Math 445 Construction Portfolio Part 1

Instructions:  Do each construction with compass and unmarked straightedge.  Label key points so that others can read and understand the steps in your work.

This part of the Construction Portfolio is devoted to orthogonal circles, power of a point, and inversion.  Most of the constructions are supposed to be exercises based on your prior lab work rather than the solving of new problems.

The two constructions that may deserve hints are #2 and #12.

Hint for #2.  Visualize all the circles through A and B to get an idea of which one or ones are tangent.  If the circle is constructed, how are tangent lengths related to lengths involving A and B?  You may wish to experiment with Sketchpad.

Hint for #12.  This hint is a labor-saving suggestion and also a check on your work.  Namely, once you have the inversion image $f'$ of a line or circle $f$, the inversion image of any point $F$ on $f$ can be found by drawing the line $OF$ and considering the intersections of $OF$ with $f'$.
Construction 1

Construct a circle centered at $A$ that is orthogonal to the given circle $c$ (whose center is $P$).
Construction 2

Construct all circles through A and B that is/are tangent to the given line m.
Construction 3

Construct a circle passing through A and B that is orthogonal to the given circle c (whose center is P).
Constructions 4

Construct a circle passing through A and B that is orthogonal to the given circle c (whose center is P).
Construction 5

Construct a circle $e$ orthogonal to both circles $c$ and $d$ so that the center of $e$ is a point $E$ on line $m$. Hint: $E$ must be on the radical axis of the two circles.
Construction 6

Construct a circle $e$ orthogonal to both circles $c$ and $d$ so that the center of $e$ is a point $E$ on line $m$. Hint: $E$ must be on the radical axis of the two circles.
Construction 7

Construct a circle f orthogonal to all 3 circles c, d, e.
Construction 8

Construct a circle $f$ orthogonal to all 3 circles $c$, $d$, $e$ ($c$ and $d$ are tangent at $T$ as they appear to be).
Construction 9

Construct the images by inversion in m of the segments that are the sides of rectangle ABCD. O is the center of m.
Construction 10

Construct the images by inversion in p of the segments that are the sides of the equilateral triangles ABC and A'B'C'. P is the center of p. The sides of triangle A'B'C' are tangent at points A, B, C as they appear to b.

Note: Point out the relationships between the images and the original triangles.
Construction 11

Construct the images by inversion in \( p \) of the 3 circles \( a, b, c \) that pass through both \( P \) and \( Q \).
(The circles \( a, b, c \) all pass through \( P \) and \( Q \); their centers are \( A, B, \) and \( C \).
The circle \( p \) has center \( P \) and passes through \( Q \).)
Construction 12

Construct the images by inversion in $m$ of the circle $c$ and the two lines tangent to $c$ at the endpoints of the diameter $AB$. Also construct the images of $A$ and $B$. ($O$ is the center of circle $m$.)