

308F D

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I have tried to set this up so that the answers are available Tuesday when the quiz is no longer turned on. Please try it and give me feedback.

JK

Total points:
20/20

The following questions are true/false. On the midterm there may be true/false questions; but on the exam you will be asked for a reason that the statement is true or a counterexample, so it may be useful practice to think in these terms.

2/2 If W is a subspace of \mathbb{R}^n and \mathbf{x} and \mathbf{y} are vectors in \mathbb{R}^n such that $\mathbf{x} + \mathbf{y}$ is in W , then \mathbf{x} is in W and \mathbf{y} is in W .

- ☐ True
☒ False

Correct

Answer:
False

Feedback:
Writing vectors as rows instead of columns.

Counterexample: $\mathbf{x} = \mathbf{e}_1 = [1 \ 0]$ and $\mathbf{y} = \mathbf{e}_2 = [0 \ 1]$. W is the set of $[u_1 \ u_2]$ with $u_1 - u_2 = 0$. Then $\mathbf{x} + \mathbf{y}$ is in W but neither \mathbf{x} nor \mathbf{y} is in W .

2/2 If W is a subspace of \mathbb{R}^n and $a\mathbf{x}$ in W , where a is a nonzero scalar, then \mathbf{x} is in W .

- ☒ True
☐ False

Correct

Answer:

True

Feedback:

Writing vectors as rows instead of columns.

Reason: $\mathbf{x} = (1/a)a\mathbf{x}$ is in W .

2/2 If $S = \{\mathbf{x}_1, \dots, \mathbf{x}_k\}$ is a subset of \mathbb{R}^n and $k \leq n$, then S is a linearly independent set.

- ☐ True
☒ False

Correct

Answer:

False

Feedback:

Writing vectors as rows instead of columns.

Counterexample: $S = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$. The converse of this statement is true, but this statement is false.

2/2 If $S = \{\mathbf{x}_1, \dots, \mathbf{x}_k\}$ is a subset of \mathbb{R}^n and $k > n$, then S is a linearly dependent set.

- ☒ True
☐ False

Correct

Answer:

True

Feedback:

Writing vectors as rows instead of columns.

Counterexample: Writing a matrix A with these vectors as columns, the equation $Ax = 0$ has a non-zero solution, since the rank is less than n , which is the number of variables $= k$.

2/2 If $S = \{x_1, \dots, x_k\}$ is a subset of \mathbb{R}^n and $k < n$, then S is not a spanning set for \mathbb{R}^n .

☒ True☐ False**Correct****Answer:**

True

Feedback:

Writing vectors as rows instead of columns.

Counterexample: Writing a matrix A with these vectors as columns, the equation $Ax = y$ is inconsistent for some y , since the rank is less than or equal to k , so less than n .

2/2 If $S = \{x_1, \dots, x_k\}$ is a subset of \mathbb{R}^n and $k \geq n$, then S is a spanning set for \mathbb{R}^n .

☐ True☒ False**Correct****Answer:**

False

Feedback:

Writing vectors as rows instead of columns.

Counterexample: It is true that any spanning set has at least n elements, but not every set with a lot of elements is a spanning set. For example, all of the vectors could be multiples of a single vector.

2/2 Let A be an $(m \times r)$ matrix and B is an $(r \times n)$ matrix. Then the null space of B is contained in the null space of AB .

- ☒ True
☐ False

Correct

Answer:

True

Feedback:

Reason: If x is any vector in the null space of B , then $Bx = 0$. So also $(AB)x = A(Bx) = A0 = 0$. Thus any vector in the null space of B is also in the null space of AB .

2/2 Let A be an $(m \times r)$ matrix and B is an $(r \times n)$ matrix. Then the range of AB is contained in the range of B .

- ☐ True
☒ False

Correct

Answer:

False

Feedback:

Counterexample: The range of AB is in \mathbb{R}^m but the range of B is in \mathbb{R}^r , so these ranges are not even in the same space. (By the way, the range of A is also in \mathbb{R}^m and the range of A contains the range of AB , but that was not the question.)

2/2 Let A be an $(m \times n)$ and let B be the row-reduced echelon form of A . Then the

null space of A is the same set as the null space of B.

- ☒ True
☐ False

Correct

Answer:

True

Feedback:

Reason: The set of solutions of $Ax = 0$ is the same as the set of solutions of $Bx = 0$, a fact that we have used since week 1.

2/2 Let A be an $(m \times n)$ and let B be the row-reduced echelon form of A. Then the range of A is the same set as the range of B.

- ☐ True
☒ False

Correct

Answer:

False

Feedback:

This is false that the ranges are the same set. What is true that the ranges have the same dimension but the sets of vectors are different

Counterexample: Let $A =$ the column vector $[1 \ 1 \ 1]^T$; then $B = [1 \ 0 \ 0]^T$. So the range of A consists of the set of scalar multiples of the column vector $[1 \ 1 \ 1]^T$; but the range of B the set of scalar multiples of the column vector $[1 \ 0 \ 0]^T$.

Total points:
20/20

Questions or Comments?

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