Do all problems. Note that 1A and 1B are each worth 16 points. The remaining problems 2, 3, 4, 5 are each worth 17 points for a total of 100.

1A Spaces from a Matrix

	1	0	-1	2	1]
Let $A =$	0	0	2	0	6.
	2	0	0	4	8

Find a basis for each of the following spaces. For full credit BOX and label your answers clearly.

a. Range of A

b. Null Space of A

c. Range of A^T

d. Null Space of A^T

1B Matrix Relationships

For the matrix A in the previous problem, write down

a. Rank of A ______
b. Nullity of A ______

- c. Rank of A^T_____
- d. Nullity of A^T_____

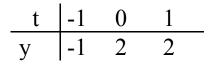
Write down 3 equations expressing relationships among the 4 quantities of rank and nullity of M and M^{T} that hold for ANY m x n matrix M.

i.	
ii.	
iii.	

Then verify that these relationships hold for your answers to a, b, c, d in this problem.

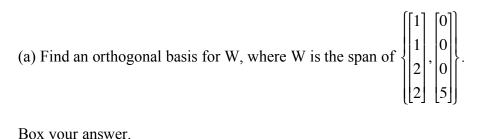
i	
ii. _.	
iii.	

Find the least squares linear fit to the given data.



Your answer should be the equation of a line. Box your answer.

3. Orthogonal Basis



Box your answer.

(b) Find an orthonormal basis for W.

Box your answer.

4. Linear Transformations

(a) For X in R³ define
$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = (x_1 - x_2 + 2x_3) \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Is T a linear transformation? Yes ____ No ____

If yes, what is the matrix of T?

(b) For X in R² define
$$S\left(\begin{bmatrix} x_1\\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1+2\\ 1+x_2 \end{bmatrix}$$
.

Is S a linear transformation? Yes ____ No ____

- If yes, what is the matrix of S?
- (c) Let P(X) be the rotation of a point X in R^2 counterclockwise around center of rotation 0 by an angle of 90 degrees. What is the matrix of P?

(d) U is a linear transformation from R^2 to R^2 so that

$$U\begin{pmatrix} \begin{bmatrix} 1\\1 \end{bmatrix} = \begin{bmatrix} 2\\0 \end{bmatrix}$$
 and $U\begin{pmatrix} \begin{bmatrix} 0\\1 \end{bmatrix} = \begin{bmatrix} 0\\1 \end{bmatrix}$.

What is the matrix of U?

5. Short Answer

 $X^{T}v_{1} = 2, X^{T}v_{2} = 2, X^{T}v_{3} = 2$

(a) Let A be a 3 x 15 matrix.

What are the possible values of the rank of A?

What are the possible values of the nullity of A?

(b) True or False. If B is a matrix with nullity = 0, the columns of B are linearly independent. True or False _____

Why?

(c]) Let
$$v_1 = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$$
. Suppose that X is a vector in R³ and that we know the

dot products of X with these vectors, namely:

$$X^T v_1 = 2, X^T v_2 = 2, X^T v_3 = 2.$$

What is the vector X? (This should be an answer X = some vector with only numbers in it, not letters. Hint: There may be some orthogonal vectors here somewhere.)