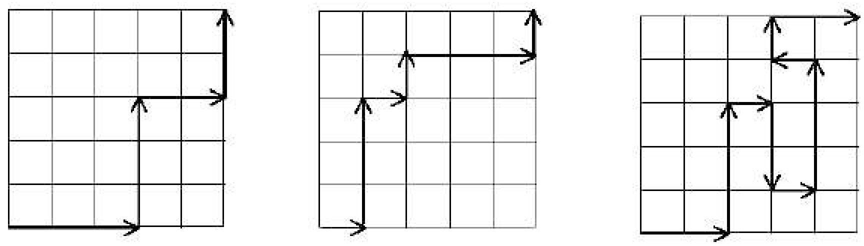
Session 5: Paths, Medals, and Art

Ms. Zappella likes to take a different route to work every day. She will quit her job the day she has to repeat her route. Her home and work are pictured in the grid of streets below.



Trips: valid, valid, invalid.

56. If Ms. Zappella never backtracks (she only travels north or east), how many days will she work at this job?

57. How many more days can Ms. Zappella work if she moves two blocks further away? Does it matter in which direction she moves?

58. Hey, what's the sum of the numbers in the fifth row of Pascal's Triangle? (That's the 1, 5, . . . row — OK, so it's not a hard question.)

59. Hey, what's the sum of the *squares* of the numbers in the fifth row of Pascal's Triangle? (See, a little harder.) Is your answer in Pascal's triangle?

60. Asa, Bahir, and Constance are the only contestants for an Olympic event. How many ways can they be assigned the gold, silver, and bronze medals?

61. Now suppose that the Olympics allows ties (so they have multiple medals available). There's still some structure: there could be a gold medal and two silvers, but not three bronzes. Now how many ways can Asa, Bahir, and Constance be assigned medals?

62. The judge from Berzerkistan does not believe that all three contestants should necessarily be given medals. Using his rules, how many possible outcomes for the event where Asa, Bahir, and Constance are the contestants.

63. Without a calculator, expand $(h + t)^5$.

64. You flip a coin five times. How many ways are there to flip two heads and three tails?

65. How many trains of length 24 are made with exactly 5 rods of length 2, 3 rods of length 3, and 1 rod of length 5?

66. How many "words" can be made from ARTMABBOTT?

67. If you randomly rearrange the letters in ARTMABBOTT, find the probability that...

    (a) ... the first letter is T.
    (b) ... the first *two* letters are T.
    (c) ... the first four letters are T.
    (d) ... the letters spell out BRATATTOMB?

68. Find all train lengths you *can't* make if you only have rods of lengths 3 and 8.

69. Find all train lengths you *can't* make if you only have rods of lengths 3 and 9.

Tough Stuff

70. Work out the Simplex lock problem for one that has just three buttons. Give the buttons names, like Asa, Bahir, and Constance.

71. In row 7 of Pascal's Triangle, the numbers 7, 21, and 35 appear consecutively. Interestingly, these three numbers are in arithmetic sequence. Does this ever happen again? If so, find the next three times it happens. If not, prove it can't happen again.

72. Prove that if p is a prime number, then $\binom{pa}{pb}$ has the same remainder as $\binom{a}{b}$ when you divide by p. For example, consider $\binom{5}{2} = 10$. For the prime 7, you have you have $\binom{5}{2} = 10 = 1 \cdot 7 + 3$. Multiplying the two numbers by 7 gives $\binom{35}{14} = 2,319,959,400 = 331,422,771 \cdot 7 + 3$.