Math 300A Midterm – Winter 2011

PRINT NAME _____

Question 1 (20 points)

(a) Write a truth table for both (i) A ∧ ~B and for (ii) ~ (A ⇒ B). (You can make a single table that includes both, but include every column needed.)

(b) Tell from your table whether (i) and (ii) are equivalent or not.

Equivalent _____. Not equivalent _____

Question 2 (20 points)

Check a box to tell whether the statement is True or False or to give a short answer.

a) If A = {1, 2, a, b, c}, then 2 ⊂ A.

 True _____ False _____

b) What is the cardinal number of the set {1, {1, 2}, 1, {1, 2, 3, 4, 5}}?

 0____ | 1____ | 2____| 3____ | 4____ | 5____ | 6____ | 7____ | 8____ | Other ____

c) The logical statement ~(A∧(~B)) is equivalent to (~A)∧B.

 True _____ False _____

d) The set of all positive rational numbers is a countable set.

 True _____ False _____

e) The set of all subsets of the set of positive integers is a countable set.

 True _____ False _____

Question 3 (12 points)

Write the contrapositive of each statement below.

a) If a number is divisible by 6, then it is divisible by 3.

b) If a triangle is a right triangle, then two of the angles of the triangle are complementary.

c) If you are taking this exam, then you are enrolled in Math 300.

Question 4 (12 points)

Give examples as asked in the questions. No proofs or explanation needed, just write an example for each – and more than one correct answer possible.

a) Give an example of an infinite set that is countable.

b) Give an example of an infinite set that is uncountable.

c) Give a specific example of two sets A and B so that

$$A \cup B = \{1, 2, 3, \{1\}, a, b\} \text{ and } A \cap B = \{\{1\}, 2, a\}$$
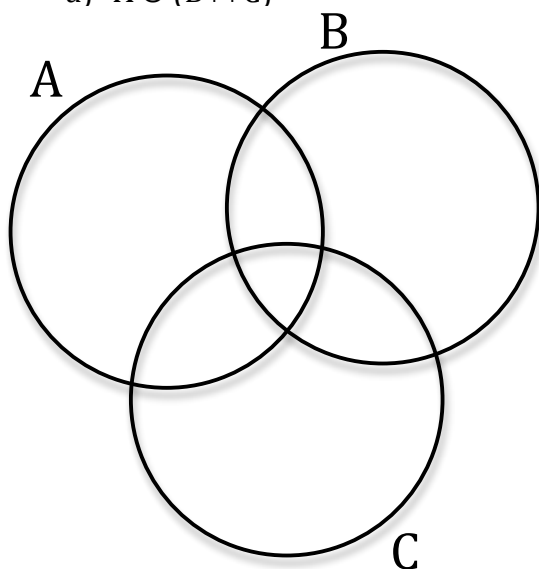
A = _____

B = _____

Question 5 (20 points)

Prove that the sum of any 3 consecutive odd integers is divisible by 3.
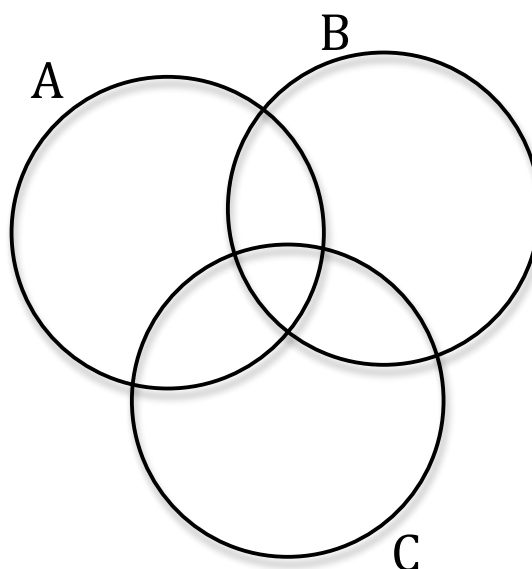
Question 6 (16 points)

Below is a collection of diagrams, each with a set operation above it. The circles in the diagrams below represent the sets A, B, C. For each diagram, shade in the region that represents the set defined by the expression, (Make the shading very clear. You may want to do some scratch work elsewhere before committing to a final answer in a diagram on this page.)
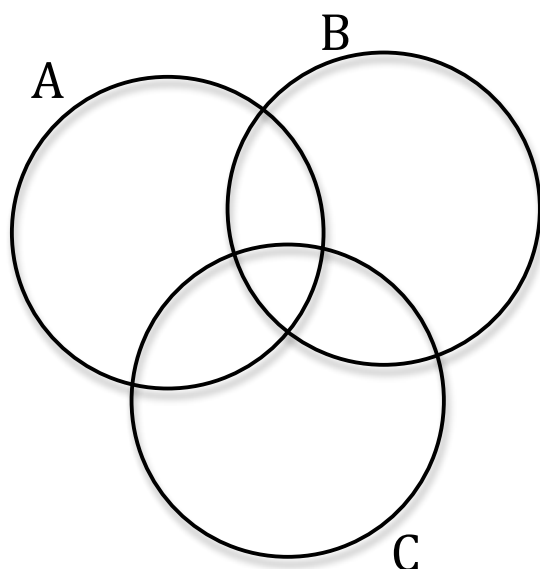
a) $A \cup (B \cap C)$

B
A

C

b) $A - (B \cup C)$

B
A

C

c) $(A \cup B) \cap C$

B
A

C

d) $(A - (A \cap B)) \cap (A - C)$

B
A

C