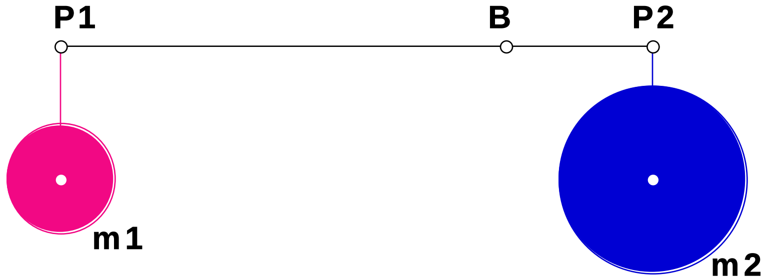
Center of Mass of Two Weights

Suspend a weight from each end of a beam. If the weight at endpoint P1 has mass m1 and the weight at endpoint P2 has mass m2, where is the balance point B?



According to Archimedes, if B is the balance point, the ratio of lengths is inversely proportional to the ratio of the masses. In other words, if P1B = L1 and BP2 = L2,

L1/L2 = m2/m1

The point B is called the **center of mass** of the system. Label the lengths: P1P2 = L; P1B = L1; BP2 = L2. By experiment and/or computation, fill in the empty cells in the table.

m1	m2	m1 + m2	L1	L2	L
2	1	3	12	24	36
1	1		(1/2)L	(1/2)L	L
3	3				L
1	3				10
3	1				24
2	3				L
2	5				L
113	26				L
7		15			L
.35		1			L
s		1			L
	t	1			L
u		u+v			L
m1	m2				L

Analysis of Your Method

Write down a step-by-step explanation of how you compute L1 and L2, given the two masses. Illustrate this method by applying it to the case m1 = 2, m2 = 5. L = L.

Your method should still work when the masses are expressed as letters and not as specific numbers. If m1 = s and m2 = t, use your method to produce a formula for L1 and L2 in terms of s and t. L = L.

Center of Mass on the Real Number Line

Consider the special case when the points P1 and P2 are on the real number line. For example, suppose that P1 = 33 and P2 = 173. If m1 = 2 and m2 = 5, find the center of mass B in this case. (B will be a real number.) When you have your answer, read the three student explanations following. Analyze these explanations. Do you agree that they are all correct? Does one of them match with your explanation, or is your method a fourth way.

Your Explanation:

Student E's explanation: In an earlier problem, we have see that L1 = (5/7)L and L2 = (2/7)L. In this case, L = 173-33 = 140, so L1 = 100 and L2 = 40. If we begin at point P1 = 33, we want to travel (5/7)140 = 140 in the direction of P2=173, so we add 33 + 100 to get B = 133.

Student F's explanation: From the table, L1 = (5/7)L and L2 = (2/7)L. Since L = 173-33 = 140, we can begin at 173 and move in the negative direction (to the left) by (2/7)140 = 40 units to reach 173 - 40 = 133. So B = 133.

Student G's explanation: I think of the center of mass B as a blend of P1 and P2. The total mass is 2+5 = 7, so the contribution from P1 is 2/7 of the total and the contribution of P2 is 5/7 of the total. Thus B is the blend (2/7)33 + (5/7)173 = (66 + 865)/7 = 931/7 = 133.

Your Response to the explanations of Students E, F, G

General Case of Two Masses on the Real Number Line

Suppose the two points are on the real number line. P1 is the number a1 and P2 is the number a2 and the masses are m1 and m2.

What are L, L1 and L2?

Follow in turn each of the methods of thinking of Students E, F and G to get 3 derivations of a formula for B in terms of a1, a2, m1, and m2. Each method should produce a formula that looks different at the outset. Are the 3 formulas really the same?

Computation by Student E method:
Computation by Student F method:
Computation by Student G method

One formula? Explain why the 3 formulas are really the same.

In the special case when m1 + m2 = 1, what do the formulas look like?