EXERCISES

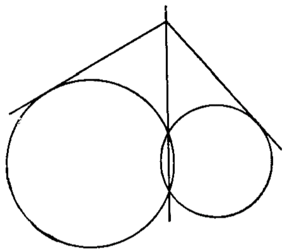

Fig. 26

1. Prove that if two circles intersect, every point on the common chord, and on the common chord extended, has the same power with respect to both circles. See Fig. 26.

2. Prove that if two circles are externally tangent, every point on the common internal tangent has the same power with respect to both circles.

3. State and prove the corresponding theorem for two circles which are internally tangent.

RADICAL AXIS

All points whose powers with respect to two non-intersecting circles are equal lie on a straight line perpendicular to the line of centers of the given circles. This perpendicular is called the RADICAL AXIS of the circles.

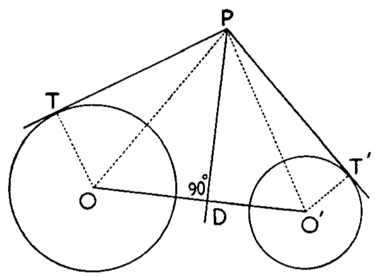

Fig. 27

We shall prove this by showing that whatever point of equal power is chosen, the perpendicular drawn through it to the line of centers always meets the latter at the same point. See Fig. 27.

For $(OD)^2=(PO)^2-(PD)^2=(OT)^2+(TP)^2-(PD)^2$,

and $(DO')^2=(PO')^2-(PD)^2=(O'T')^2+(TP)^2-(PD)^2$;

whence $(OD)^2-(DO')^2=(OT)^2-(O'T')^2$. Therefore the product $(OD-DO')\ (OD+DO')$ is constant; and since $(OD+DO')$ is constant, $(OD-DO')$ must be constant also.

$$OD+DO'=k_1$$
$$OD-DO'=k_2$$
$$2(OD)=k_1+k_2$$
$$\text{and } OD=\frac{k_1+k_2}{2}, \text{ another constant.}$$

That is, D, the foot of the perpendicular from P to OO', is the same for all possible positions of the point P. Therefore all the points P must lie on the perpendicular to OO' at D, the radical axis of the two circles.

EXERCISES

1. Show that the foregoing proof can be applied to two intersecting circles, and that the line PD must be the common chord (extended) of the two circles.

2. If now we agree to extend the meaning of "radical axis" to apply also to the common chord (extended) of two intersecting circles, what shall we mean by the radical axis of two circles that are externally tangent? Of two circles that are internally tangent?

3. What is the locus of all points having the same power with respect to two spheres?

INVERSION

Two points are said to be inverses of each other with respect to a circle when the product of their distances from the center is equal to the radius squared. In Fig. 28, $OP \times OP'=r^2$; P' is the inverse of P, and P is the inverse of P'. The circle is called the CIRCLE OF INVERSION; its center, the CENTER OF INVERSION.

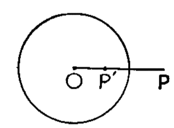

Fig. 28

The inverse of a point inside the circle is outside the circle; and conversely. The inverse of a point on the