

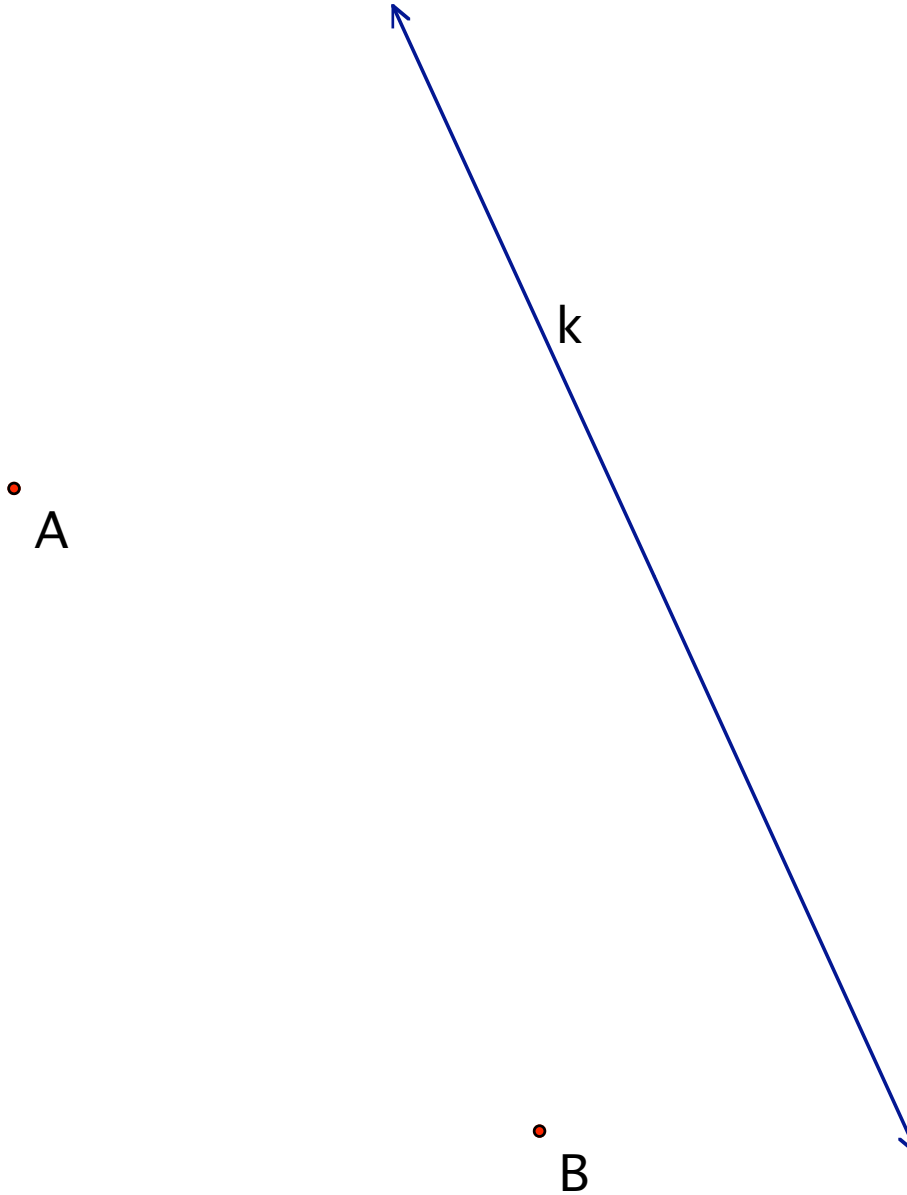
## Construction Portfolio #4

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- 27. *Light Path* 2
- 28. *Triangular Billiards* 3
- 29. *Triple Line Reflection (parallels)* 4
- 30. *Triple Line Reflection (concurrent)* 5
- 31. *Constructions using transformations: equilateral triangle* 6
- 32. *Constructions using transformations: segments with given midpoint* 7
- 33. *Composition of two point symmetries* 8
- 34. *Center of a Rotation or Invariant Line of a GR* 9
- 35. *Center of a Product of Rotations* 10
- 36. *Glide Reflection as product of 3 Line Reflections* 11
- 37. *Product of a Rotation and a Line Reflection* 12
- 38. *Image of an Isometry* 13

