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Math 308 A

Electrical Networks

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Abstract

An interesting application of linear algebra can be found in electrical engineering and specifically electrical networks. I have found that linear algebra is a useful tool in analyzing electric circuits in terms of organization and saving time. The aspect of linear algebra that is used is the matrix equation and the basic idea of reducing a matrix to reduced echelon form in order to solve for each variable. In this report, I have given background information on the topic of electrical networks and how it relates to linear algebra. I have also included an example of how to analyze a circuit using linear algebra.

Background

When dealing with electrical engineering and circuit analysis, matrices and linear algebra are used as a way of organizing and simplifying calculations. A simple electric circuit consists of voltage sources, wire with current running through it, and various types of resistors (including capacitors, inductors, etc) that cause a voltage drop in the circuit. In order to tell how much the voltage drops across a resistor, one must use Ohm's law. This law simply states that $V = IR$, where V is voltage across the resistor (volts), I is the current through the resistor (amperes), and R is the resistance of the resistor (ohms).

When analyzing an electric circuit, there are many techniques that may be used. One simple technique uses the idea of Kirchhoff's voltage law (KVL). Kirchhoff's voltage law states that "the algebraic sum of the voltages around any closed path in a circuit is identically zero for all time" (Dorf 69). Simply put, this means that around any path in a circuit, the sum of the voltage sources must equal the sum of the voltage drops. In order to use Kirchhoff's voltage law, the method of mesh current analysis must be used to examine the circuit. A mesh is a single loop that doesn't contain any other loops within it. Thus, the mesh current is the current that flows within each individual loop. The direction of current flow in each mesh is arbitrary, so it is easiest to choose the current flow in a clockwise direction. Analyzing the circuit now involves finding out the current flow in each mesh given the values of the voltage sources and resistances. In order to analyze the circuit, Kirchhoff's voltage law must be applied to each mesh. It is important to note that if a resistor is shared by two meshes, the voltage across this resistor is (current in mesh being analyzed – current in other mesh) x (resistance). For example, if we were analyzing mesh 1 below, the voltage across R_2 would be $(I_1 - I_2) \times R_2$. After

summing the voltages around a circuit with n meshes, there will be n equations with n unknowns. One way to solve this would be to solve each equation in terms of one variable and then substitute into the others. This method would work, but would be far too time consuming. This is where linear algebra comes in. Using Ohm's law, each equation can be written in the form $RI = V$ (similar to $Ax = b$). For example, the equations in the model below would turn out to be:

