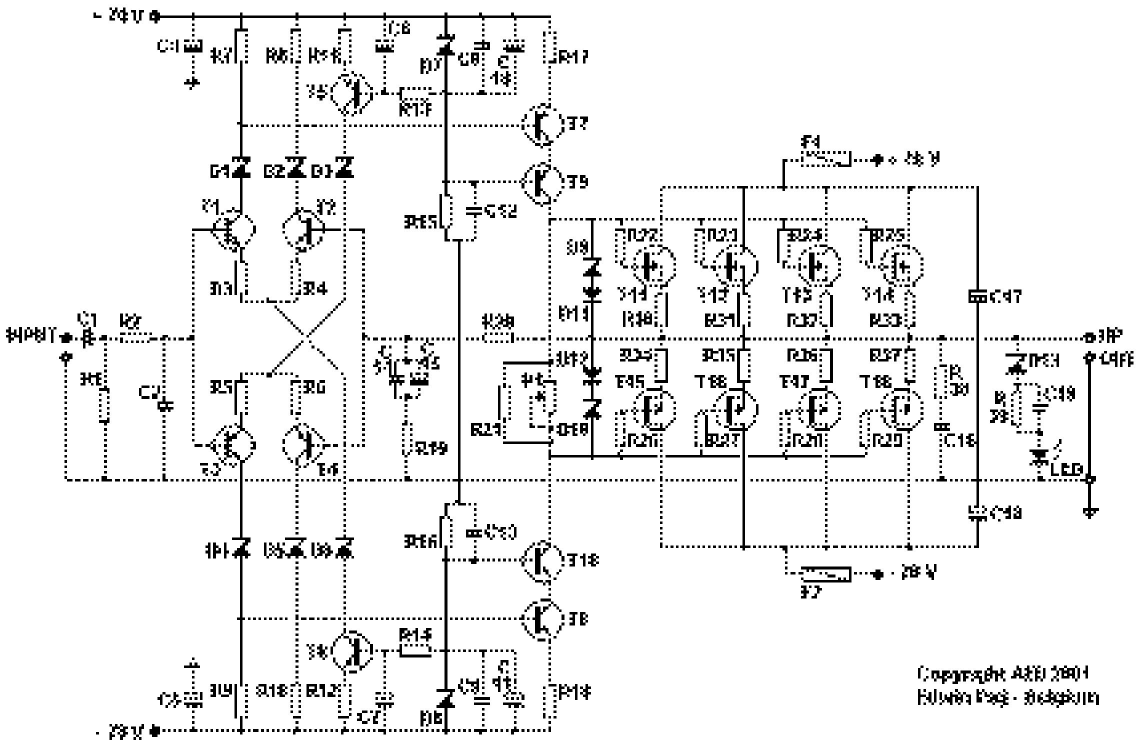
Application of
Linear Algebra
in



Electrical Circuits

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Linear Algebra in Electrical Circuits

Perhaps one of the most apparent uses of linear algebra is that which is used in Electrical Engineering. As most students of mathematics have encountered, when the subject of systems of equations is introduced, math class is temporarily converted into a crash course in electrical components. There, the resistor, voltage source and capacitor take the stage as well as their accompanying language consisting of Kirchoff and Ohm. With the basic concepts down, math class is resumed and students can look forward to playing with n number of equations with n number of unknowns. To solve for the currents and voltages, students can use simplification and substitutions, but with many equations, this task quickly becomes very time consuming and tedious.

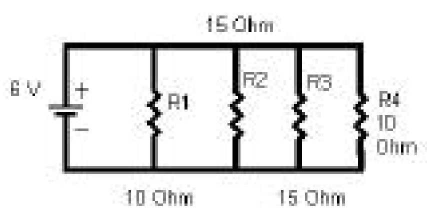
However, using Gaussian Elimination along with computers, engineers are able to efficiently calculate unknown values of extremely large and complex systems without performing hundreds of calculations and exhaustive bookkeeping of values.

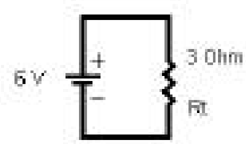
Electrical Circuits

Today more than ever, electronics are an integral part of our everyday lives. They contribute to every aspect of our way of life from lighting the space around our work environments, to exploring uncharted territories. But behind each and every electrical appliance or device, no matter what task it was designed for, lies a vast system of electrical components that must function as a whole. Each component (resistors, capacitors, inductors, etc.) has specifications of their own, as does the final product that they are a part of, so engineers must design their devices to meet not only their intended purpose, but so that the individual components are within their tolerances. Vital to this is the analysis of currents and voltages throughout the electrical circuit.