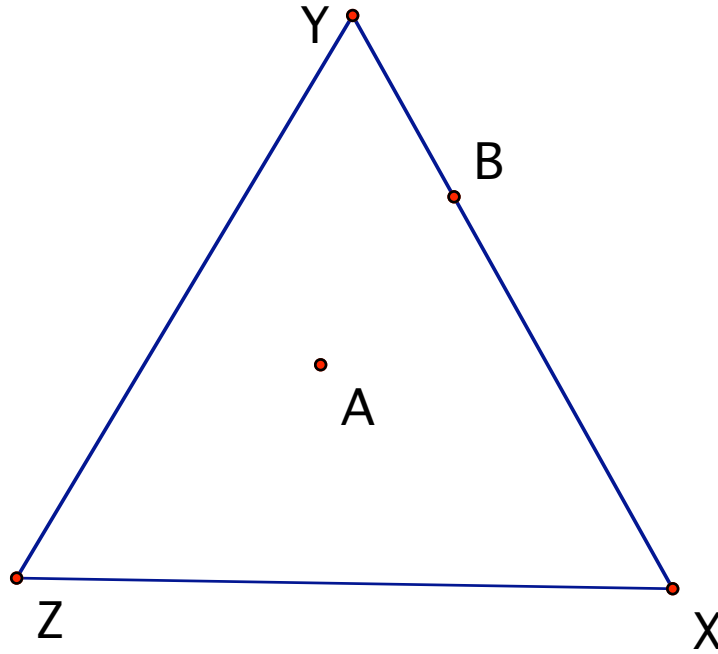
**27. Light Path**

Construct a point  $C$  on line  $k$  so that the path from  $A$  to  $C$  to  $B$  is the shortest possible. Specifically, the sum of lengths  $AC + CB$  should be smaller than for any other point  $C$  on  $k$ . This is the path a beam of light would take from  $A$  to  $B$  if reflected off a mirror  $k$ .



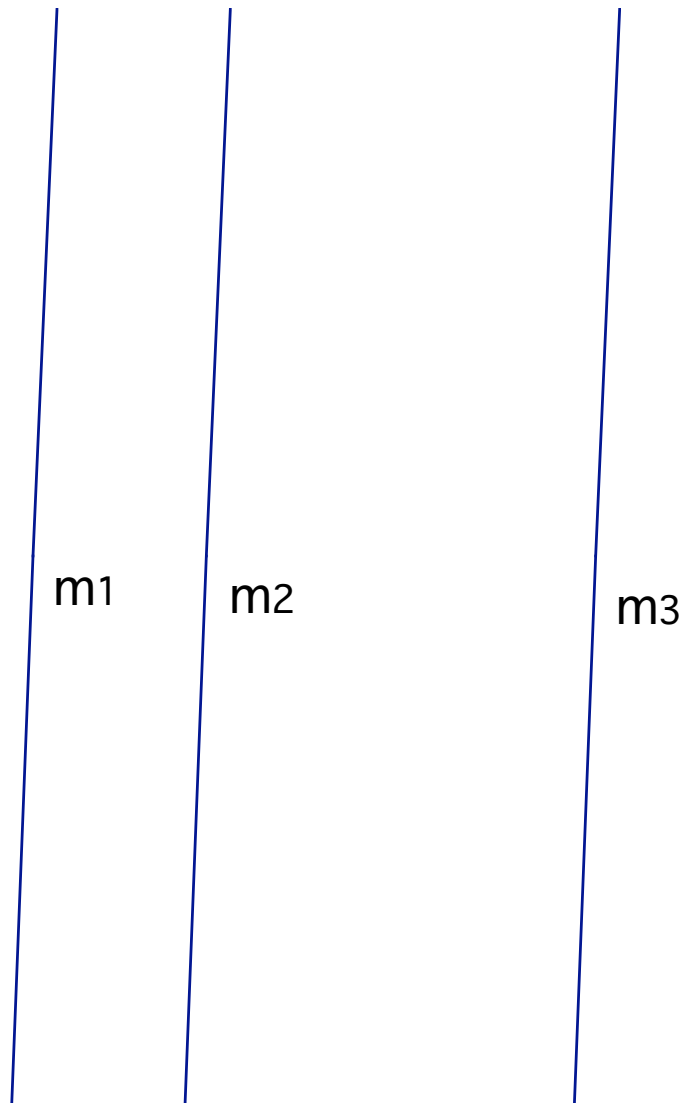
**28. Triangular Billiards**

Imagine that  $XYZ$  is a triangular billiard table. Construct the path of a billiard ball that is banked first off side  $XZ$  and then off side  $YZ$  before it reaches  $B$ .



