Math 308 Midterm 1
Problem 1

Range

(a) State an algebraic specification for a vector y to be in the range of A, where

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & 1 \\ 3 & 2 & 5 \\ 2 & 0 & 4 \end{bmatrix}$$

(b) Find a basis for the range of A.

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Problem 2

Inverse

a) Find a matrix C that is the inverse of matrix $B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 1 & 0 & 0 \end{bmatrix}$ (if the inverse exists).

b) Test that your matrix C is really the inverse of B by checking the definition.

c) Suppose M and P are invertible matrices and N is a matrix so that MNP = I.
Write the inverse N^{-1}, if it exists, as a product involving some or all of M, P, M^{-1}, and P^{-1} (but the product does not include N).

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Problem 3

Matrix M

Let $M = \begin{bmatrix} 0 & 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

a) Solve the equation Mx = 0 and put the solution in vector form.

b) Find a basis for N(M), the null space of M.

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Problem 4

Linearly Independent

a) Write a complete sentence defining “linearly independent.”

b) Determine whether or not the rows of this matrix are linearly independent. Give a brief reason or show work; just an unsupported “yes” or “no” is not sufficient.

$$G = \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 2 & 1 & 1 \\ 1 & 2 & 1 & 2 \end{bmatrix}$$

c) Determine whether or not the columns of the same matrix are linearly independent. Give a brief reason or show work; just an unsupported “yes” or “no” is not sufficient.

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Problem 5

True or false?

Give a brief reason as well as T or F.

a) If $v_1, v_2, \ldots, v_k$ are vectors in R^n and k < n, then the set of these vectors must be linearly dependent.

b) If $v_1, v_2, \ldots, v_k$ are vectors in R^n and one of the vectors is the zero vector, then the set of these vectors must be linearly dependent.

c) If A is an nxn matrix such that the range of A is R^n, then the set of columns of A must be linearly independent.