Milton Recolizado
St. ID: 0032477
Math 308 A Project
Fall 2001

Geology and Geography

Abstract

In the areas of geology and geography, linear algebra can be applied. Linear models are often used for modeling terrain, glacial cirques, soil pH, erosion surfaces, and for grain size analysis. Trend surface, a least-squares fit method, is used.

Explanation

Assume an experiment has two independent variables, *u* and *v*, and one dependent variable, *y*. Then an equation that predicts *y* from *u* and *v* has the form:

$$y = \beta_0 + \beta_1 u + \beta_2 v.$$

A more general prediction equation used for modeling the different geological and geographical quantities has the form:

$$y = \beta_0 + \beta_1 u + \beta_2 v + \beta_3 u^2 + \beta_4 uv + \beta_5 v^2.$$

These two equations lead to a linear model. Both are linear in the unknown parameters even though *u* and *v* are multiplied. A linear model occurs whenever *y* is predicted by an equation of the form:

$$y = \beta_0 f_0(u, v) + \beta_1 f_1(u, v) + \dots + \beta_k f_k(u, v)$$

with $f_{0,\dots,}\ f_k$ being any kind of known functions and $\beta_{0,\dots,}\beta_k$ being unknown weights.

Example

In one case, trend surface is used in the analysis of grain size in order to study sand dunes. Dunes are formed, depending on wind velocity and direction. The analysis of grain size data, as well as entrainment analysis, predicts a northerly decreasing critical shear velocity. The wind blows sand from the south over an increasing amount of land and a corresponding decreased wind velocity. Also, sand typically shows an increased thickness trending northward. To test this hypothesis, grain size data for the dunes could be examined.

Grain size is proportional to the shear velocity of the transporting medium. Modeling the paleowind strength via analysis of grain size variations over the length of dunes could be done. It is believed that the dunes will exhibit trends corresponding to a fining up to the north end of the dunes. The trend would be the result of the sand being blown northward from the south-southwest, over an increasing amount of land, rather than over the sea. The fining up of the grains would be the result of decreasing wind strength over the land.

Entrainment by Wind

The critical shear velocity to entrain a 1mm quartz grain by wind with no saltation in progress is 46 cm/s or 16.6 km/h. Variables that are used include critical shear velocity, grain diameter, gravitational acceleration, density of fluid, and density of particle. Using these variables, the critical wind speed required for entrainment of a single grain can be made.

Trend surface analysis

For application of trend surfaces to the grain size data, UTM coordinates are used as the x and y variables, while average grain size for the core (subtracting out soil profile data) is the z variable. The equations below are used to find interpolated z values, in the first, second, third, and fourth order trend surfaces.

$$Z = A + Bx + Cy$$

$$Z = A + Bx + Cy + Dx^2 + Ey^2 + Fxy$$

$$Z = A + Bx + Cy + Dx^2 + Ey^2 + Fxy + Gx^3 + Hy^3 + Ix^2y + Jxy^2$$

$$Z = A + Bx + Cy + Dx^2 + Ey^2 + Fxy + Gx^3 + Hy^3 + Ix^2y + Jxy^2 + Kx^4 + Ly^4 + Mx^3y + Nxy^3 + Ox^2y^2$$

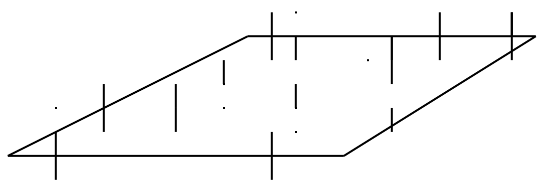
The coefficients, A-O, are solved using the least squares method. These coefficients attempt to minimize error.

The analysis of grain size data via trend surfacing and entrainment velocities provides a detailed description about grain size trends and their implications. The trends that resulted support the hypothesis that sand was blown in from the south, then accumulated in the north.

As with geology, linear models can also be used in geography.

Example

Local models of terrain are created from the data $(u_1, v_1, y_1), \ldots, (u_n, v_n, y_n)$, where u_j, v_j, and y_j are latitude, longitude, and altitude, respectively. Describe the linear model based on the given equation $y = \beta_0 + \beta_1 u + \beta_2 v$ that gives a least-squares fit to such data. The solution is called the least-squares plane.



Solution

The expected data has to satisfy the following equations:

$$y_1 = \beta_0 + \beta_1 u_1 + \beta_2 v_1 + \quad {}_1$$

$$y_2 = \beta_0 + \beta_1 u_2 + \beta_2 v_2 + \quad {}_2$$

$$\vdots$$

$$y_n = \beta_0 + \beta_1 u_n + \beta_2 v_n + \quad {}_n.$$

This system has the matrix form $\mathbf{y} = X\beta + \in$, where

Observation vector   Design matrix   Parameter vector   Residual vector

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ y_n \end{bmatrix}, \quad X = \begin{bmatrix} 1 & u_1 & v_1 \\ 1 & u_2 & v_2 \\ . & . & . \\ . & . & . \\ 1 & u_n & v_n \end{bmatrix} \quad \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}, \quad \in = \begin{bmatrix} {}_1 \\ {}_2 \\ . \\ . \\ {}_n \end{bmatrix}$$

This example shows the linear model for multiple regression. Linear algebra helps us recognize the general principle of all the linear models. No matter how many variables are involved, the normal equations for β have the same matrix form when X is properly defined. The least-squares β is given by $(X^T X)^{-1} X^T \mathbf{y}$ for any linear model where $X^T X$ is invertible.

Bibliography

Krumbein, W.C., and F.A. Graybill, *An Introduction to Statistical Models in Geology*. (New York: McGraw-Hill, 1965).

Lay, David C. *Linear Algebra and Its Applications*. New York: Addison-Wesley, 2000.

Unwin, David J., *An Introduction to Trend Surface Analysis*, Concepts and Techniques in Modern Geography, No. 5 (Norwich, England: Geo Books, 1975).