Problem 1: Solving a linear equation

Given matrix $A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ -3 & -7 & 0 & 0 \\ -1 & -4 & 0 & 10 \end{bmatrix}$ and vector $y = \begin{bmatrix} 5 \\ -8 \\ 9 \end{bmatrix}$.

(a) Solve Ax = y (if the equation is consistent) and write the general solution x in (vector) parametric form.

Write a basis for the null space of A. **Basis = ____________________**

(b) What is the dimension of the range of A? **Dimension = _____**

(c) Is y in the span of the row vectors of A? **Yes? ___ No? ___**

Problem 2: Conclusions from echelon form.

In each case, we start with a matrix A and vector and tell what one will get by reducing the augmented matrix of the system Ax = y to echelon form. Answer the questions in each case using this information. **It is possible some equations will have no solution**.

A	**y**	**Echelon form of augmented matrix of Ax = y.**
$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 3 & 2 \\ 4 & 5 & 4 & 4 \end{bmatrix}$	$y = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 1 & 0 & -5 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$

(a) Write the general solution for Ax = y in (vector) parametric form
Solution:

(b) What is the dimension of the null space of A? **Dimension = ____**

(c) Write down a basis for the null space of A. **Basis =**

(d) Is y in the range of A? **Yes? __ No? __**

(e) What is the dimension of the range of A? **Dimension = ______**

(f) Write down a basis of the range of A. **Basis =**

(g) Are the rows of A independent? **Yes? __ No? ___**

(h) What is the dimension of the row space of A? **Dimension = ____**

B	z	Echelon form of augmented matrix of Bx = z.
$B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \\ 3 & 5 \\ 4 & 8 \end{bmatrix}$	$z = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(a) Write the general solution for Bx = z in (vector) parametric form
Solution:

(b) What is the dimension of the null space of B? **Dimension = ______**

(c) Write down a basis for the null space of B. **Basis =**

(d) Is z in the range of B? **Yes? No?**

(e) What is the dimension of the range of B? **Dimension = ______**

(f) Write down a basis of the range of B. **Basis =**

(g) Are the columns of B independent? **Yes? No?**

C	Reduced row echelon form of C.
$C = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 4 \\ 1 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

(a) Is C invertible? **Yes? ___ No? ___**

(b) Are the columns of C independent? **Yes? ___ No? ___**

(c) Write down a basis for the null space of C.

Problem 3: Compute matrix products and inverses

Compute the stated matrix products (if defined) for these matrices.

$$A = \begin{bmatrix} 5 & 1 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 3 \end{bmatrix}, C = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 2 & 0 & -1 \end{bmatrix}$$

Compute each of the following matrix products or other matrices (if defined):

A^{-1}	C^{-1}
AB	BA
CD	BC
CD^T	C^TD

Problem 4: Find the eigenvalues and eigenvectors

Find the eigenvalues and eigenvectors of matrix $F = \begin{bmatrix} 0 & 5 \\ -5 & 10 \end{bmatrix}$.

If possible, diagonalize F, i.e., write F = UDV, where D is diagonal.

U =______________________ D = ________ V = _____________

Problem 5: Given the eigenvalues find the eigenvectors

Given that **1 and -3 are the eigenvalues** of the matrix $C = \begin{bmatrix} -3 & 8 & 2 \\ -1 & 9 & 2 \\ 4 & -8 & -1 \end{bmatrix}$, find the **eigenvectors** of this matrix. **Hint**: First find whether there is a third eigenvalue. If you compute the determinant of C, knowing 2 eigenvalues, you can find the third root of the characteristic polynomial without computing det(C - λI).

If possible, diagonalize C, i.e., write C = MDN, where D is diagonal. You **DO NOT** need to compute the inverse of any matrix in this problem. If a matrix is the inverse of a known matrix, just write it as the inverse.

M =_________ D = _____________________ N = ____________________

Problem 6: Compute orthogonal projections

Let $h = \begin{bmatrix} 6 \\ 12 \\ 3 \end{bmatrix}, \quad u = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad v = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$

(a) Compute m = **the projection of h on Span({u}).** (The formula should be computed numerically, but you need not simplify fractions, etc., in your answer.)

(b) Compute g = **the projection of h on Span({u, v}).** (The formula should be computed numerically, but you need not simplify fractions, etc., in your answer.)

Problem 7: Matrix of Linear Transformation

(a) If T is the linear transformation of R^2 that rotates the plane by an angle so that the point (1, 0) is rotated to (3/5, 4/5). What is the matrix A of this transformation?

(b) Is the matrix A an orthogonal matrix? **Yes? ___ No? ___**

(c) Is the matrix 2A an orthogonal matrix? **Yes? ___ No? ___**
Show why in both cases.

(d) Given the vector $u = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$, if x is a vector in R^3, then let T(x) = $(u \cdot x)u$. Is T a linear transformation? If so, what is its matrix (with respect to the standard basis).

Problem 8: Least squares solution

You have the following data for variables x, y, z

x	y	z
0	0	3
1	0	4
0	1	4
1	1	1

(a) Now suppose that you want to fit this data to a function $z = a + bx + cy$, where, a, b, c are some constants that you need to solve for. Write a system of equations for variables **a, b, c** that can be solved if there is an exact fit of the function to the data.

(b) Find the lease-squares "solution" a, b, c to this system and check the values of $z = a + bx + cy$ at the data points to see how close a fit it is.

Least squares solution: z = ___ + ___ x + ___ y