PROBLEM 2 OF SECTION 1.4

The flows at each of the four nodes give a system of four equations in (x_1, x_2, x_3, x_4); this is the augmented matrix of this system. This system is the answer to part (a) of the problem.

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1200 \\ 1 & 1 & 0 & 0 & 1000 \\ 0 & 1 & 1 & 0 & 800 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

The matrix reduces to this reduced row echelon matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1200 \\ 0 & 1 & 0 & -1 & -200 \\ 0 & 0 & 1 & 1 & 1000 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So here is the solution in vector form, with x_4 a free variable.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1200 - x_4 \\ -200 + x_4 \\ 1000 - x_4 \\ x_4 \end{bmatrix}$$

To answer part (b), we set $x_4 = 100$ and substitute to get this solution:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1100 \\ -100 \\ 900 \\ 100 \end{bmatrix}$$

To answer part (c), we set each $x_i \geq 0$ and to get four inequalities in x_4. all of which must be true for the four variables to be non-negative.

(1) $$1200 - x_4 \geq 0$$

(2) $$-200 + x_4 \geq 0$$

(3) $$1000 - x_4 \geq 0$$

(4) $$x_4 \geq 0$$

Now inequalities (2) and (4) imply both $200 \leq x_4$ and $0 \leq x_4$, so both are true if $200 \leq x_4$. But inequalities (1) and (3) imply that $x_4 \leq 1200$ and $x_4 \leq 1000$, both of which are true if $x_4 \leq 1000$. So all four variables are non-negative precisely when $200 \leq x_4 \leq 1000$. Thus the minimum value of x_4 for which all the variables are non-negative is $x_4 = 200$ and the maximum value is $x_4 = 1000$.