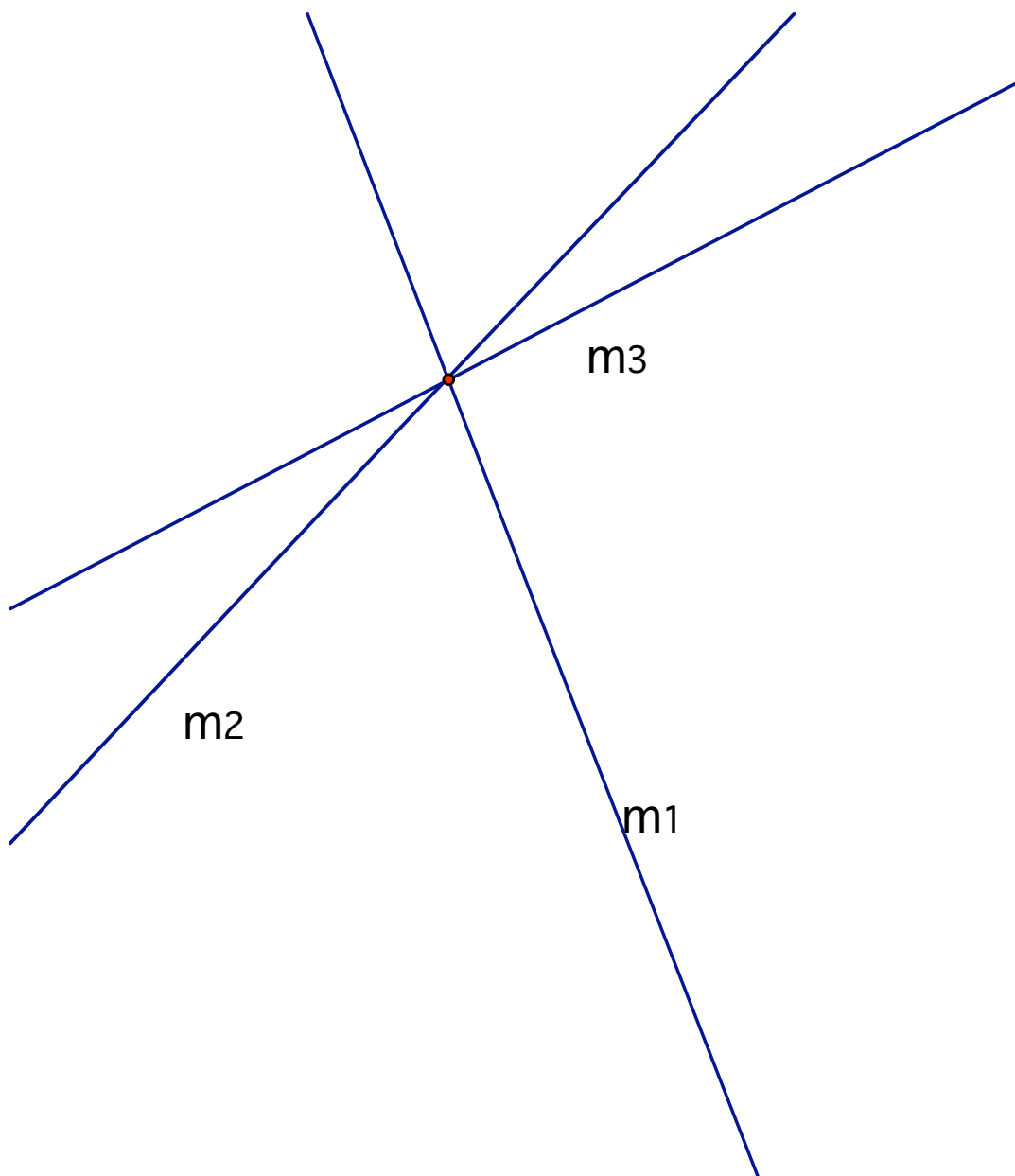
**29. Triple Line Reflection (parallels)**

Let reflection in parallel lines  $m_1$ ,  $m_2$ ,  $m_3$  be  $M_1$ ,  $M_2$ ,  $M_3$ . Construct a line  $n$  so that reflection in  $n$  is the same transformation as the composition  $M_3 M_2 M_1$ .