$$\text{Mesh 1: } (R_1 + R_2) I_1 - R_2 I_2 = V_1$$

$$\text{Mesh 2: } -R_2 I_1 + (R_2 + R_3 + R_4) I_2 - R_4 I_3 = 0$$

$$\text{Mesh 3: } -R_4 I_2 + (R_5 + R_4) I_3 = -V_2$$

Model Circuit

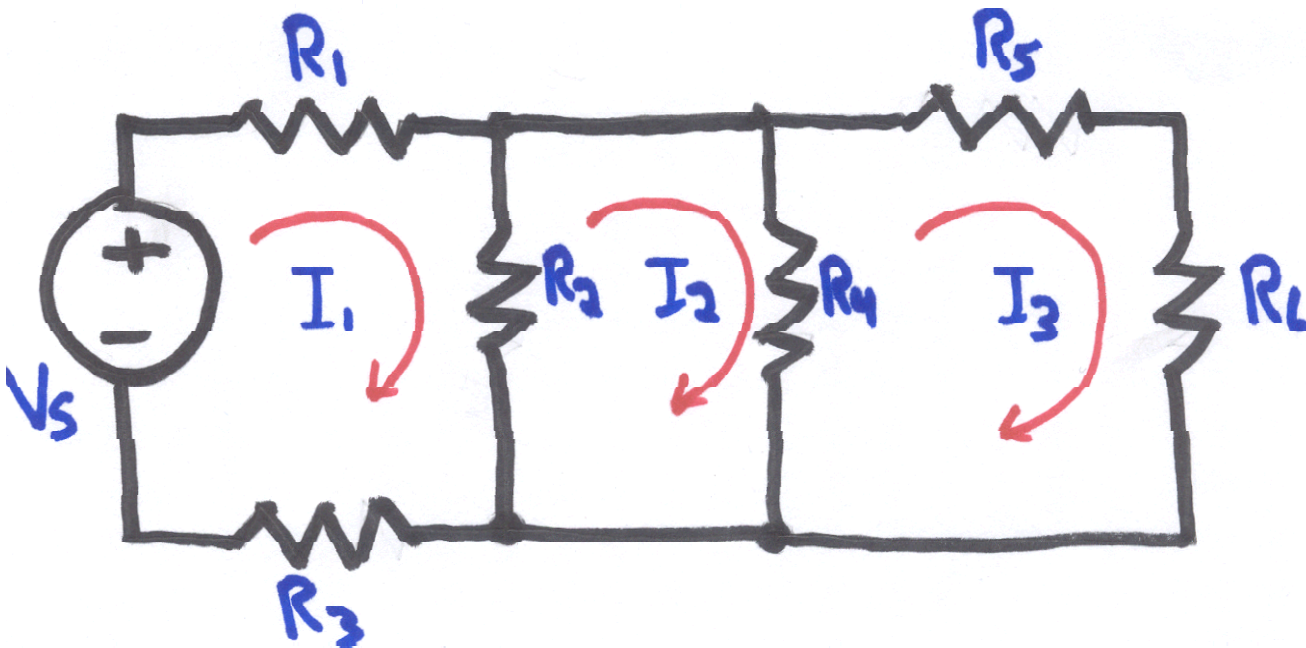


Putting these equations into matrices lets the engineer solve for each current by using the matrix equation $RI = V$. By solving the augmented matrix (reducing the matrix to reduced echelon form), values of I_1 , I_2 , and I_3 can be found. The matrix equation can be very useful when large circuits with many loops are too be analyzed. Also, because of Kirchhoff's voltage law, the matrix is consistent every time, so there will always be a solution. This is due to the fact that if there is any voltage drop in a mesh, there must be some sort of voltage source in the loop (in order to satisfy Kirchhoff's voltage law).

Solving a Problem

In order to analyze the simple circuit below (find the amount of current flowing in each loop), the technique of mesh analysis will be combined with the use of linear algebra.

Example Circuit



$$V_s = 120V, R_1 = 2, R_2 = 5, R_3 = 2, R_4 = 4, R_5 = 15, R_6 = 5$$

Step 1:

The first step in solving this problem is to write the sum of voltages around each loop. (Note that in this case, we arbitrarily choose current direction as clockwise). The voltage sums of each loop must equal zero, thus giving the following equations:

$$\text{Mesh 1: } V_s - I_1 R_1 - (I_1 - I_2) R_2 - I_1 R_1 = 0$$

$$\text{Mesh 2: } -(I_2 - I_3) R_4 - (I_2 - I_1) R_2 = 0$$

$$\text{Mesh 3: } -I_3 R_5 - I_3 R_6 - (I_3 - I_2) R_4 = 0$$

Step 2:

Substitute in all known values, put all unknowns (current values) on one side of the equation, and finally simplify. Doing so yields the following equations:

$$\text{Mesh 1: } 9 I_1 - 5 I_2 + 0 I_3 = 120$$

$$\text{Mesh 2: } -5 I_1 + 9 I_2 - 4 I_3 = 0$$

$$\text{Mesh 3: } 0 I_1 - 4 I_2 + 24 I_3 = 0$$

Step 3:

Put the three equations into matrix form $Ax = b$. In this case, values of resistance will make up the coefficient matrix, current will be the unknown x , and voltages will be the product of resistances and currents. The three equations from above will have the form $RI = V$. Once the mesh equations are put into matrices, they will look like:

$$\begin{array}{ccc|ccc} 9 & -5 & 0 & | I_1 | & | 120 | \\ -5 & 9 & -4 & | I_2 | & = | 0 | \\ 0 & -4 & 24 & | I_3 | & | 0 | \end{array}$$

Step 4:

The next step is to put the values of resistance and voltage into an augmented matrix. Using basic concepts of linear algebra (learned in chapter 1), this matrix is then reduced

to reduced echelon form. The augmented matrix and reduced echelon matrix are shown below:

$$\begin{array}{ccc|c} 9 & -5 & 0 & 120 \\ -5 & 9 & -4 & 0 \\ 0 & -4 & 24 & 0 \end{array} \sim \begin{array}{ccc|c} 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 12 \\ 0 & 0 & 1 & 2 \end{array}$$

Augmented Matrix

Reduced Echelon Matrix

(Note: Each step in the reduction of the matrix is not shown because the calculations involved many ugly fractions. Also, there is an assumed knowledge of basic matrix reduction that was learned in chapter 1 of the course).

Step 5:

The last step is to simply read off the values for each unknown. Since the voltages are linear combinations of the columns of the resistance (coefficient) matrix, the values of each current are found in the reduced echelon matrix. These values are shown below:

Solution: $I_1 = 20$ A, $I_2 = 12$ A, $I_3 = 2$ A

Bibliography

Dorf, Richard and James A. Svoboda. Introduction to Electric Circuits. New York: John Wiley & Sons, Inc, 2001.

Evaluation

I enjoyed working on this project, even though it took up a great deal of my time. I took an electrical engineering class last year (before I knew a thing about matrices) and solving each circuit took a long time. I used to get frustrated because the only time consuming part was solving a system of equations. Now that I know about matrices and have experience in linear algebra, I know that I can solve those problems easily. I think

that having a knowledge of matrices and linear algebra is extremely valuable in many fields of study. This project was a good chance to find out how useful linear algebra is in many different areas of math and science.

During my research, I also discovered that linear algebra is used extensively throughout electrical engineering and electrical networks. Some of these specific areas of application are: Finding the total response in a circuit, linear transformations used for differential operators in circuits, matrices used for impedance and admittance in networks, etc. The reason I choose only to discuss linear algebra in relation to Kirchhoff's voltage law and mesh current analysis is because I have no real knowledge of any other areas regarding electrical networks. Since I took an electrical engineering class last year, I became extremely familiar with mesh current analysis. I wanted to see how linear algebra could relate to and expand my knowledge of a subject I already knew about. I thought this would be more valuable to me, than to learn about a completely new topic that I knew nothing of, and then write a report on that.