## Math 445 Assignment \#1 (Due Wednesday 1/12/2005)

## Part A

The questions $1-5$ in this part refer to this figure from the lab. Since there are many parts to this, please organize your response with plenty of white space, brief but convincing and clear reasons, and clearly labeled answers. Since some of the answers have multiple parts, and the reasoning may be repeated, a good answer will organize the ideas so as to state an idea once and then apply it. It is not good style to repeat the same lines over and over.


1. The three shaded triangles are similar to triangle ABC , what is the scaling factor (ratio of similitude) in each case? How do you know this?
2. If the area of triangle ABC is T , what are the areas of each of the 3 triangles and the 3 quadrilaterals into which ABC is dissected in the figure? The answers should be in terms of a, b, c, x, y, z and T. Explain.
3. Use algebra to show that the 6 areas add up to the area of ABC (use your formulas from 2).
4. P divides each of the 3 parallel segments. Write down these 3 ratios (using $x, y$, z, a, b, c): PC2/PB1, PA2/PC1, PA1/PB2. Explain.
5. Let the lines AP, BP, CP intersect the opposite sides of the triangle ABC in points $\mathrm{A}^{\prime}, \mathrm{B}^{\prime}, \mathrm{C}^{\prime}$. What are the ratios $\mathrm{A}^{\prime} \mathrm{B} / \mathrm{A}^{\prime} \mathrm{C}, \mathrm{B}^{\prime} \mathrm{C} / \mathrm{B}^{\prime} \mathrm{A}, \mathrm{C}^{\prime} \mathrm{A} / \mathrm{C}^{\prime \prime} \mathrm{B}$ ? Explain.

## Part B: Barycentric Coordinates

This figure is built up of congruent copies of the small triangle in the corner (as we did in class with the cut-out triangles). Each side of ABC is divided into 5 equal parts. The points inside are formed from parallels to the sides through the division points.

In this figure, the barycentric coordinates $(x, y, z)$ are indicated in order for each of the vertices of ABC . Use the same order when answer the questions.


## Questions: Show your work or explain briefly your reasons.

6. What are the barycentric coordinates of these points: D, E, F, G, H, I, J?
7. What are the barycentric coordinates of the midpoint of DF?
8. What are the barycentric coordinates of the point P that is the intersection of BE and AD ?
9. What are the barycentric coordinates of the point Q that is the intersection of DF and AB ?
10. What is the equation in the form $\mathrm{ax}+\mathrm{by}+\mathrm{cz}=\mathrm{d}$ for the line CH ? For the line DF?

## Part C

## 11. Barycentric coordinates of intersections

Given a triangle ABC and points $\mathrm{A}^{\prime}$ on BC and $\mathrm{B}^{\prime}$ on CA , let P be the intersection of $\mathrm{AA}^{\prime}$ and $\mathrm{BB}^{\prime}$. Suppose the ratio $\mathrm{BA}^{\prime} / \mathrm{A}^{\prime} \mathrm{C}=2 / 3$ and ratio $\mathrm{CB}^{\prime} / \mathrm{B}^{\prime} \mathrm{A}=1 / 5$.

Show your work with reasoning.

- What are the barycentric coordinates of P ?
- If $\mathrm{C}^{\prime}$ is the intersection of CP with AB , what is the ratio $\mathrm{AC}^{\prime} / \mathrm{C}^{\prime} \mathrm{B}$ ?
- If $Q$ is the intersection of line $A^{\prime} B^{\prime}$ with line $A B$, what is the ratio $A Q / Q B$ ?

12. Plotting points from barycentric coordinates

Construct an equilateral triangle ABC and plot the points $\mathrm{P}, \mathrm{Q}, \mathrm{R}$ (you can use a marked ruler or you can construct).

- the barycentric coordinates of P are $(1 / 7,4 / 7,2 / 7)$
- the barycentric coordinates of Q are $(1 / 6,1 / 2,1 / 3)$
- the barycentric coordinates of R are $(-1 / 3,6 / 3,-2 / 3)$
- write the equation of the line QR

