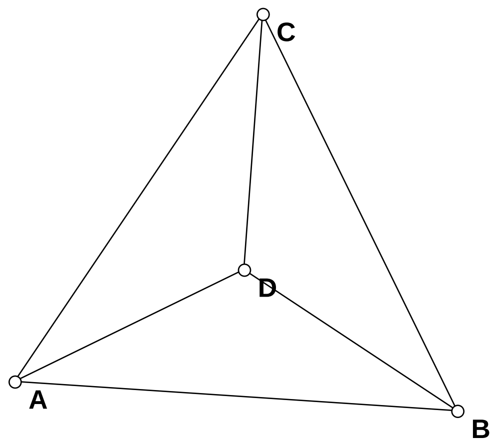
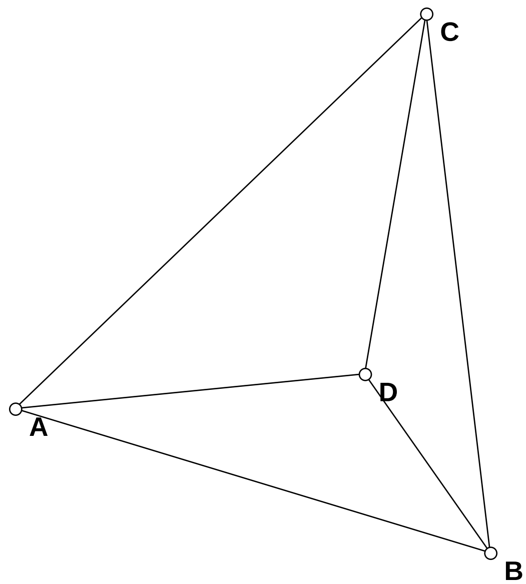
Tetrahedra and Parallel Projection - Friday, 10/12

Students should work in groups of 2 or 3. Bring the Polydrons from the office workroom, and also a supply or rubber bands.

Midpoints and Tetrahedra

1. Make a regular tetrahedron with Polydrons. Study it. Introduce terminology of vertices, edges and faces. How many of each? If some finish early, have them make an octahedron also.
2. Students should draw parallel projections of the "wireframe" shadow of the edges and vertices. Consider the image when the projection direction is vertical and the tetrahedron is resting on a desk. It looks regular like this the first figure. If the tetrahedron is not parallel to the desk, the projection looks something like the second figure.





3. Imagine cutting off a corner, by marking points A', B', C' which are 1/3 of the distance from D to each of the other vertices and then cutting on the plane of these 3 points. The edges of the cut form a regular triangle A'B'C' in space. You can model this with the rubber bands, drawing them tight.   Then draw the projection of A'B'C' in the figure above. It is possible to make the same kind of cut at the other vertices. What does the projection of the analogous triangle cut at corner A look like? Note the parallel line segments in space and in the projection.
4. Puzzle: Pose this problem to students and wait for them to chew on it for a while. Find a plane cut through the tetrahedron so that the cut is not a triangle but a square.
5. When most of them have the answer, hold the tetrahedron with one edge on top and parallel to the desk. Ask them to sketch the parallel projection by the vertical

direction. Answer: The shadow is a square with its diagonals (note this makes 6 segments).

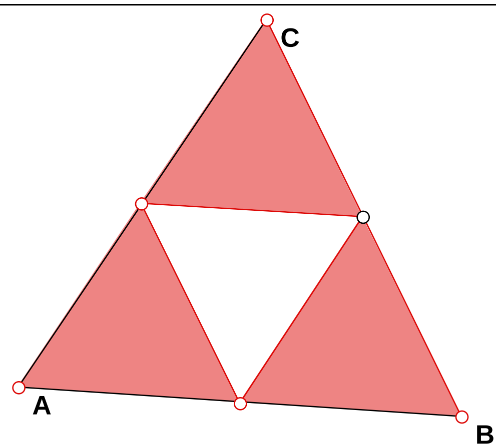
6. Have the students draw the projections of ALL the square projections in the figures above. How many square cross-sections are there? Note: This links directly to the first problem in Lab 2 (on the web).
7. Finally, hold the tetrahedron with one edge on top parallel to the desk again. Note the location of the 6 midpoints of edges. Have the students look at their projection figure again and visualize it a different way, as an equatorial belt of 4 points, with a north and a south pole, making 6 points. Now have students make octahedra if they have not already, and notice that the midpoints are the vertices of an octahedron.
8. Finally, have students draw a quadrilateral on paper or file folder and cut it out. Fold the quad along one diagonal, to make a space figure. Have students notice (1) the midpoint figure of the quad is still a parallelogram and (2) the folder space quad forms the vertices and two of the faces (and all but one of the edges) of an irregular tetrahedron.

Big tetrahedra, volume and holes

Students will need to work together on this, to have enough tetrahedra.

Place 3 tetrahedra on the desk so that the 3 bases form the corner midpoint triangles of an equilateral triangle like this figure. Then rest a fourth tetrahedron on the to form a tetrahedron double the size of the original ones.

Questions for the students: (1) What is the ratio of the volume of the doubled tetrahedron to the volume of one of the original tetrahedra? (2) What is the shape of the hole in the middle? Is it a tetrahedron or something else. What shape are the faces of the hole? Given them time to answer, but then use one of he already built octahedra to fill the hole. Finally, notice that the vertices of this octahedron are the same midpoints as observed before.



If there is extra time, you can make cubes and talk about projections. Find a hexagonal cut and a hexagonal projection. Also, by connecting 4 non-adjacent vertices of the cube, you get a regular tetrahedron.