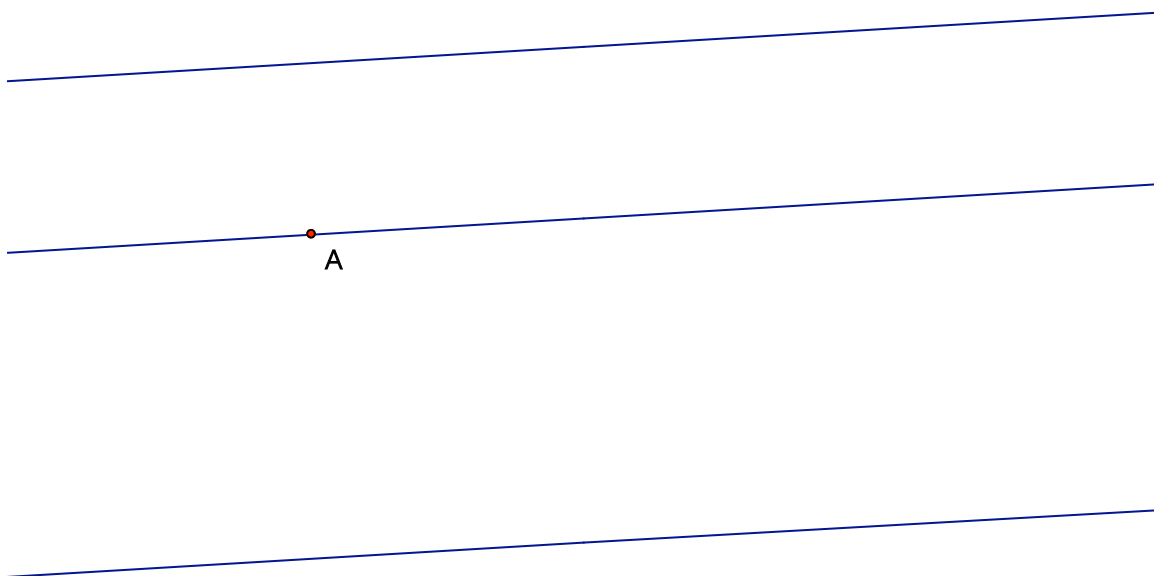
**30. Triple Line Reflection (concurrent)**

Let reflection in concurrent lines  $m_1$ ,  $m_2$ ,  $m_3$  be  $M_1$ ,  $M_2$ ,  $M_3$ . Construct a line  $n$  so that reflection in  $n$  is the same transformation as the composition  $M_3 M_2 M_1$ .



