Quiz 5 ANSWERS:

Question 1

Let the path C consist of two line segments: the first segment from $(0,0,0)$ to $(1,2,-1)$ and the second segment from $(1,2,-1)$ to $(3,2,0)$.

Compute $\int_C xy^2dx + xdy + zdz$.

Answer

Let C_1 be the segment from $(0,0,0)$ to $(1,2,-1)$. It can be parametrized as $(1-t)(0,0,0)+t(1,2,-1) = (t, 2t, -t)$, for $0 \le t \le 1$. So the line integral becomes

$$\int_{C_1} xy^2dx + xdy + zdz = \int_0^1 4t^3\,dt + t\,d2t + (-t)d(-t) = \int_0^1 (4t^3+3t)dt = t^4 + \frac{3}{2}t^2|_0^1 = 5/2$$

Let C_2 be the segment from $(1,2,-1)$ to $(3,2,0)$. It can be parametrized as $(1-t)(1,2,-1)+t(3,2,0) = (1+2t, 2, t-1)$, for $0 \le t \le 1$. So the line integral becomes

$$\int_{C_2} xy^2dx + xdy + zdz = \int_0^1 4(1+2t)\,d(1+2t) + 0 + (t-1)d(t-1) = \int_0^1 (8+16t+t-1)dt$$

$$= 7t + \frac{17}{2}t^2|_0^1 = 31/2$$

Then the line integral over C is the sum $\frac{5+31}{2} = 18$.

Question 2

Let S be the upper half of the circle $x^2+y^2 = 25$ (i.e., the part of the circle with $y \ge 0$).

Compute $\int_S yds$.

Answer: Use polar coordinates. Then $y = 5\sin\theta$ and

$$(x,y) = (5\cos\theta, 5\sin\theta)$$

and

$$((\frac{dx}{d\theta})^2 + (\frac{dy}{d\theta})^2)^{1/2} = ((-5\sin\theta)^2 + (5\cos\theta)^2)^{1/2} = 5$$

so

$$\int_S yds = \int_0^\pi (5\sin\theta)\,5d\theta = 25\int_0^\pi \sin\theta\,d\theta = 50$$