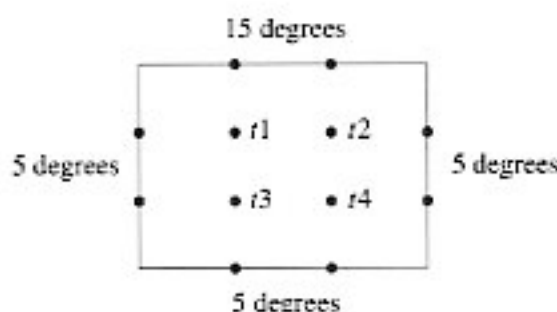


1.4.2 Estimating Inside Temperatures from Outside

Suppose we have a thin plate whose temperatures along its outside edges are known, and we want to know the temperatures at its interior points. (We will assume that the temperatures are in a *steady state*; that is, they are not changing with time.)

Example D The rectangular plate below has temperatures of 15 degrees along the top edge and 5 degrees along the other three edges. Find the temperatures at the four indicated interior points: t_1, t_2, t_3, t_4 .



Solution There are several ways to estimate the temperatures at the interior points, and we will pick the simplest:

At each interior grid point, assume that the temperature is the average of the temperatures at the four surrounding grid points.

For example, we assume the temperature t_1 is the average of the temperatures 5 (at left), 15 (above), t_2 (at right), and t_3 (below). This method leads to a system of four linear equations in the four unknown interior temperatures:

- ▶ Let *eqn1* be $t_1 = \frac{5 + 15 + t_2 + t_3}{4}$.
- ▶ Let *eqn2* be $t_2 = \frac{t_1 + 15 + 5 + t_4}{4}$.
- ▶ Let *eqn3* be $t_3 = \frac{5 + t_1 + t_4 + 5}{4}$.
- ▶ Let *eqn4* be $t_4 = \frac{t_3 + t_2 + 5 + 5}{4}$.
- ▶ Let M = augmented matrix for the linear system consisting of the equations *eqn1*, *eqn2*, *eqn3*, *eqn4* and whose unknowns are the variables t_1, t_2, t_3, t_4 :

$$M = \begin{bmatrix} 1 & -1/4 & -1/4 & 0 & 5 \\ -1/4 & 1 & 0 & -1/4 & 5 \\ -1/4 & 0 & 1 & -1/4 & 5/2 \\ 0 & -1/4 & -1/4 & 1 & 5/2 \end{bmatrix}$$

- ▶ Compute the solution of the linear system whose augmented matrix is M :

$$M = \begin{bmatrix} 35/4 \\ 35/4 \\ 25/4 \\ 25/4 \end{bmatrix}$$

Therefore, our estimates of the interior temperatures are $t_1 = 8.75$, $t_2 = 8.75$, $t_3 = 6.25$, $t_4 = 6.25$. ■

7. Temperature grid

Estimate the temperatures at the six interior points of the plate pictured below, using the method of Example D.