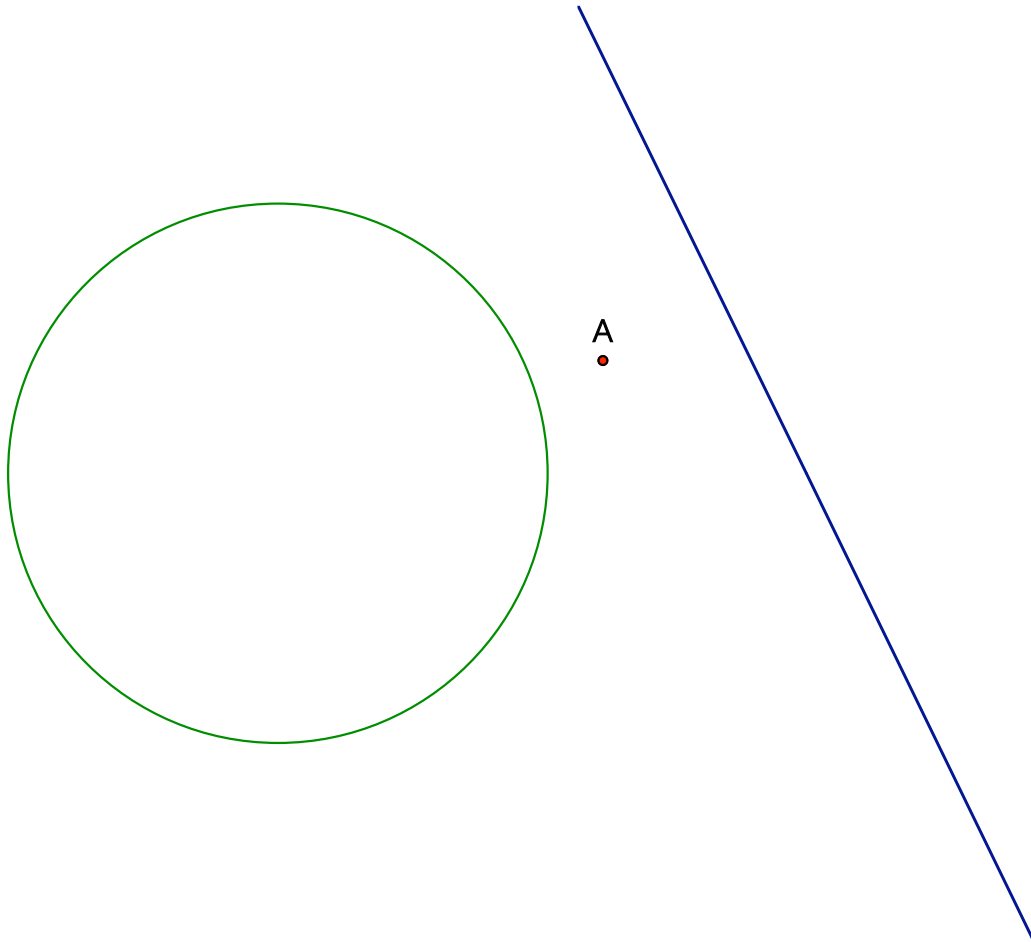
**31. Constructions using transformations: equilateral triangle**

Construct points B and C so that ABC is an equilateral triangle with one vertex on each of the 3 parallel lines.



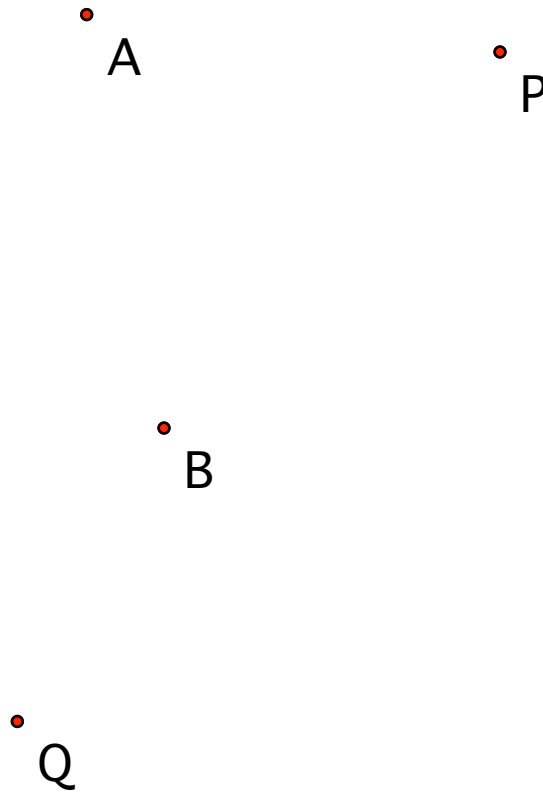
**32. Constructions using transformations: segments with given midpoint**

Construct ALL segments PQ so that P is on the line, Q is on the circle, and A is the midpoint of PQ.