Simple Series or Parallel Circuits

For simple circuits, such as those used in math textbooks to introduce systems of equations, it is often sufficient to use series and parallel relationships to simplify circuits.





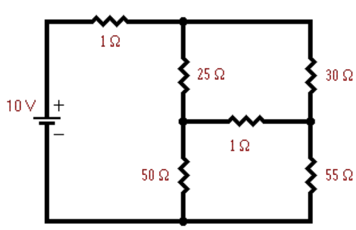
With this done, Ohm's Law (V=RI) can be used to find voltages or currents.

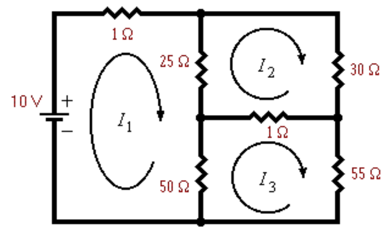
Vs=Vt	Vt=Rt*I	$I=I_1=I_2=I_3=I_4$	V=RI
6V=6V	6V=3Ohm*I	2A=2A=2A=2A=2A	V_1=20V
	I=2A		V_2=30V
			V_3=30V
			V_4=20V

Larger circuits though, are a problem, as this method is no longer efficient. It becomes far too time consuming to analyze and reduce circuits. Instead a new method of determining voltages and currents is used called Nodal Voltage Analysis and Loop Current Analysis.

Nodal Voltage Analysis and Loop Current Analysis

Using Nodal or Loop Analysis, we end up with systems of equations with unkown variables.





$$\begin{cases} 1i_1 + 25(i_1 - i_2) + 50(i_1 - i_3) = 10 \\ 25(i_2 - i_1) + 30i_2 + 1(i_2 - i_3) = 0 \\ 50(i_3 - i_1) + 1(i_3 - i_2) + 55i_3 = 0 \end{cases}$$

$$\begin{cases} 76i_1 - 25i_2 - 50i_3 = 10 \\ -25i_1 + 56i_2 - 1\,i_3 = 0 \\ -50i_1 - 1\,i_2 + 106i_3 = 0 \end{cases}$$

By simplifying and manipulating these equations, eventually all the unknowns will be solved assuming there were the same number of equations as there were unknowns.

i_1=(1/76)(25i_2+50i_3+10) -> -25((1/76)(25i_2+50i_3+10)) + 56 i_2 - i_3 = 0
(-625/76) i_2 – (1250/76) i_3 – (250/76) + 56i_2 - i_3 = 0
(3631/76) i_2 – (663/38) i_3 = (250/76) -> i_2 = (1326/3631) i_3 + (250/76)

-50(1/76)(25 [(1326/3631) i_3 + (250/76)] + 50i_3+10) - (1326/3631) i_3 + (250/76) + 106i_3 = 0

i_3 = 0.117 , i_2 = 0.111 , i_1 = 0.245

This method too, has its pitfalls, as circuits with many loops or nodes will require many substitutions, not to mention the large task of keeping track of all the variables.

Gaussian Elimination

To fix the problem of dealing with all the bookkeeping of variables, a simple change of notation is required. That is, to place the equations into a matrix form.

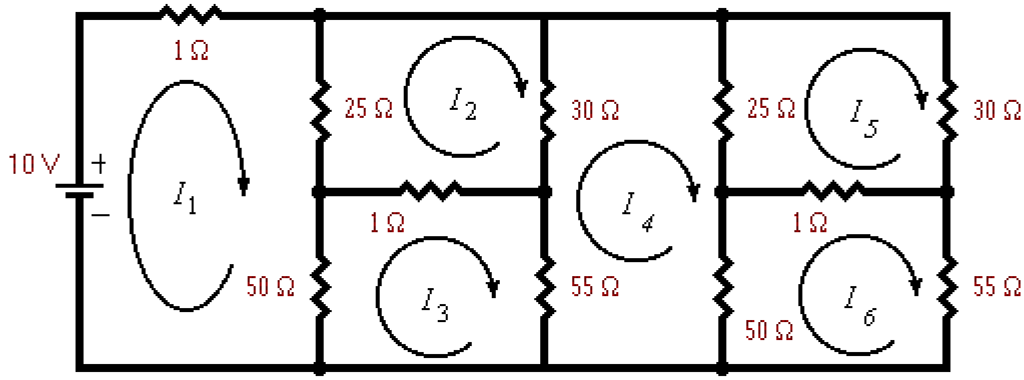
$$\begin{bmatrix} 76 & -25 & -50 & 10 \\ -25 & 56 & -1 & 0 \\ -50 & -1 & 106 & 0 \end{bmatrix}$$

Since the columns are of the same variable, it is easy to see that row operations can be done to solve for the unknowns.

$$\begin{bmatrix} 1 & 0 & 0 & 0.245 \\ 0 & 1 & 0 & 0.111 \\ 0 & 0 & 1 & 0.117 \end{bmatrix}$$

This method is known as Gaussian Elimination. Now, for large circuits, this will still be a long process to row reduce to echelon form, but it's a know fact that computers love matrices. With the help of a computer and the right software, ridiculously large circuits consisting of hundreds of thousands of components can be analyzed in a relatively short span of time. Today's computers can perform billions of operations a second, and with the developments in parallel processing, analyses of larger and larger electrical systems in a short time frame are very feasible.

