Math 308 Midterm 2
PRINT NAME _____

Do all problems. Note that 1A and 1B are each worth 16 points. The remaining problems 2, 3, 4, 5 are each worth 17 points for a total of 100.

1A Spaces from a Matrix

Let $A = \begin{bmatrix} 1 & 0 & -1 & 2 & 1 \\ 0 & 0 & 2 & 0 & 6 \\ 2 & 0 & 0 & 4 & 8 \end{bmatrix}$.

Find a basis for each of the following spaces. For full credit BOX and label your answers clearly.

a. Range of A

b. Null Space of A

c. Range of A^T

d. Null Space of A^T

1B Matrix Relationships

For the matrix A in the previous problem, write down

a. Rank of A _____

b. Nullity of A _____

c. Rank of A^T_____

d. Nullity of A^T_____

Write down 3 equations expressing relationships among the 4 quantities of rank and nullity of M and M^T that hold for ANY m x n matrix M.

i. _____

ii. _____

iii. _____

Then verify that these relationships hold for your answers to a, b, c, d in this problem.

i. _____

ii. _____

iii. _____

2. Linear Fit

Find the least squares linear fit to the given data.

t	-1	0	1
y	-1	2	2

Your answer should be the equation of a line. Box your answer.

3. Orthogonal Basis

(a) Find an orthogonal basis for W, where W is the span of $\left\{\begin{bmatrix}1\\1\\2\\2\end{bmatrix}, \begin{bmatrix}0\\0\\0\\5\end{bmatrix}\right\}$.

Box your answer.

(b) Find an orthonormal basis for W.

Box your answer.

4. Linear Transformations

(a) For X in R^3 define $T(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}) = (x_1 - x_2 + 2x_3)\begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

Is T a linear transformation? Yes ____ No ____

If yes, what is the matrix of T?

(b) For X in R^2 define $S\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2 \\ 1 + x_2 \end{bmatrix}$.

Is S a linear transformation? Yes ____ No ____

If yes, what is the matrix of S?

(c) Let P(X) be the rotation of a point X in R^2 counterclockwise around center of rotation 0 by an angle of 90 degrees. What is the matrix of P?

(d) U is a linear transformation from R^2 to R^2 so that

$U\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ and $U\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

What is the matrix of U?

5. Short Answer

$X^T v_1 = 2, X^T v_2 = 2, X^T v_3 = 2$

(a) Let A be a 3 x 15 matrix.

What are the possible values of the rank of A?

_____

What are the possible values of the nullity of A?

_____

(b) True or False. If B is a matrix with nullity = 0, the columns of B are linearly independent. True or False _____

Why?

(c]) Let $v_1 = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$. Suppose that X is a vector in R^3 and that we know the dot products of X with these vectors, namely:

$X^T v_1 = 2, X^T v_2 = 2, X^T v_3 = 2.$

What is the vector X? (This should be an answer X = some vector with only numbers in it, not letters. Hint: There may be some orthogonal vectors here somewhere.)