**33. Composition of two point symmetries**

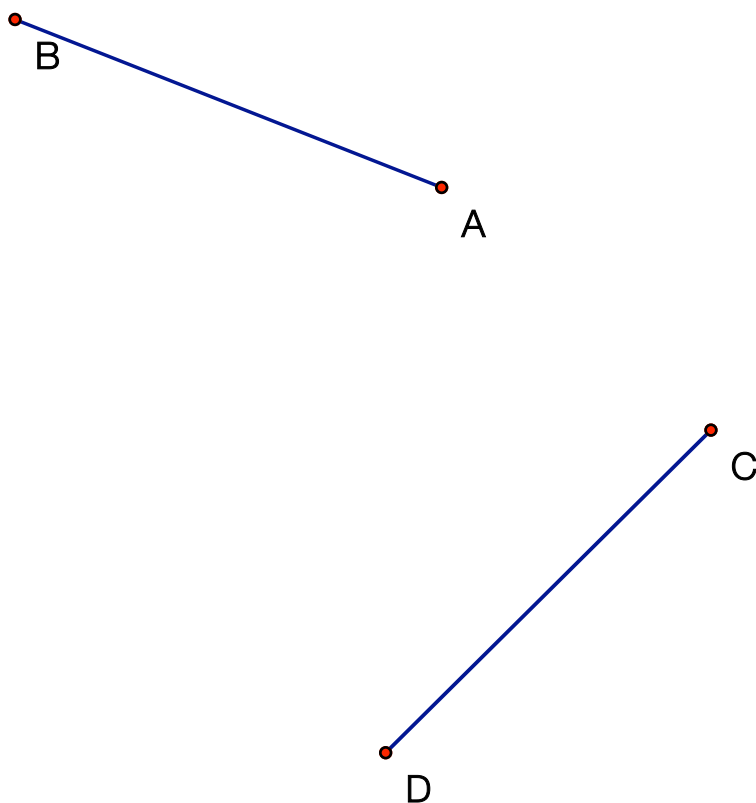
Given the points  $A$  and  $B$ , let  $H_A$  and  $H_B$  denote the point reflections with centers  $A$  and  $B$ . Let  $S$  be the composition  $H_B H_A$ . Construct points  $P' = S(P)$  and  $Q' = S(Q)$ . Note: You are not required to construct  $H_A$  and  $H_B$  of any points unless you find it necessary.





**34. Center of a Rotation or Invariant Line of a GR**

- (1) Construct the center  $O$  of the rotation that takes  $A$  to  $C$  and  $B$  to  $D$ .
  - (2) Construct the invariant line  $g$  of the glide reflection that takes  $A$  to  $C$  and  $B$  to  $D$ .
- Be sure to label  $O$  and  $g$  very clearly as well as showing construction steps.



**35. Center of a Product of Rotations**

Given the points A and B below; let S be rotation with center A by 60 degrees and let T be rotation with center B by 180 degrees.

- a) Construct the center C of the rotation  $U = ST$ . Write down the angle of rotation.
- b) Construct the center D of the rotation  $V = TS$ . Write down the angle of rotation.



