Problem 73? Use some triangle thing we passed out on Friday to expand $(a+b)^6$ without actually multiplying everything out.

75. Use a TI-*n*spire to expand $(0.25r + 0.75w)^5$. [Some technical assistance: To have the calculator expand an expression, you can go through this sequence: "menu", "3:Algebra", "3:Expand" or type out the word with the green letter keys. Then type the expression inside a second set of parentheses (use green letter keys for r and w).]

76. You take an exam in Japanese with five multiple-choice questions. Each question has four possible answers, and one is right. The only problem is — you don't know any Japanese, so you're stuck making complete and utter random guesses.
    (a) Find the probability of getting all five questions right.
    (b) Find the probability of getting all five questions wrong.
    (c) Find the probability of getting exactly two right.
    (d) Is it more likely for you to get two questions right, or three questions right? Explain how you know.

77. On a ten-question true-or-false test, how many different *ways* are there to answer the test and get exactly seven questions right? Is there a notation for this?

78. Use some triangle to find the number of different ways there are to answer a ten-question true-or-false test and get at least seven questions right.

79. What is the sum of the numbers in the 10th row of Pascal's Triangle? How is this related to a ten-question true-or-false test?

80. You can calculate the difference of two cubes if you want to. Come on, it's fun:
$$3^3 - 2^3 = 27 - 8 = 19$$
$$4^3 - 3^3 = 64 - 27 = 37$$
$$5^3 - 4^3 = 125 - 64 = 61$$
    Starting with $1^3 - 0^3 = 1$, find the sum of the first 100 differences of cubes. (The last one is $100^3 - 99^3$.)

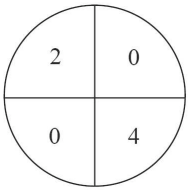
81. The function $f(x) = x^3$ is given below.

    (a) For each output, find the *common difference* between consecutive inputs. The notation for this is the Δ operator. Fill in just the Δ column first.

Input	Output	Δ	Δ^2	Δ^3
0	0	1	6	
1	1	7		
2	8			
3	27			
4	64			
5	125	91		
6	216			
7	343			

    (b) Continue taking common differences for $f(x) = x^3$ until a constant value is found. (Δ^2 means the difference of the Δ column, etc.)

82. A number spinner is marked with four numbers like this:



    All the regions are equally likely to be landed on. If you spin the spinner three times, what is the most likely *sum* of the three numbers? What sum is the next most likely?

83. Use a TI-*n*spire to expand $(2 + x^2 + x^4)^3$. So what?

84. What is the most likely sum if you spin this spinner seven times?

Tough Stuff

85. You're standing on the edge of a pool, facing away from it, and holding a bag with 4 white balls and 4 red balls. You pick a ball without replacement. If it's a white ball, take a step forward. If it's a red ball, take a step back (into the pool, sadly). If you survive, draw another ball and keep going until either

    (a) ... you draw all the balls, or

    (b) ... you're in the pool.

    Find the number of different ways you could draw all the balls without entering the pool. Generalize to n balls of each color.