Section 3.8 #28 Answer (two answers actually)

We are given that $\{u_1, \dots, u_p\}$ is orthonormal. This means that $u_i^T u_j = 0$ for I not equal to j, but equals 1 if i = j. (this is the definition; check it out).

We are also given that $v = a_1 u_1 + \dots + a_p u_p$. We are asked to compute

$$\|v\|^2 = v^T v = \left(a_1 u_1 + \dots + a_p u_p\right)^T \left(a_1 u_1 + \dots + a_p u_p\right).$$

But this is just a matrix product. These products obey the distributive law (just like numbers) so if we expand the products out we get the sum of p^2 matrix products, each of which is of the form $a_i u_i^T a_j u_j = a_i^2$ if i= j, but = 0 otherwise. So the only nonzero terms in the product are these p terms.

$$\|v\|^2 = v^T v = a_1 u_1^T a_1 u_1 + \dots + a_p u_p^T a_p u_p = a_1^2 + \dots + a_p^2.$$

Note: This is just the Pythagorean theorem with p terms. If you are having trouble seeing all the terms, just try the case where p = 3. You can write each term when you have a specific number.

Alternate MATRIX solution:

If a is the vector in R^p with the a_i coordinates, and U is the matrix with the columns u_i, then v = Ua.

Then $\|v\|^2 = (Ua)^T Ua = a^T \left(U^T U\right) a = a^T I a = a^T a$

And $a^T a = a_1^2 + \dots + a_p^2$

The important point is the fact that the set being orthonormal is the same as saying $U^T U = I$.

Section 3.7 #18

This is projection onto the first coordinate axis.

Here is how to see this. First solve the two equations and get a basis of W is

$$\{e_1\} = \left\{\begin{bmatrix}1\\0\\0\end{bmatrix}\right\}$$

Then use the projection formula $T(v) = (v^T e_1)e_1 = ae_1 = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}$. Note. The denominator in the formula = 1 since the length of e_1 is 1.

This is the answer to the algebraic problem. The picture is a bit of work on a computer. The figure on page 129 is close. Each point is projected to the (x, y, 0) plane and then to the x-axis. If you ignore the point in the middle of the L-shaped paths and draw a segment directly from P(2,3,2) to the point on the x-axis, you have the picture.

Another way to think of this is an example like the one by a door in my house. There are a bunch of marks on the vertical jamb of the basement door, indicating how tall each of my kids was at various times. This was a projection from the top of the head of the kid to the vertical line of the jamb. So this was a projection using e_3 instead of e_1.