A

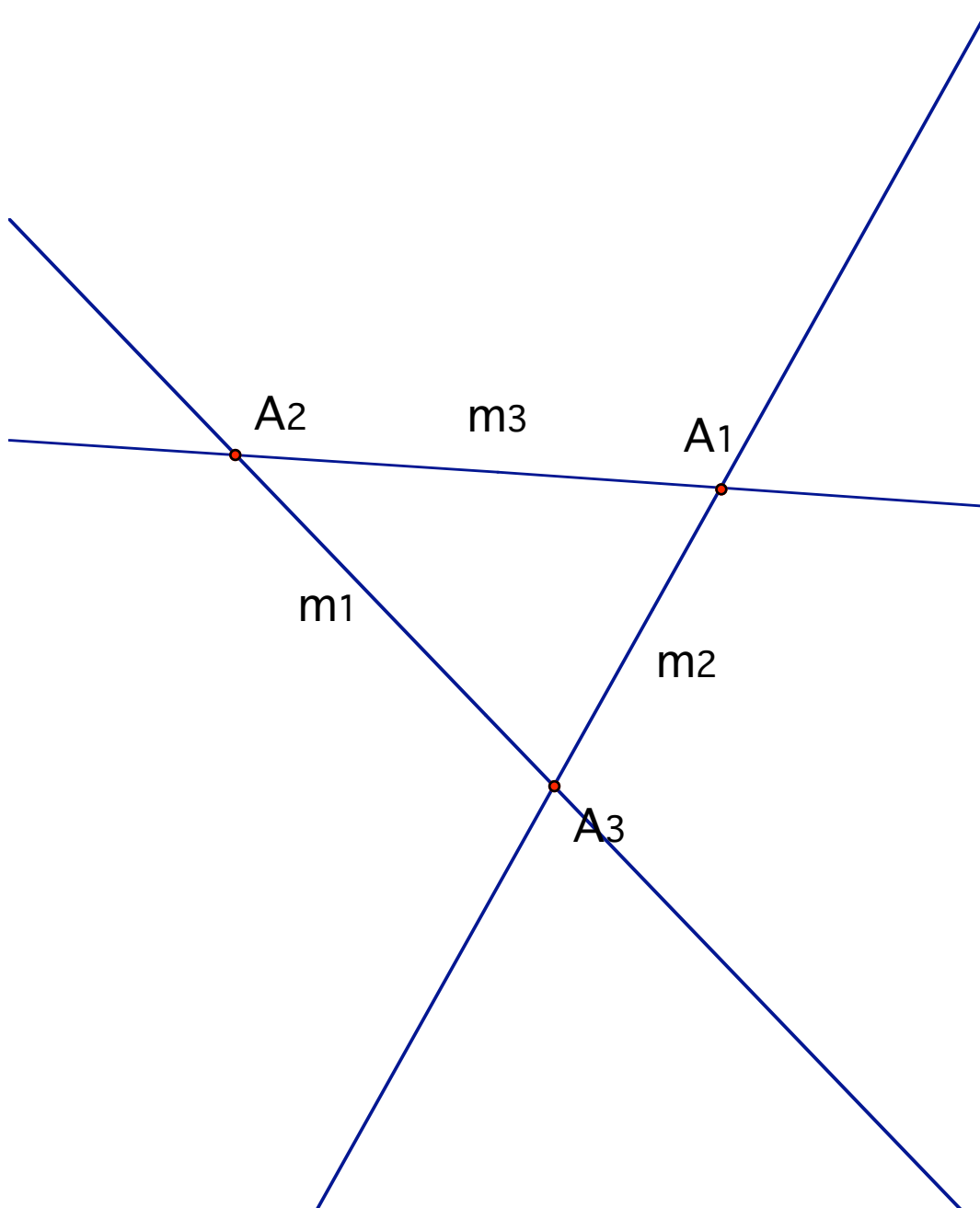


B

**36. Glide Reflection as product of 3 Line Reflections**

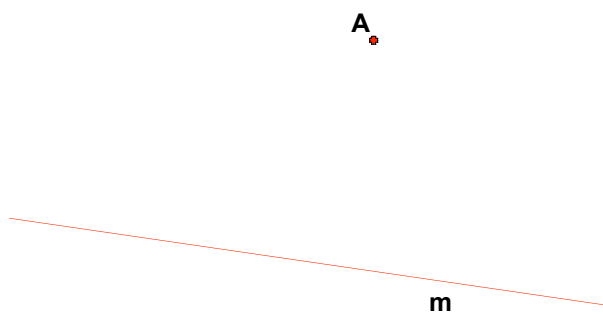
Let  $M_1, M_2, M_3$  be line reflections in the lines  $m_1, m_2, m_3$  below. Let  $N = M_1 M_2 M_3$  and let  $P = M_3 M_2 M_1$ .

- Construct the invariant (special) line of the glide reflection  $N$  and also a glide vector  $XY$ .
- Construct the invariant (special) line of the glide reflection  $P$  and also a glide vector  $UV$ . Question to Ponder: How are  $N$  and  $P$  related?



**37. Product of a Rotation and a Line Reflection**

Let  $E$  be rotation with center  $A$  and angle 90 degrees and let  $M$  be reflection in line  $m$ .  
Construct the geometric defining data of  $ME$ .



**38. Image of an Isometry**

In the figure are given congruent quadrilaterals  $ABCD$  and  $A'B'C'D'$ . There is a unique isometry  $T$  that takes  $ABCD$  to  $A'B'C'D'$ , i.e.,  $A'B'C'D'$  is  $T(ABCD)$ , the image of  $ABCD$ .

Construct the quadrilateral  $A''B''C''D''$  that is  $T(A'B'C'D')$ , the  $T$  image of  $A'B'C'D'$ .

