Construction Portfolio #4

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27. **Light Path**

Construct a point C on line k so that the path from A to C to B is the shortest possible. Specifically, the sum of lengths AC + CB should be smaller than for any other point C on k. This is the path a beam of light would take from A to B if reflected off a mirror k.
28. Triangular Billiards

Imagine that XYZ is a triangular billiard table. Construct the path of a billiard ball that is banked first off side XZ and then off side YZ before it reaches B.
29. **Triple Line Reflection (parallels)**

Let reflection in parallel lines $m_1$, $m_2$, $m_3$ be $M_1$, $M_2$, $M_3$. Construct a line $n$ so that reflection in $n$ is the same transformation as the composition $M_3 M_2 M_1$. 

\[ \begin{array}{ccc} & m_1 & m_2 \hline m_3 & & \end{array} \]
30. Triple Line Reflection (concurrent)

Let reflection in concurrent lines m1, m2, m3 be M1, M2, M3. Construct a line n so that reflection in n is the same transformation as the composition M3 M2 M1.
31. Constructions using transformations: equilateral triangle

Construct points B and C so that ABC is an equilateral triangle with one vertex on each of the 3 parallel lines.
32. Constructions using transformations: segments with given midpoint

Construct ALL segments PQ so that P is on the line, Q is on the circle, and A is the midpoint of PQ.
33. Composition of two point symmetries

Given the points A and B, let $H_A$ and $H_B$ denote the point reflections with centers A and B. Let S be the composition $H_B \ H_A$. Construct points $P' = S(P)$ and $Q' = S(Q)$. Note: You are not required to construct $H_A$ and $H_B$ of any points unless you find it necessary.
34. **Center of a Rotation or Invariant Line of a GR**

(1) Construct the center $O$ of the rotation that takes $A$ to $C$ and $B$ to $D$.
(2) Construct the invariant line $g$ of the glide reflection that takes $A$ to $C$ and $B$ to $D$

Be sure to label $O$ and $g$ very clearly as well as showing construction steps.
35. Center of a Product of Rotations

Given the points A and B below; let S be rotation with center A by 60 degrees and let T be rotation with center B by 180 degrees.

a) Construct the center C of the rotation $U = ST$. Write down the angle of rotation.

b) Construct the center D of the rotation $V = TS$. Write down the angle of rotation.
36. Glide Reflection as product of 3 Line Reflections

Let $M_1$, $M_2$, $M_3$ be line reflections in the lines $m_1$, $m_2$, $m_3$ below. Let $N = M_1 M_2 M_3$ and let $P = M_3 M_2 M_1$.

a) Construct the invariant (special) line of the glide reflection $N$ and also a glide vector $XY$.

b) Construct the invariant (special) line of the glide reflection $P$ and also a glide vector $UV$. Question to Ponder: How are $N$ and $P$ related?
37. *Product of a Rotation and a Line Reflection*

Let $E$ be rotation with center $A$ and angle 90 degrees and let $M$ be reflection in line $m$. Construct the geometric defining data of $ME$. 
38. Image of an Isometry

In the figure are given congruent quadrilaterals ABCD and A'B'C'D'. There is a unique isometry T that takes ABCD to A'B'C'D', i.e., A'B'C'D' is T(ABCD), the image of ABCD.

Construct the quadrilateral A"B"C"D" that is T(A'B'C'D'), the T image of A'B'C'D'.