Example 1- Loop Current Analysis Using Gaussian Elimination



Loop Equations:

$1i_1 + 25(i_1-i_2) + 50(i_1-i_3) = 10$

$25(i_2- i_1) + 30(i_2- i_4) + 1(i_2- i_3) = 0$

$50(i_3- i_1) + 1(i_3- i_2) + 55(i_3-i_4) = 0$

$55(i_4- i_3) + 30(i_4- i_2) + 25(i_4-i_5) + 50(i_4- i_6) = 0$

$25(i_5- i_4) + 30\ i_5+ 1(i_5-i6) = 0$

$50(i_6- i_4) + 1(i_6- i_5) + 55\ i_6 = 0$

Collect terms:

$76i_1 - 25i_2 – 50i_3 + 0\ i_4 + 0i_5 + 0\ i_6 = 10$

$-25i_1 + 56\ i_2 – 1\ i_3 – 30\ i_4 + 0i_5 + 0\ i_6 = 0$

$-50i_1 –1\ i_2 + 106\ i_3 – 55\ i_4 + 0\ i_5 + 0\ i_6 = 0$

$0i_1 –30\ i_2 – 55\ i_3 +160\ i_4 – 25i_5 – 50\ i_6 = 0$

$0i_1 + 0\ i_2 + 0\ i_3 –25\ i_4 + 56i_5 – 1i_6 = 0$

$0i_1 + 0i_2 + 0i_3 – 50\ i_4 – 1i_5 + 106\ i_6 = 0$

Write as Augmented Matrix:

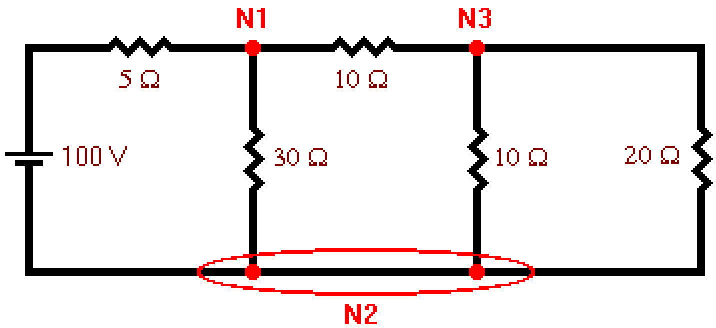
76	–25	–50	0	0	0	10
-25	56	–1	–30	0	0	0
-50	–1	106	–55	0	0	0
0	–30	–55	160	–25	–50	0
0	0	0	–25	56	–1	0
0	0	0	–50	–1	106	0

Row reduce using calculator or program:

1	0	0	0	0	0	.478
0	1	0	0	0	0	.348
0	0	1	0	0	0	.353
0	0	0	1	0	0	.239
0	0	0	0	1	0	.109
0	0	0	0	1	0	.114

$i_1 = .478$ A, $i_2 = .348$ A, $i_3 = .353$ A, $i_4 = .239$ A, $i_5 = .109$ A, $i_6 = .114$ A

Example 2- Nodal Voltage Analysis Using Gaussian Elimination



N2 is the reference node as so the voltage is 0V.

Node Equations:

$(V_1/30) + (V_1\text{-}100)/5 + (V_1\text{-}V_3)/10 = 0$

$(V_3 - V_1)/10 + V_3/10 + (V_3\text{-}100)/20 = 0$

Collect terms:

$[(1/30) + (1/5) + (1/10)]\ V_1 – (1/10)\ V_3 = 20$

$(-1/10)\ V_1 + [(1/10) + (1/10) + (1/20)]\ V_3 = 5$

$(1/3)\ V_1 – (1/10)\ V_3 = 20$

$(-1/10)\ V_1 + (1/4)\ V_3 = 5$

Write as Augmented Matrix:

1/3	-1/10	20
-1/10	1/4	5

Row Reduce to Echelon Form:

1	0	75
0	1	50

$V_1 = 75$ V, $V_3 = 50$ V

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