Quantitative Analysis of Water Distribution in China:

An Application of Linear Algebra

Math 308 A

Prof. James King

3 December 2001

In the field of resource management, linear algebra is highly applicable in analyzing acquired data. Currently in China, a major concern for the welfare of its people and its economy is the disparate distribution of water, with too much water present in the South and too little in the North. China’s water supply is separated into nine river systems, according to the Nanjing Institute of Hydrology and Water Resources, but one can simplify the situation into the North, South and the Chang-Jiang Basin regions. The Chang-Jiang Basin region is in its own category, even though it’s considered to be part of the South, because it is not only situated between the North and the South, but there are currently ideas to transfer water from this area to the North. One must note that the area occupied by the Chang-Jiang region is small compared to the entire Northern region. Data from 1993 shows that the total amount of water utilized for all of China is around 518 billion cubic meters. Also, this total is divided into different sectors of the economy, such as industry, urban water supply, and agriculture, in which agriculture is made up of irrigation, forestry, pastures, fishery, and rural water supply.

The issue can use linear by establishing a model, which relates total amount of water used for industry, urban water supply, and all of agriculture to the total amount of water used for each region, which are the North, the Chang-Jiang Basin region, and the South. This model shows that the data can be written into a system of equations, which allows for examination. This is done by the matrix equation A**x**=**b**. “A” represents standard matrix that is 3 by 3 with its columns representing each economic sector and each column’s entries representing fraction of that regions amount of utilized water to the total amount of utilized water in that economic sector. Also, “**x**” is a 3 by 1 vector in which x1 equals how much water was utilized for industry, x2 equals how much water was utilized for the urban water supply, and x3 equals how much water was utilized for all agriculture. Then, "**b**" is the 3 by 1 vector in which b1, b2, b3 equals how much water was utilized for the North, Chang-Jiang Basin, and the South region, respectively. Finally, this paper seeks to show how a possible water transfer would affect the total Southern water supply, if China's total amounts of utilized water were kept constant.

In 1993, a study was conducted on the division of water throughout regions in China. The following figure shows the tabulated results, in which the total amounts for the regions and the total amounts for the economic sectors is calculated with the data given by the Nanjing Institute of Hydrology and Water Resources.

Figure 1. Water consumed by economic sector in China, 1993 (billions of cubic meters)

Region	Industry	Urban Water Supply	All Agriculture	Total
North	27.66	10.64	149.88	188.18
Chang-Jiang Basin	40.92	7.23	116.06	164.16
South	20.27	6.23	139.73	166.22
Total	88.85	24.1	405.67	518.56

By taking each entry in the columns and dividing this value by the total amount for each economic sector, the new value for each in the column is equal to the fraction of the amount used per economic sector. This gives the following set of information, as shown in Figure 2.

Figure 2. Fraction of water consumed in each economic sector in China, 1993

Region	Industry	Urban Water Supply	All Agriculture	Total
North	0.31131	0.44149	0.36946	0.36289
Chang-Jiang Basin	0.46055	0.30000	0.28609	0.31657
South	0.22814	0.25851	0.34444	0.32054
Total	1.00000	1.00000	1.00000	1.00000

This new set of figures is understandable through linear algebra. The potential of linear algebra is to show information in a new way. As described above, the set of figures is described by A**x**=**b**. The entries in the columns of A correspond to the values in the columns of industry, urban water supply, and all of agriculture. For original situation given in Figure 1, the entries in **x** are equal to the total amount of water used in industry, urban water supply, and all of agriculture for that year. Then, "**b**" is the total amount of water utilized for each region. The next example more visibly shows the relationships between components of the matrix equation. Since A**x**=**b**, a linear combination of the columns of A with some **x** yields some **b** gives the following set of equations:

(1) $(0.31131)x1+(0.44149)x2+(0.36946)x3=b1$

(2) $(0.46055)x1+(0.30000)x2+(0.28609)x3=b2$

(3) $(0.22814)x1+(0.25851)x2+(0.34444)x3=b3$

These equations model the correlation between total quantity of water per economic sector (**x**) with the total quantity of water for a particular region (**b**). So, the values from Figure 1 show that the above set of equations correctly model the given situation:

$$\begin{bmatrix} .31131 & .44149 & .36946 \\ .46055 & .30000 & .28609 \\ .22814 & .25851 & .34444 \end{bmatrix} \begin{bmatrix} 88.85 \\ 24.1 \\ 405.67 \end{bmatrix} = \begin{bmatrix} 188.18 \\ 164.16 \\ 166.22 \end{bmatrix}.$$

This model is for a fixed fraction in the amount of water that is utilized each year, where the values for the total amount of water in each economic sector may vary from year to year and the final figures for the total amount of water in each region is dependent on these values.

In order to analyze the effects of a water transfer from the Chang-Jiang Basin to the North in the described linear system, let the total amount of water used per year by each economic sector be fixed. According to Prof. Steve Harrell on November 21, 2001, water transfer from the Chang-Jiang Basin may only yield a total increase of 6 percent in the North. Neglecting water loss from the system by evaporation and other reasons, one may let the total amount of water in each economic sector remain constant. Then, in order to calculate this 6 percent increase, the values for the North region are multiplied by 1.06 and the corresponding values, for each economic sector, in the Chang-Jiang Basin are calculated where the North's original value is subtracted from 1.06 times that original value. Then, this difference is subtracted from the corresponding value in the Chang-Jiang region to give the new value in the region's economic sector. The results are shown in Figure 3.

Figure 3. Potential water utilization by economic sector in China (billions of cubic meters)

Region	Industry	Urban Water Supply	All Agriculture	Total
North	29.32	11.28	158.87	199.47
Chang-Jiang Basin	39.26	6.59	107.7	152.87
South	20.27	6.23	139.73	166.22
Total	88.85	24.1	405.67	518.56

Then, the transferred 11.29 billion cubic meters of water is more beneficial to the North, even though it would account for only a 6 percent increase. This is shown by the fraction of this amount of transferred water to the total amount of water utilized in that region. Now, the total amount of water utilized in that region for the South is a combination of the Chang-Jiang Basin and the South. The original division between the two regions is now evident because the conservative system had to be organized in this manner, where the Chang-Jiang Basin's relationship with the North could be analyzed separately. The fraction for the North equals .06, while the fraction for the South equals to .035. The water transfer is much more advantageous for North, when considering the total area that would benefit from this process and this analysis shows that the Chinese government should further explore ways to physically transport this water to the North. Linear algebra shows that one can use acquired data and create a system of equations to effectively demonstrate the relationships between variables.

References

Prof. Steve Harrell. Class lecture on the Chinese Environment. November 21, 2001

Heilig, G.K. (1999): ChinaFood. "Can China Feed Itself?" IIASA, Laxenburg (CD-ROM Vers. 1.1). Nanjing Institute of Hydrology and Water Resources (1996). http://www.iiasa.ac.at/Research/LUC/ChinaFood/data/water/wat_7.htm

Lay, David C. *Linear Algebra and its Applications*. 2nd Edition. New York: Addison-Wesley, 2000.

Smil, Vaclav. *China's Environmental Crisis*. New York: M.E. Sharpe, Inc., 1993.